CLOSED-FORM RANGE-BASED POSTURE ESTIMATION BASED ON DECOUPLING TRANSLATION AND ORIENTATION

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ABSTRACT

For estimating the posture, i.e., position and orientation, of an *extended target* based on range measurements, a new closed-form solution is proposed, which is based on decoupling position and orientation. For decoupling, any procedure for range-based localization of *point targets*, i.e., for mere position estimation, can be used. The new solution is suboptimal, but nevertheless provides good accuracy and is very practical from an application point of view.

1. INTRODUCTION

This paper is concerned with estimating the posture, i.e., position and orientation, of a target object with respect to an external coordinate system based on measured ranges between reference points in the external world and reference points attached to the target. This kind of problem typically arises in acoustic localization systems, where for example loudspeakers placed in the environment emit signals picked up by microphones attached to the target. When the audio signals emitted by the loudspeakers are additionally transmitted through a different medium, e.g. wireless radio, to the microphones, the acoustic propagation delays can be converted to ranges between loudspeakers and microphones. These propagation delays are usually determined by means of correlation techniques [1].

Several solutions for range-based localization have been reported in [2–4]. These methods, however, only estimate the position and not the orientation. They use a geometry-based approach [2], closed-form solutions [3] or solutions of the nonlinear measurement equation by means of Taylor-series expansions [4] or gradient descent procedure.

This paper introduces a new closed-form solution for the range-based posture estimation problem based on decoupling of position and orientation. Decoupling is achieved by expressing the target reference points with respect to the external coordinate system and vice versa. For that purpose, any localization procedure for converting range measurements to positions of a point target can be employed. Here, a practical closed-form solution will be reviewed and used for that purpose.

The structure of the paper is as follows. A mathematical formulation of the range-based localization problem is given in Section 2. An appropriate localization procedure for point targets used for decoupling position and orientation is reviewed in Section 3. Section 4 is then concerned with the new closed-form posture estimation procedure. A solution sketch is given in Section 4.1. The corresponding derivations are performed in Section 4.2. The performance of the new closed-form solution compared to the optimal iterative solution is evaluated in Section 5. In Section 6, an acoustic tracking system is presented, which makes use of the proposed new algorithm. Conclusions and some details on future investigations are given in Section 7.

2. PROBLEM FORMULATION

We consider the problem of estimating the posture, i.e., position and orientation, of a target frame with respect to a world coordinate system. The relationship between a point ${}^{T}\underline{P}$ in the target coordinate system and its representation ${}^{W}\underline{P}$ in the world coordinate system is described by a nonlinear equation

$$ {}^{W}\underline{P} = {}^{W}\mathbf{T}_{T}\,{}^{T}\underline{P} + \underline{T} \quad , \tag{1} $$

where ${}^{W}\mathbf{T}_{T}$ is the rotation matrix, which comprises the orientations. $\underline{T}$ contains the translations, which correspond to the position of the target frame.

Estimation is based on ranges R_{ij} measured between N reference points ${}^{T}\underline{C}_i$, $i = 1, \ldots, N$, attached to the target and given with respect to the target frame and M reference points ${}^{W}\underline{P}_j$, $j = 1, \ldots, M$, given with respect to the world frame. The ranges are related to ${}^{W}\mathbf{T}_{T}$ and $\underline{T}$ according to

$$ R_{ij} = \left\| {}^{W}\underline{P}_j - \left({}^{W}\mathbf{T}_{T}\,{}^{T}\underline{C}_i + \underline{T} \right) \right\|_2 \quad . $$

A maximum of $N \cdot M$ ranges R_{ij} for $i = 1, \ldots, N$, $j = 1, \ldots, M$, is available.

3. CLOSED-FORM SOLUTION FOR POSITION

In this section, we consider estimating the position of a single point target at position $\underline{x}$. For that purpose, a set of ranges

$$ R_j = \left\| \underline{x} - {}^{W}\underline{P}_j \right\|_2 \tag{2} $$

measured between the target and the reference points ${}^{W}\underline{P}_j$, $j = 1, \ldots, M$, given with respect to the world coordinate system are available.

Several options for estimating $\underline{x}$ including iterative nonlinear optimization procedures have been proposed. Here, we will review a closed-form solution given in [3].

The first step is to square the nonlinear measurement equation according to

$$ R_j^2 = (\underline{x} - {}^{W}\underline{P}_j)^{\mathrm{T}}(\underline{x} - {}^{W}\underline{P}_j) \quad , $$

which gives

$$R_j^2 = \|\underline{x}\|_2^2 - 2\,{}^W\underline{P}_j^{\mathrm{T}}\underline{x} + \left\| {}^W\underline{P}_j \right\|_2^2 \ .$$

This equation can be rewritten in vector-matrix notation as

$$\underline{\delta} + \|\underline{x}\|_2^2\,\underline{1}_M = \mathbf{H}\underline{x} \tag{3}$$

with

$$\underline{\delta} = \begin{bmatrix} \| {}^W\underline{P}_1 \|_2^2 - R_1^2 \\ \vdots \\ \| {}^W\underline{P}_M \|_2^2 - R_M^2 \end{bmatrix} \ , \ \mathbf{H} = 2 \begin{bmatrix} {}^W\underline{P}_1^{\mathrm{T}} \\ \vdots \\ {}^W\underline{P}_M^{\mathrm{T}} \end{bmatrix} \ , \ \underline{1}_M = \underbrace{\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}}_{\text{(length } M)} \ ,$$

which is linear once the distance $\|\underline{x}\|_2$ is known. Before we obtain a position estimate from (3), this distance has to be estimated. For that purpose, the formal least-squares solution of $\underline{x}$ in terms of $\|\underline{x}\|_2^2$ is written as

$$\hat{\underline{x}}_1 = \hat{\underline{\alpha}}_1 + \|\underline{x}\|_2^2\,\hat{\underline{\beta}}_1$$

with

$$\hat{\underline{\alpha}}_1 = \mathbf{G}_1\underline{\delta} \ , \ \hat{\underline{\beta}}_1 = \mathbf{G}_1\underline{1}_M \ , \ \mathbf{G}_1 = (\mathbf{H}^{\mathrm{T}}\mathbf{E}_1^{-1}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{E}_1^{-1} \ ,$$

where $\mathbf{E}_1$ is an appropriate weighting matrix. The relationship $\|\underline{x}\|_2^2 = \underline{x}^{\mathrm{T}}\underline{x}$ is then used to obtain a quadratic equation for $\|\underline{x}\|_2^2$ given by

$$\hat{\underline{\beta}}_1^{\mathrm{T}}\hat{\underline{\beta}}_1(\|\underline{x}\|_2^2)^2 + (2\hat{\underline{\alpha}}_1^{\mathrm{T}}\hat{\underline{\beta}}_1 - 1)\,\|\underline{x}\|_2^2 + \hat{\underline{\alpha}}_1^{\mathrm{T}}\hat{\underline{\alpha}}_1 = 0 \ . \tag{4}$$

Upon replacing $\|\underline{x}\|_2^2$ by the abbreviation r, a positive root[1] $\hat{r}$ of (4) can be used as an estimate for $\|\underline{x}\|_2^2$ in (3) and a weighted least-squares estimate $\hat{\underline{x}}_2$ for the target position $\underline{x}$ is obtained as

$$\hat{\underline{x}}_2 = \hat{\underline{\alpha}}_2 + \hat{r}\hat{\underline{\beta}}_2$$

with

$$\hat{\underline{\alpha}}_2 = \mathbf{G}_2\underline{\delta} \ , \ \hat{\underline{\beta}}_2 = \mathbf{G}_2\underline{1}_M \ , \ \mathbf{G}_2 = (\mathbf{H}^{\mathrm{T}}\mathbf{E}_2^{-1}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{E}_2^{-1} \ ,$$

where $\mathbf{E}_2$ is an appropriate weighting matrix.

4. CLOSED-FORM SOLUTION FOR POSTURE

Based on the closed-form solution for the position of *point targets* given in Section 3, known reference points given with respect to a certain coordinate system can be converted to reference points with respect to a different coordinate system. This feature is now exploited for deriving a new closed-form solution for calculating the desired posture of *extended targets* by decoupling position and orientation.

[1] In the case of two positive roots, the ambiguity must be resolved by incorporating additional information.

4.1. Sketch of Solution

The closed-form solution for the target posture is performed in two steps.

In the first step, the known reference points ${}^T\underline{C}_i$, $i = 1, \dots, N$, with respect to the target frame and the N measured ranges R_{ij} between these points and the known reference point ${}^W\underline{P}_j$ are used to estimate the unknown point ${}^T\underline{P}_j$ with respect to the target frame. This will be done for all unknown points ${}^T\underline{P}_j$, $j = 1, \dots, M$, by means of the solution given in Section 3. Similarly, the unknown points ${}^W\underline{C}_i$, $i = 1, \dots, N$, will be estimated by using the known reference points ${}^W\underline{P}_j$, $j = 1, \dots, M$, and the M measured ranges R_{ij} between these points and the point ${}^T\underline{C}_i$.

In the second step (Section 4.2), translation and rotation are decoupled based on the results of the first step. These results are then used to derive two overdetermined systems of *linear* equations for the estimation of the translation vector and the rotation matrix, which can easily be solved by means of standard techniques.

4.2. Decoupling of Translation and Rotation

The decoupling of translation and rotation is based on the relationship (1), which converts points given in the target frame to the world coordinate system. The relation between the known reference points ${}^T\underline{C}_i$ with respect to a target frame and the estimated points ${}^W\underline{C}_i$ with respect to a world coordinate system is given by

$$\underbrace{{}^W\underline{C}_i}_{\text{estimated}} = {}^W\mathbf{T}_T \underbrace{{}^T\underline{C}_i}_{\text{given}} + \underline{T} \ . \tag{5}$$

A similar transformation for the known reference points ${}^W\underline{P}_j$ is given by

$$\underbrace{{}^W\underline{P}_j}_{\text{given}} = {}^W\mathbf{T}_T \underbrace{{}^T\underline{P}_j}_{\text{estimated}} + \underline{T} \ . \tag{6}$$

4.2.1. Translation

Now a closed-form solution for the translation between the two coordinate systems will be presented. To separate the translation vector from the rotation matrix, we use (5) and obtain

$${}^W\underline{C}_i - \underline{T} = {}^W\mathbf{T}_T\,{}^T\underline{C}_i \ .$$

Squaring this equation gives

$$\left({}^W\underline{C}_i - \underline{T}\right)^{\mathrm{T}} \left({}^W\underline{C}_i - \underline{T}\right) = {}^T\underline{C}_i^{\mathrm{T}} \underbrace{{}^W\mathbf{T}_T^{\ \mathrm{T}}\,{}^W\mathbf{T}_T}_{\text{identity matrix}} {}^T\underline{C}_i \ ,$$

and hence

$${}^W\underline{C}_i^{\mathrm{T}}\,{}^W\underline{C}_i - 2\,{}^W\underline{C}_i^{\mathrm{T}}\underline{T} + \underline{T}^{\mathrm{T}}\underline{T} = {}^T\underline{C}_i^{\mathrm{T}}\,{}^T\underline{C}_i \ . \tag{7}$$

A similar result is obtained for (6)

$$\left({}^W\underline{P}_j - \underline{T}\right)^{\mathrm{T}} \left({}^W\underline{P}_j - \underline{T}\right) = {}^T\underline{P}_j^{\mathrm{T}} \underbrace{{}^W\mathbf{T}_T^{\ \mathrm{T}}\,{}^W\mathbf{T}_T}_{\text{identity matrix}} {}^T\underline{P}_j \ ,$$

which gives

$${}^W\underline{P}_j^{\mathrm{T}}\,{}^W\underline{P}_j - 2\,{}^W\underline{P}_j^{\mathrm{T}}\underline{T} + \underline{T}^{\mathrm{T}}\underline{T} = {}^T\underline{P}_j^{\mathrm{T}}\,{}^T\underline{P}_j \ . \tag{8}$$

Subtracting (8) from (7) gives a relationship between the known reference points, the estimated reference points, and the unknown translation vector

$$\begin{aligned} {}^{W}\underline{C}_i^{\mathrm{T}}\,{}^{W}\underline{C}_i - {}^{W}\underline{P}_j^{\mathrm{T}}\,{}^{W}\underline{P}_j - {}^{T}\underline{C}_i^{\mathrm{T}}\,{}^{T}\underline{C}_i + {}^{T}\underline{P}_j^{\mathrm{T}}\,{}^{T}\underline{P}_j = \\ = 2\left({}^{W}\underline{C}_i - {}^{W}\underline{P}_j\right)^{\mathrm{T}} \underline{T} \ . \end{aligned}$$

This can be written for $i = 1, \ldots, N$, and $j = 1, \ldots, M$, as a set of $N \cdot M$ linear equations

$$\underline{\delta}' = \mathbf{H}'\underline{T}$$

with

$$\underline{\delta}' = \begin{bmatrix} {}^{W}\underline{C}_1^{\mathrm{T}}\,{}^{W}\underline{C}_1 - {}^{W}\underline{P}_1^{\mathrm{T}}\,{}^{W}\underline{P}_1 - {}^{T}\underline{C}_1^{\mathrm{T}}\,{}^{T}\underline{C}_1 + {}^{T}\underline{P}_1^{\mathrm{T}}\,{}^{T}\underline{P}_1 \\ \vdots \\ {}^{W}\underline{C}_i^{\mathrm{T}}\,{}^{W}\underline{C}_i - {}^{W}\underline{P}_j^{\mathrm{T}}\,{}^{W}\underline{P}_j - {}^{T}\underline{C}_i^{\mathrm{T}}\,{}^{T}\underline{C}_i + {}^{T}\underline{P}_j^{\mathrm{T}}\,{}^{T}\underline{P}_j \\ \vdots \\ {}^{W}\underline{C}_N^{\mathrm{T}}\,{}^{W}\underline{C}_N - {}^{W}\underline{P}_M^{\mathrm{T}}\,{}^{W}\underline{P}_M - {}^{T}\underline{C}_N^{\mathrm{T}}\,{}^{T}\underline{C}_N + {}^{T}\underline{P}_M^{\mathrm{T}}\,{}^{T}\underline{P}_M \end{bmatrix}$$

and

$$\mathbf{H}' = \begin{bmatrix} 2\left({}^{W}\underline{C}_1 - {}^{W}\underline{P}_1\right)^{\mathrm{T}} \\ \vdots \\ 2\left({}^{W}\underline{C}_i - {}^{W}\underline{P}_j\right)^{\mathrm{T}} \\ \vdots \\ 2\left({}^{W}\underline{C}_N - {}^{W}\underline{P}_M\right)^{\mathrm{T}} \end{bmatrix} \ .$$

A weighted least-squares estimate for the translation vector is then obtained as

$$\hat{\underline{T}} = \left(\mathbf{H}'^{\mathrm{T}}\mathbf{W}^{-1}\mathbf{H}'\right)^{-1}\mathbf{H}'^{\mathrm{T}}\mathbf{W}^{-1}\underline{\delta}' \ .$$

4.2.2. Rotation

In order to separate the rotation matrix from the translation vector, we subtract (6) from (5) and obtain

$$\underbrace{{}^{W}\underline{C}_i - {}^{W}\underline{P}_j}_{{}^{W}\underline{t}_{ij}} = {}^{W}\mathbf{T}_T \underbrace{\left({}^{T}\underline{C}_i - {}^{T}\underline{P}_j\right)}_{{}^{T}\underline{t}_{ij}} \ , \tag{9}$$

or

$${}^{W}\underline{t}_{ij} = {}^{W}\mathbf{T}_T\,{}^{T}\underline{t}_{ij} \ . \tag{10}$$

This can be written as a set of $N \cdot M$ equations

$${}^{W}\mathbf{T} = {}^{W}\mathbf{T}_T\ {}^{T}\mathbf{T} \ , \tag{11}$$

with

$${}^{W}\mathbf{T} = \begin{bmatrix} \left({}^{W}\underline{C}_1 - {}^{W}\underline{P}_1\right)^{\mathrm{T}} \\ \vdots \\ \left({}^{W}\underline{C}_i - {}^{W}\underline{P}_j\right)^{\mathrm{T}} \\ \vdots \\ \left({}^{W}\underline{C}_N - {}^{W}\underline{P}_M\right)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} , {}^{T}\mathbf{T} = \begin{bmatrix} \left({}^{T}\underline{C}_1 - {}^{T}\underline{P}_1\right)^{\mathrm{T}} \\ \vdots \\ \left({}^{T}\underline{C}_i - {}^{T}\underline{P}_j\right)^{\mathrm{T}} \\ \vdots \\ \left({}^{T}\underline{C}_N - {}^{T}\underline{P}_M\right)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \ .$$

Rotation for a two-dimensional coordinate system

In the case of a two-dimensional coordinate system, the rotation matrix is given by

$${}^{W}\mathbf{T}_T = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \ . \tag{12}$$

In order to estimate the orientation ψ, (10) is multiplied by ${}^{T}\underline{t}_{ij}^{\mathrm{T}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, which results in

$${}^{T}\underline{t}_{ij}^{\mathrm{T}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} {}^{W}\underline{t}_{ij} = {}^{T}\underline{t}_{ij}^{\mathrm{T}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} {}^{W}\mathbf{T}_T\,{}^{T}\underline{t}_{ij} \ . \tag{13}$$

Using the rotation matrix, which is given by (12), (13) can be simplified and an expression for $\cos\psi$

$${}^{T}\underline{t}_{ij}^{\mathrm{T}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} {}^{W}\underline{t}_{ij} = \cos\psi\ {}^{T}\underline{t}_{ij}^{\mathrm{T}}\ {}^{T}\underline{t}_{ij} \tag{14}$$

is obtained. Similarly, an expression for $\sin\psi$ is obtained. For that purpose, (10) is multiplied by ${}^{T}\underline{t}_{ij}^{\mathrm{T}} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Analogously, it can be simplified by using the rotation matrix and an expression for $\sin\psi$

$${}^{T}\underline{t}_{ij}^{\mathrm{T}} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} {}^{W}\underline{t}_{ij} = \sin\psi\ {}^{T}\underline{t}_{ij}^{\mathrm{T}}\ {}^{T}\underline{t}_{ij} \tag{15}$$

is obtained. Now we combine (14) and (15)

$$\frac{\sin\psi}{\cos\psi} = \frac{{}^{T}\underline{t}_{ij}^{\mathrm{T}} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} {}^{W}\underline{t}_{ij}}{{}^{T}\underline{t}_{ij}^{\mathrm{T}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} {}^{W}\underline{t}_{ij}}$$

and an expression for the orientation ψ is given by

$$\psi = \mathrm{atan}\left(\frac{{}^{T}\underline{t}_{ij}^{\mathrm{T}} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} {}^{W}\underline{t}_{ij}}{{}^{T}\underline{t}_{ij}^{\mathrm{T}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} {}^{W}\underline{t}_{ij}}\right) \ . \tag{16}$$

For calculating a unique orientation, atan2 should be used instead of atan. In order to estimate the orientation $\hat{\psi}$, a weighted average of the orientations $\hat{\psi}_{ij}$ with $i = 1, \ldots, N$, and $j = 1, \ldots, M$, is calculated.

Rotation for a three-dimensional coordinate system

In the case of a three-dimensional coordinate system, the rotation matrix comprises the three rotation angles $\begin{bmatrix} \alpha & \beta & \gamma \end{bmatrix}^{\mathrm{T}}$. By using (11), a least-squares estimate for the rotation matrix is then obtained as

$${}^{W}\hat{\mathbf{T}}_T = \left(\left({}^{T}\mathbf{T}\ {}^{T}\mathbf{T}^{\mathrm{T}}\right)^{-1}\ {}^{T}\mathbf{T}\ {}^{W}\mathbf{T}^{\mathrm{T}}\right)^{\mathrm{T}} \ .$$

5. SIMULATION RESULTS FOR A TWO-DIMENSIONAL COORDINATE SYSTEM

The performance of the new approach is evaluated by simulations in a two-dimensional coordinate system. The reference point locations ${}^{W}\underline{P}_j, j = 1, \ldots, 4$, in the world coordinate system are selected as ${}^{W}\underline{P}_1 = [0\ 0]^{\mathrm{T}}$, ${}^{W}\underline{P}_2 = [10\ 0]^{\mathrm{T}}$, ${}^{W}\underline{P}_3 = [0\ 10]^{\mathrm{T}}$, and ${}^{W}\underline{P}_4 = [10\ 10]^{\mathrm{T}}$. The reference point locations ${}^{T}\underline{C}_i, i =$

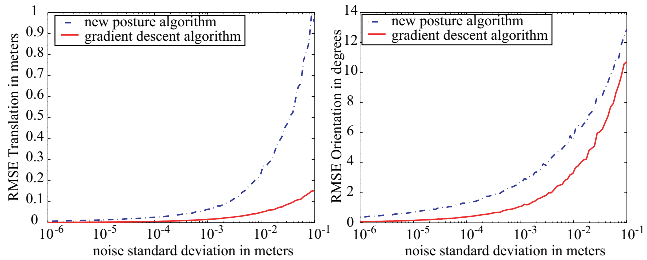

Fig. 1. RMSE versus noise standard deviation for translation and orientation ψ.

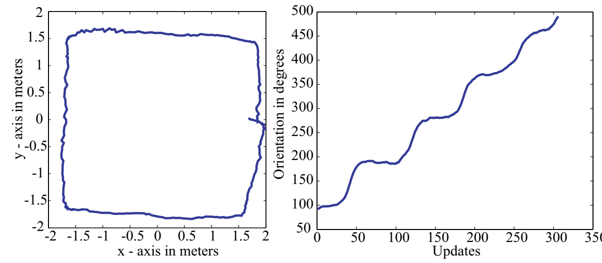

Fig. 2. The estimated translation and orientation sequences in a test run with a predefined motion trajectory.

$1, \ldots, 4$, in the target frame are selected as ${}^{T}\underline{C}_1 = [0\ 0]^{\mathrm{T}}$, ${}^{T}\underline{C}_2 = [1\ 0]^{\mathrm{T}}$, ${}^{T}\underline{C}_3 = [0\ 1]^{\mathrm{T}}$, and ${}^{T}\underline{C}_4 = [1\ 1]^{\mathrm{T}}$. Based on this arrangement, $N \cdot M = 16$ ranges are available. The reference solution uses a gradient descent procedure with initial values obtained from the proposed new posture estimation algorithm. The parameters of the gradient descent procedure are the step-size parameter given by 0.02 and the number of iterations given by 20. The simulated range estimates were generated by adding zero-mean white Gaussian noise of appropriate variance to the true range values.

The true translation is $\begin{bmatrix} x & y \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 4.5 & 6.2 \end{bmatrix}^{\mathrm{T}}$, and the true orientation is $\psi = 15°$. To compare the new algorithm with the gradient descent procedure, 1000 trials were performed at several noise levels for standard deviations ranging from 10^{-6} m to 10^{-1} m. The results are shown in Fig. 1. The Root-Mean-Square Error (RMSE) between the estimation and the true values of the posture is plotted as a function of the noise standard deviation.

For small measurement noise levels, the new posture estimation algorithm provides results very close to the reference solution. In an environment with a high noise level, the new posture estimation algorithm still provides satisfactory results. However, if the solution quality is not sufficient for the considered application, the new approach at least provides a good starting guess for the subsequent application of a gradient descent procedure.

6. EXPERIMENTAL SETUP FOR A THREE-DIMENSIONAL COORDINATE SYSTEM

The proposed new algorithm for estimating the posture of a target object is used for operator tracking in a telepresence scenario [5]. More specifically, the position and orientation of a head-mounted display is estimated, which is attached to the operator's head.

In the tracking system, several loudspeakers are placed on the ceiling around the user environment at fixed positions in the world coordinate system. The loudspeakers simultaneously emit wide band audio signals, which are generated by a Blackfin-DSP-System manufactured by Analog Devices. For the discrimination of the signals, each signal spectrum is spread with an orthogonal Gold Code of length 32. The signals are picked up by microphones attached to the head-mounted display at fixed positions with respect to the target frame. The Times of Arrival are estimated by means of cross-correlation and then converted to ranges between microphones and loudspeakers. The resulting ranges are used as input for the new algorithm in order to compute a starting guess for a gradient descent algorithm. This procedure provides 15 updates per seconds for both translation and orientation. For demonstrating the results of the procedure, a test run has been performed with a predefined motion trajectory, where the operator walks on a rectangular path. The corresponding estimated translation and orientation sequences are shown in Fig. 2. A comparison of these results with a few hand-measured reference postures revealed a sufficient accuracy of the tracking algorithm for both translation and orientation.

7. CONCLUSIONS AND FUTURE WORK

A new localization procedure has been introduced, which provides a closed-form conversion from measured ranges to the desired posture of a target. Since it is based on decoupling position and orientation, the proposed solution is suboptimal. However, it is much more practical than the usual numerical approaches based on iterative optimization, which require a good starting guess in order to ensure convergence to the optimal solution. Furthermore, the required number of calculations and, hence, the convergence time of iterative solutions depends upon the parameter values involved. This is not the case for the proposed procedure, which provides solutions after a fixed number of computations. In addition, the provided accuracy is sufficient for typical applications. In any case, the new solution approach provides a very good starting solution for iterative optimization.

Future work is concerned with an error analysis of the proposed localization procedure in order to calculate optimal weighting matrices in the required least-squares solutions.

8. REFERENCES

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