LINE PARAMETERS ESTIMATION BY ARRAY PROCESSING METHODS.

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Abstract

The high resolution methods of array processing lead to an improvement of the results obtained for source localization. By adopting specific conventions, it is possible to employ high resolution methods to characterize straight lines in an image. In this paper we propose an original method that leads to the estimation of the parameter "offset" of the straight lines. The proposed method is fast and effective compared an existing method. An extension to non rectilinear contours is developed.

Introduction

The array processing methods aim at characterizing sources. The so-called high resolution methods allowed to improve the spatial resolution for source localization [1]. By adopting some conventions it is possible to apply these methods to the characterization of straight lines in an image, by their parameters angle and offset. Some methods have already been proposed in [3]. Nevertheless none of these methods leads to an entire characterization of the straight lines by means of high resolution methods: either only the angles are estimated, or the Extension of the Hough Transform is employed for the estimation of the offsets. In this paper, a coherent set of high resolution methods is proposed. We will show that specific formalism and methods lead to the estimation of the offsets. We will emphasize on the advantages of our method respect to the Extension of the Hough Transform. In particular, this method will be applied to images containing a roughly aligned set of points and to real grey level images. An extended procedure is dedicated to the characterization of non rectilinear contours.

1. THE DATA MODEL

Let $I(x, y)$ represent image (Figure 1). We consider that $I(x, y)$ is compound of d straight lines and an additive uniformly distributed noise. Moreover, in this model we suppose that the numerical image $I(x, y)$ contains only binary pixels. The pixels $'1'$ form the straight lines, they are called "useful pixels", whereas the $'0'$ pixels are associated to the background. The image size is $N \times N$. Each straight line in an image is associated to an offset x_0 on the X axes and an angle θ, between this line and the line of equation $x = x_0$ (Figure 1). It is possible to generate some signals out of the image data: In order to establish the analogy between the localization of sources in array processing and recognition of lines in image processing, we consider the N lines of the image-matrix as the N outputs of a linear array compound of N equidistant sensors ranged along the image side. The signal received by each sensor can be considered as the result of the pixels of the corresponding line in the matrix. We can therefore define the signal received by the i^{th} sensor as the superposition of the useful pixels belonging to the corresponding line. So there are d non zero pixels on the i^{th} line of the image-matrix, localized on the columns $x_1, \cdots, x_d$ respectively; the signal received by the sensor in front of the i^{th} line, is [4]:

$$z_i = \sum_{k=1}^{k=d} e^{-j\mu x_k} \tag{1}$$

Where μ is a parameter that can be constant or variable -constant or variable parameter propagation scheme-. Figure 1 illustrates the case of one line with angle θ and offset x_0. In the presence of d different straight lines in the image and an additive noise, the signal received on the sensor i is:

$$z_i = \sum_{k=1}^{k=d} e^{j\mu i \tan(\theta_k)} e^{-j\mu x_{0k}} + n_i, \qquad i = 0, \cdots, N-1 \tag{2}$$

Starting from this signal, the ESPRIT [1] or Propagator [2] methods can be used to estimate the orientations $\{\theta_k\}$ of the straight lines [3]. We propose in the following a method for the estimation of the offsets.

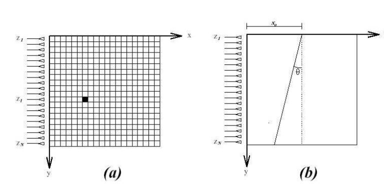


Fig. 1. (a) The image-matrix provided with the coordinate system and the rectilinear array of N equidistant sensors. (b) A straight line characterized by its angle θ and its offset x_0.

2. ESTIMATION OF THE OFFSETS

There exists a method for the estimation of the offsets which is the Extension of the Hough Transform [3]. Considering the polar parametrization of straight lines, the distances $\{\rho_k\}$ of the corresponding normals are estimated by projecting the image along the orientation θ_k and by retrieving, for $k = 1, \ldots, d$:

$$\rho_k = \underset{-\sqrt{2}N \leq \rho \leq \sqrt{2}N}{\operatorname{argmax}} \sum_{i=1}^{i=N_p} c\left(\rho - x_i \cos\theta_k - y_i \sin\theta_k\right) \tag{3}$$

where N_p is the number of useful pixels having components (x_i, y_i), contained in the image and c is a truncated cosine function. The complexity of the method, with a given step for the variation of ρ, is: $O(d*(10*N*N_p + 10*N)) \simeq O(d*10*N*N_p)$.
The numerical cost of the algorithm is rather elevated when the number of non zero valued pixels is elevated.

2.1. The variable speed propagation scheme

We propose another method that behaves better in terms of complexity with complex images.

Signal generation The two main properties of the formalism used in the case of offset estimation are the following: the propagation speed is variable in function of the line, and a high resolution method is applied several times -for each orientation value- in order to retrieve offset values. The signal received on sensor i is then, when the first orientation value is considered:

$$z_i = \sum_{k=1}^{k=d_1} e^{-j\tau x_{0k}} e^{j\tau i tan(\theta_1)} + n_i, \qquad i = 0, \cdots, N-1 \tag{4}$$

d_1 is the number of straight lines with angle θ_1. When τ varies linearly in function of the line index the measure vector $\mathbf{z}$ contains a modulated frequency term. Indeed we set $\tau = \alpha i$.

$$z_i = \sum_{k=1}^{k=d_1} e^{-j\alpha i x_{0k}} e^{j\alpha i^2 tan(\theta_1)} + n_i \tag{5}$$

This is a sum of d_1 signals that have a common quadratic phase term but different linear terms. The first treatment consists in obtaining an expression containing only linear terms. This goal is reached by dividing z_i by the non zero term $a_i(\theta_1) = e^{j\alpha i^2 tan(\theta_1)}$. We obtain then:

$$w_i = \sum_{k=1}^{k=d_1} e^{-j\alpha i x_{0k}} + n_i^{'}, \qquad i = 0, \cdots, N-1. \tag{6}$$

The resulting signal appears as a combination of d_1 sinusoids or frequencies : $f_k = \alpha x_{0k}/2\pi, \quad k = 1, \cdots, d_1$. Consequently, the estimation of the offsets can be considered as the frequencies estimation problem [5]. In the following a high resolution algorithm, initially introduced in spectral analysis [5], for estimating the offsets is proposed.

2.2. The Modified Forward Backward Linear Prediction method:

By adopting our signal model we adapt the spectral analysis method called modified forward backward linear prediction method (MFBLP) [5] for estimating the offsets: We consider d_k straight lines with given angle θ_k, and apply the MFBLP method.
1) For a data vector $\mathbf{w}$ we form the matrix $\mathbf{Q}$ of size $2 \times (N-L) \times L$:

$$\mathbf{Q} = \begin{bmatrix} w_{L-1} & w_{L-2} & \cdots & w_0 \\ w_L & w_{L-1} & \cdots & w_1 \\ w_{L+1} & w_L & \cdots & w_2 \\ . & . & . & . \\ . & . & . & . \\ w_{N-2} & w_{N-3} & \cdots & w_{N-L-1} \\ w_1^* & w_2^* & \cdots & w_L^* \\ w_2^* & w_3^* & \cdots & w_{L+1}^* \\ w_3^* & w_4^* & \cdots & w_{L+2}^* \\ . & . & . & . \\ . & . & . & . \\ w_{N-L}^* & w_{N-L+1}^* & \cdots & w_{N-1}^* \end{bmatrix}$$

We build the size $2 \times (N-L) \times 1$ vector:

$$\mathbf{h} = \left[w_L, w_{L+1}, ..., w_{N-1}, w_0^*, w_1^*, ..., w_{N-L-1}^*\right]^T$$

L is such that:

$$d_k \leq L \leq N - d_k/2$$

2) Calculate the singular value decomposition of $\mathbf{Q}$:

$$\mathbf{Q} = \mathbf{U}\Lambda\mathbf{V}^H$$

3) Form the matrix Σ setting to 0 the $L - d_k$ smallest singular values contained in Λ.

$$\Sigma = diag\left\{\lambda_1, \lambda_2, ..., \lambda_{d_k}, 0, ..., 0, 0, 0\right\}$$

4) Form the vector $\mathbf{g}$ from the following matrix computation:

$$\mathbf{g} = [g_1, g_2, ..., g_L]^T = -\mathbf{V} * \Sigma^{'} * \mathbf{U}^H \mathbf{h}$$

The pseudo-inverse of Σ, written $\Sigma^{'}$, is obtained by inverting its non zero elements and transposing it.
5) Determine the roots of the polynomial function H, where

$$H(z) = 1 + g_1 z^{-1} + g_2 z^{-2} + ... + g_L z^{-L}$$

6) The d_k zeros of H that are located on the unit circle have as arguments the frequency values; these frequency values are proportional to the offsets, the proportionality coefficient being $-\alpha$.

2.3. Numerical complexity of the proposed algorithm:

we remind that N is the size of one side of the image, L is a parameter chosen close to N.

The successive operations and their respective complexity are the following:

- Signal generation: $O(N_p)$.

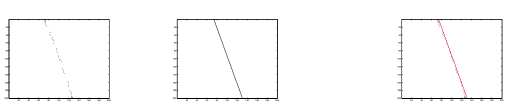
For each of the d directions found by the propagation scheme:

- Creation of the matrix $\mathbf{Q}$: $O(2*(N-L)*L) = O(N)$ since $N \approx L$,
- Singular value decomposition of $\mathbf{Q}$: $O((N-L)^3 + N^3) = O(N^3)$,
- Creation of the matrix Σ: $O(L) = O(N)$
- Creation of the vector $\mathbf{g}$: two matrix products with complexity $O(L)$ and a shift of the values of $\mathbf{g}$: $O(L)$, give a total complexity $O(L+L+L) = O(N)$
- Creation of the polynomial function H: $O(L) = O(N)$.
- Research of the zeros: the procedure "roots" is based on an eigen-decomposition of an $L \times L$ matrix. Thus the complexity is $O(N^3)$.

The total complexity of the method is finally $O(N_p + d*N^3) \simeq O(d*N^3)$. We note that the complexity of the method "variable speed propagation scheme" does not depend on the number of "useful" pixels.

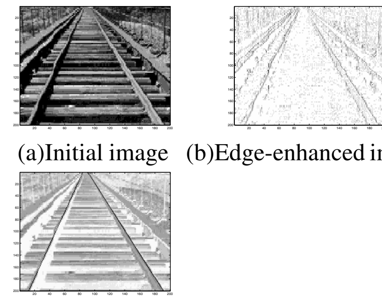
2.4. Simulation results

The images employed have size 200×200 pixels. The propagation parameters values are $\mu = 1$ and $\alpha = 2.5 * 10^{-3}$. L is equal to its maximum authorized value (close to 200). Some results concerning binary images have already been proposed [1]. Figure 2 presents the result obtained on an image containing a set of roughly aligned points. The overall orientation of these points is efficiently retrieved by the proposed method. The conventions adopted in the constant speed propagation scheme can be generalized to the problem of grey level images. The modification respect to the initial formalism is the following: the pixel values belong to the discrete interval $[0; 255]$. The propagating signal coming from one pixel is associated with an amplitude that is proportional to the value of the gradient on the pixel. The aim in the example given in Figure 3 is to find the two rails of a real grey level image. The Figure 3(c) shows that the two rails are efficiently retrieved. For the estimation of the offsets, knowing the two orientation values, the Extension of the Hough Transform lasts three minutes, the variable speed propagation scheme associated to the method MFBLP lasts five seconds.



(a)initial image (b)estimation (c)initial image, estimation

Fig. 2. The main direction of a set of points



(a)Initial image (b)Edge-enhanced image

(c)Estimation (black lines), initial image

Fig. 3. Railway

3. ESTIMATION OF NON RECTILINEAR CONTOURS IN AN IMAGE AS AN INVERSE PROBLEM

In this section, a method is proposed for the estimation of continuous non rectilinear contours. It relies on the formulation of an inverse problem over the generated signals and the determination of a phase model.

3.1. Temporal invariance of phase models in array and image processing

The determination of the phases by High Resolution methods relies on the a priori knowledge of a model. This is the case in particular for the wavefronts with incidence direction θ on the array. This phase does not depend on time: $\forall\, t, \theta(t)$ is a constant θ. The phases are then constant during the observation of the obtained signals.
The phase model introduced in the case of a straight line contained in an image is

$$\varphi_i = -\mu(i-1)\tan\theta$$

3.2. From a wavefront to a distorted wave: instability of the propagating medium

We assume here fictitious hypotheses modelling the temporal evolution of a propagating medium.

1. The $'0'$ level pixels of the image correspond to the propagating media.
2. At the instant ($t = 0$), an emitting source is localized in the propagation medium. One of the incident wavefronts on the array sensors is considered as a straight line in the image. At the beginning we will consider that the medium is homogeneous, linear and isotropic. Therefore the medium is stable.
3. At an instant t_{lim}, the propagating medium is no longer stable. The waves emitted by the source are disrupted and distorted.
4. In the course of time the medium (starting from a given instant) finds anew a stability state that

minimizes the perturbing contributions to which it is submitted. The distorted curve contained in the image to be studied is considered as a distortion of a wave that follows the medium stabilization.

3.3. The inverse problem

An initialization step consists in determining a straight line that fits a locally rectilinear portion of the curve to be studied; we aim at determining the N unknowns of the image x_k, forming a vector X_{input} and taken into account in the sensor k:

$$z_k = e^{-j\mu x_k}, \ \forall\, k = 1, \ldots, N$$

The observation vector obtained is

$$Z_{input} = \left[e^{j\varphi_1}, \ldots, e^{j\varphi_N}\right]^T \tag{7}$$

with $\varphi_k = -\mu x_k$. We aim at minimizing

$$J(X_l) = \left|Z_{input} - Z_{estimated\ for\ X_l}\right|^2$$

For this purpose we use gradient with fixed step type methods. The gradient is estimated by finite differences. We stop when the gradient is under a threshold. The series converges ($k \to +\infty$) towards a vector $\widehat{X}$ such that

$$Z_{input} = Z_{\widehat{X}}$$

that is to say $\widehat{X}$ is a local minimum argument of J. There exist an N-uplet of relative integers written p_l such that

$$X_{input} = \widehat{X} + \frac{2\pi}{\mu}[p_1, p_2, \ldots, p_N]^T \tag{8}$$

The next step concerns the determination of X_{input}. The uniqueness of the correct N-uplet for the reconstruction of the distorted wave requires to determine at least one of the components x_l of X. At this stage of the method the choice of an initialization by a convenient straight line steps in.

3.4. Simulation results

Figure 4 presents the main steps of the method.

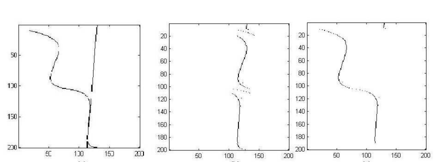


Fig. 4. (a)Initialization of the method (b)- Determination of the vector $\hat{X}$ (c)-junction of the different parts of the curve by determining the coefficients p_l

4. CONCLUSION

By adopting a specific formalism, it is possible to apply array processing methods for the estimation of both angle and offset of straight lines in an image. An efficient method for the estimation of the offsets was proposed. Therefore we got a coherent set of methods based on array processing that leads to the parameters of straight lines in an image. Until now the offset values were obtained with the Extension of the Hough Transform. The proposed method is efficient compared to this method; it performs well with roughly aligned points; moreover by applying this method to real grey level images, we showed that its computational cost is lower than the cost of the Extension of the Hough Transform. The case of non rectilinear contours was examined. Starting from the results of the previous method, an extended method enables the characterization of non rectilinear curves.

5. REFERENCES

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