ASYMPTOTIC PERFORMANCE FOR SUBSPACE BEARING METHODS WITH SEPARABLE NUISANCE PARAMETERS IN PRESENCE OF MODELING ERRORS

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ABSTRACT

This paper provides an asymptotic (in the number of snapshots) closed form expression of the bias and RMS (Root Mean Square) error of the estimated DOA (Direction Of Arrival) for the algorithm recently introduced in [1]. This algorithm provides a 1-D DOA's estimation in a multi-parameters context where the DOAs have to be estimated with some separable nuisance parameters. Results are based on a second order approximation of the criterion. DOA estimation errors are then expressed as a ratio of Hermitian forms of multi-variate complex random variables. Theoretical results are validated by simulations in a self-calibration context.

1. INTRODUCTION

Subspace based estimation of DOAs using an array of spatially distributed sensors [2] has been intensively studied these last decades. Recently, we introduced a new algorithm [1], which provides a low-cost 1D estimation of DOAs in a multi-parameters context where the steering vector can be factorized in the DOA and some nuisance parameter vector $\boldsymbol{\eta}$. Vector $\boldsymbol{\eta}$ depends on the context. In wide-band context [3], $\boldsymbol{\eta}$ corresponds to the frequency or the bandwidth of the sources. In self-calibration with mutual coupling [4], $\boldsymbol{\eta}$ may be composed of coupling matrix coefficients. In diverse polarisation [5], $\boldsymbol{\eta}$ may be the cross-polarization of the sources etc... The purpose of this paper is to provide a closed form expression of the asymptotic (in the number of snapshots) performance (bias and RMS error) of this algorithm (denoted in this paper MSP algorithm for Music Separable Parameters algorithm) due to modeling errors. For conciseness and clarity of presentation, this paper investigates the single nuisance parameter case. Results are based on a second order approximation of the criterion, recently introduced in [6][7]. Following the general approach presented in [7], DOA estimation errors are then expressed as a ratio of Hermitian forms of multi-variate complex random variables. Theoretical results are validated by simulations in a self-calibration context.

2. SIGNAL MODELING AND PROBLEM FORMULATION

A noisy mixture of a known number M of narrow-band sources with DOAs θ_m ($1 \leq m \leq M$) and associated nuisance parameter η_m, is assumed to be received by an array of N sensors. The associated observation vector, $\mathbf{x}(t)$, whose components $x_n(t)$ ($1 \leq n \leq N$) are the complex envelopes of the signals at the output of the sensors, is thus given by

$$\mathbf{x}(t) = \sum_{m=1}^{M} \widetilde{\mathbf{b}}\left(\theta_m, \eta_m\right) s_m\left(t\right) + \mathbf{n}(t) = \widetilde{\mathbf{B}}\,\mathbf{s}\left(t\right) + \mathbf{n}(t), \tag{1}$$

where $\widetilde{\mathbf{b}}\left(\theta, \eta\right)$ is the steering vector of a source, with DOA θ and nuisance parameter η, $\widetilde{\mathbf{B}}$=[$\widetilde{\mathbf{b}}\left(\theta_1, \eta_1\right)$...$\widetilde{\mathbf{b}}\left(\theta_M, \eta_M\right)$] and $s_m\left(t\right)$ is the complex envelope of the m^{th} source. $\mathbf{n}(t)$ is the noise vector, supposed to be spatially white.

The estimation problem under consideration is to estimate the M DOA parameters $\theta_1, \ldots, \theta_M$ with the MSP algorithm [1] where the steering vector $\mathbf{b}\left(\theta, \eta\right)$ depends on a single parameter η such that:

$$\begin{aligned} \mathbf{b}\left(\theta, \eta\right) &= \mathbf{U}\left(\theta\right)\boldsymbol{\Phi}\left(\eta\right), & (2)\\ \boldsymbol{\Phi}\left(\eta\right) &= [1\ \eta]^T,\ \mathbf{U}\left(\theta\right) = [\mathbf{u}_1\left(\theta\right)\ \mathbf{u}_2\left(\theta\right)], & (3) \end{aligned}$$

where ^{T} denotes the transpose operator. For example, in cases of diverse polarization and mutual coupling contexts, expression of $\boldsymbol{\Phi}\left(\eta\right)$ in (3) is given in [1].

The modeling errors $\mathbf{e}_m$ of the m^{th} source is defined by:

$$\mathbf{e}_m = \widetilde{\mathbf{b}}\left(\theta_m, \eta_m\right) - \mathbf{b}\left(\theta_m, \eta_m\right). \tag{4}$$

In these conditions the matrix $\widetilde{\mathbf{B}}$ depends on the vectors $\mathbf{e}_m$ for $1 \leq m \leq M$ such that:

$$\widetilde{\mathbf{B}}\left(\mathbf{E}\right) = \mathbf{B} + \mathbf{E} \text{ with } \mathbf{E} = [\mathbf{e}_1 \ldots \mathbf{e}_M], \tag{5}$$

where $\mathbf{B}$=[$\mathbf{b}_1$...$\mathbf{b}_M$] = $\widetilde{\mathbf{B}}$ ($\mathbf{E}$=$\mathbf{0}$) and $\mathbf{b}_m$=$\mathbf{b}\left(\theta_m, \eta_m\right)$. The covariance matrix $\mathbf{R}_\mathbf{x}\left(\mathbf{E}\right)$=E[$\mathbf{x}(t)\mathbf{x}(t)^H$] is perfectly estimated in asymptotic conditions (^{H} defines the conjugate-transpose). $\mathbf{R_x}(\mathbf{E})$ can be expressed as:

$$\mathbf{R_x}(\mathbf{E}) = \widetilde{\mathbf{B}}(\mathbf{E})\,\mathbf{R_s}\widetilde{\mathbf{B}}(\mathbf{E})^H + \sigma^2\mathbf{I}_N, \qquad (6)$$

where $\mathbf{R_s}$= E[$\mathbf{s}(t)\,\mathbf{s}(t)^H$] and $\mathbf{I}_N$ is the $N \times N$ identity matrix. However, $\widetilde{\mathbf{B}}(\mathbf{E})$ and $\mathbf{R_s}$ must be full rank. The N eigenvalues λ_m ($1 \leq m \leq M$) of $\mathbf{R_x}(\mathbf{E})$ check:

$$\mathbf{R_x}(\mathbf{E}) = \sum_{k=1}^{N} \lambda_k\, \mathbf{w}_k \mathbf{w}_k^H, \qquad (7)$$

where $\lambda_1 \geq ... \geq \lambda_{M+1}$= ...=$\lambda_N$=$\sigma^2$ and $\mathbf{w}_k$ is the eigen vector associated to λ_k. Under asymptotic assumption of this paper, the noise projector $\mathbf{\Pi}(\mathbf{E})$ checks:

$$\mathbf{\Pi}(\mathbf{E}) = \mathbf{W}_n\mathbf{W}_n^H = \mathbf{I}_N - \widetilde{\mathbf{B}}(\mathbf{E})\,\widetilde{\mathbf{B}}(\mathbf{E})^{\#}, \qquad (8)$$

where $\mathbf{W}_n$=[$\mathbf{w}_{M+1}$...$\mathbf{w}_N$] and $^{\#}$ defines the Moore Penrose pseudo-inverse such that: $\widetilde{\mathbf{B}}^{\#}\widetilde{\mathbf{B}} = \mathbf{I}_M$. We have proposed in [1], for computational cost reasons, the MSP criterion $J_{\mathbf{E}}(\theta)$:

$$\begin{aligned} J_{\mathbf{E}}(\theta) &= \frac{\det(\mathbf{U}(\theta)^H\,\mathbf{\Pi}(\mathbf{E})\,\mathbf{U}(\theta))}{\det(\mathbf{U}(\theta)^H\,\mathbf{U}(\theta))} && (9)\\ &= \det\left(\mathbf{U}(\theta)^{\#}\,\mathbf{\Pi}(\mathbf{E})\,\mathbf{U}(\theta)\right), && (10)\end{aligned}$$

where $\det(\mathbf{A})$ is the determinant of $\mathbf{A}$. In the following, we note $\widehat{\theta}_m$ the M DOAs estimates of θ_m and $\Delta\theta_m$=$\widehat{\theta}_m - \theta_m$ the DOA estimation error of θ_m.

3. RELATION BETWEEN $\Delta\theta_M$ AND E

The MSP criterion (10) in θ=θ_m can be rewritten as:

$$\begin{aligned} J_{\mathbf{E}}(\theta_m) &= \varphi_{\mathbf{E}}(\mathbf{v}_{1m},\mathbf{u}_{1m})\varphi_{\mathbf{E}}(\mathbf{v}_{2m},\mathbf{u}_{2m}) \\ &\quad -\varphi_{\mathbf{E}}(\mathbf{v}_{1m},\mathbf{u}_{2m})\varphi_{\mathbf{E}}(\mathbf{v}_{2m},\mathbf{u}_{1m}), && (11)\end{aligned}$$

$$\text{where}\quad \varphi_{\mathbf{E}}(\mathbf{v},\mathbf{u}) = \mathbf{v}^H\,\mathbf{\Pi}(\mathbf{E})\,\mathbf{u}, \qquad (12)$$

[$\mathbf{u}_{1m}\ \mathbf{u}_{2m}$]=$\mathbf{U}_m$=$\mathbf{U}(\theta_m)$ and $\mathbf{V}_m^H$=$\mathbf{U}_m^{\#}$=[$\mathbf{v}_{1m}\ \mathbf{v}_{2m}$]^H. After a second order Taylor expansion in θ of $J_{\mathbf{E}}(\theta_m)$ around θ=θ_m, the expression of $\Delta\theta_m$ becomes:

$$\Delta\theta_m \approx -\frac{\dot{J}_{\mathbf{E}}(\theta_m)}{\ddot{J}_{\mathbf{E}}(\theta_m)}, \qquad (13)$$

where $\dot{J}_{\mathbf{E}}(\theta)$ and $\ddot{J}_{\mathbf{E}}(\theta)$ respectively indicate the first and second derivatives of the criterion $J_{\mathbf{E}}(\theta)$ versus θ. The first and second derivatives of $\varphi_{\mathbf{E}}(\mathbf{v}(\theta),\mathbf{u}(\theta))$ versus θ check:

$$\begin{aligned} \dot{\varphi}_{\mathbf{E}}(\mathbf{v}(\theta),\mathbf{u}(\theta)) &= \varphi_{\mathbf{E}}(\dot{\mathbf{v}}(\theta),\mathbf{u}(\theta)) + \varphi_{\mathbf{E}}(\mathbf{v}(\theta),\dot{\mathbf{u}}(\theta)),\\ \ddot{\varphi}_{\mathbf{E}}(\mathbf{v}(\theta),\mathbf{u}(\theta)) &= \varphi_{\mathbf{E}}(\ddot{\mathbf{v}}(\theta),\mathbf{u}(\theta)) + \varphi_{\mathbf{E}}(\mathbf{v}(\theta),\ddot{\mathbf{u}}(\theta))\\ &\quad +2\varphi_{\mathbf{E}}(\dot{\mathbf{v}}(\theta),\dot{\mathbf{u}}(\theta)), && (14)\end{aligned}$$

where $\dot{\mathbf{u}}(\theta)$ and $\dot{\mathbf{v}}(\theta)$ are the first derivatives at θ of $\mathbf{u}(\theta)$ and $\mathbf{v}(\theta)$ and $\ddot{\mathbf{u}}(\theta)$ and $\ddot{\mathbf{v}}(\theta)$ are the second derivatives. Using expressions (11)(14), the expression of $\dot{J}_{\mathbf{E}}(\theta)$ and $\ddot{J}_{\mathbf{E}}(\theta_m)$ becomes in θ=θ_m:

$$\begin{aligned} \dot{J}_{\mathbf{E}}(\theta_m) &= \sum_{i=1}^{4} f\left(\mathbf{M}^i_{1122,m}\right) - f\left(\mathbf{M}^i_{1221,m}\right), && (15)\\ \ddot{J}_{\mathbf{E}}(\theta_m) &= 2\sum_{i=1}^{4}\sum_{j=i+1}^{4} f\left(\mathbf{M}^{ij}_{1122,m}\right) - f\left(\mathbf{M}^{ij}_{1221,m}\right)\\ &\quad +\sum_{i=1}^{4} f\left(\mathbf{M}^{ii}_{1122,m}\right) - f\left(\mathbf{M}^{ii}_{1221,m}\right), && (16)\end{aligned}$$

where the I^{th} column of $\mathbf{M}^I_{ijkl,m}$ and $\mathbf{M}^{II}_{ijkl,m}$ are respectively the first and the second derivative in θ=θ_m of the I^{th} column of $\mathbf{M}_{ijkl,m}$=[$\mathbf{v}_{im}, \mathbf{u}_{jm}, \mathbf{v}_{km}, \mathbf{u}_{lm}$], the I^{th} and J^{th} columns of $\mathbf{M}^{IJ}_{ijkl,m}$ are the first derivatives in θ=θ_m of the I^{th} and J^{th} columns of $\mathbf{M}_{ijkl,m}$ and

$$\begin{aligned} f(\mathbf{M}) &= \varphi_{\mathbf{E}}(\mathbf{v},\mathbf{u})\varphi_{\mathbf{E}}(\mathbf{y},\mathbf{x}),\\ \text{with}\quad \mathbf{M} &= [\mathbf{v}\ \mathbf{u}\ \mathbf{y}\ \mathbf{x}]. && (17)\end{aligned}$$

The others columns of $\mathbf{M}^I_{ijkl,m}$, $\mathbf{M}^{II}_{ijkl,m}$ and $\mathbf{M}^{IJ}_{ijkl,m}$ are made up of the other columns of $\mathbf{M}_{ijkl,m}$.

In order to obtain a tractable expression, let's now rewritte $\Delta\theta_m$ as a ratio of Hermitian forms. A second order Taylor expansion of the projector $\mathbf{\Pi}(\mathbf{E})$ in **E=0** [7][8] gives:

$$\begin{aligned} \mathbf{\Pi}(\mathbf{E}) &= \mathbf{\Pi}^2(\mathbf{E}) + o(\|\mathbf{E}\|^2),\\ \mathbf{\Pi}^2(\mathbf{E}) &= \mathbf{\Pi}_0 + \Delta^1\mathbf{\Pi}(\mathbf{E}) + \Delta^2\mathbf{\Pi}(\mathbf{E})/2, && (18)\end{aligned}$$

where $\mathbf{\Pi}_0$=$\mathbf{\Pi}(\mathbf{0})$. Using Appendix B of [9], the derivatives of $\mathbf{\Pi}(\mathbf{E})$ are:

$$\begin{aligned} \Delta^1\mathbf{\Pi}(\mathbf{E}) &= -\mathbf{U}_0 - \mathbf{U}_0^H,\\ \Delta^2\mathbf{\Pi}(\mathbf{E})/2 &= \mathbf{U}_0^H\mathbf{U}_0 - \mathbf{U}_0\mathbf{U}_0^H + \mathbf{V}_0 + \mathbf{V}_0^H, && (19)\end{aligned}$$

where

$$\mathbf{U}_0 = \mathbf{\Pi}_0\,\mathbf{E}\,\mathbf{B}^{\#} \ \text{ and }\ \mathbf{V}_0 = \mathbf{\Pi}_0\,\left(\mathbf{E}\,\mathbf{B}^{\#}\right)^2. \qquad (20)$$

A second order Taylor expansion in **E=0** of $\varphi_{\mathbf{E}}(\mathbf{v},\mathbf{u})$ = $\tilde{\varphi}_{\mathbf{E}}(\mathbf{v},\mathbf{u})$ + $o(\|\mathbf{E}\|^2)$ can be rewritten in the following hermitian form [7][8]:

$$\begin{aligned} \tilde{\varphi}_{\mathbf{E}}(\mathbf{v},\mathbf{u}) &= \mathbf{v}^H\mathbf{\Pi}^2(\mathbf{E})\,\mathbf{u} = \varepsilon^H\mathbf{Q}(\mathbf{u},\mathbf{v})\,\varepsilon && (21)\\ \mathbf{Q}(\mathbf{u},\mathbf{v}) &= \begin{bmatrix} q & -\mathbf{q}_{12}^H & \mathbf{0}^T\\ -\mathbf{q}_{21} & \mathbf{Q}_{22} & \mathbf{Q}_{23}\\ \mathbf{0} & \mathbf{Q}_{32} & \mathbf{Q}_{33}\end{bmatrix} \text{ and } \varepsilon = \begin{bmatrix}1\\ \mathbf{e}\\ \mathbf{e}^*\end{bmatrix}\end{aligned}$$

where

$$\begin{aligned} \mathbf{e} &= vec(\mathbf{E}) = [\mathbf{e}_1^T ... \mathbf{e}_M^T]^T,\ q = \mathbf{v}^H\mathbf{\Pi}_0\mathbf{u},\\ \mathbf{q}_{12} &= \Phi(\mathbf{u},\mathbf{v}), \mathbf{q}_{21} = \Phi(\mathbf{v},\mathbf{u}),\end{aligned}$$

with $\Phi(\mathbf{x},\mathbf{y}) = \left((\mathbf{B}^{\#}\mathbf{x})^{*} \otimes (\mathbf{\Pi}_0\mathbf{y})\right)$,

$$\begin{aligned}
\mathbf{Q}_{22} &= \Psi(\mathbf{B}^{\#},\mathbf{B}^{\#},\mathbf{\Pi}_0),\\
\mathbf{Q}_{23} &= \Psi(\mathbf{B}^{\#},\mathbf{\Pi}_0,(\mathbf{B}^{\#})^{H})\mathbf{P},\\
\mathbf{Q}_{32} &= \mathbf{P}^{H}\Psi(\mathbf{\Pi}_0,\mathbf{B}^{\#},\mathbf{B}^{\#}),\\
\mathbf{Q}_{33} &= \mathbf{P}^{H}\Psi(\mathbf{\Pi}_0,\mathbf{\Pi}_0,\mathbf{B}^{\#}\mathbf{B}^{\#H})\mathbf{P},
\end{aligned}$$

where $\Psi(\mathbf{X},\mathbf{Y},\mathbf{Z}) = \left((\mathbf{X}\mathbf{v})^{*}(\mathbf{Y}\mathbf{u})^{T}\right) \otimes \mathbf{Z}$, $\otimes$ is the Kronecker product and $\mathbf{P}$ the permutation matrix defined by $vec(\mathbf{E}^T)=\mathbf{P}vec(\mathbf{E})$.

Let's note $\tilde{f}_4(\mathbf{M})=\tilde{J}_{\mathbf{E}}(\mathbf{v},\mathbf{u})\tilde{J}_{\mathbf{E}}(\mathbf{x},\mathbf{y})$ verifying:

$$\tilde{f}_4(\mathbf{M}) = \varepsilon^{\otimes 2H}\,\breve{\mathbf{Q}}(\mathbf{M})\,\varepsilon^{\otimes 2}, \tag{22}$$

where $\varepsilon^{\otimes 2}=\varepsilon\otimes\varepsilon$ and $\breve{\mathbf{Q}}(\mathbf{M})=\mathbf{Q}(\mathbf{u},\mathbf{v})\otimes\mathbf{Q}(\mathbf{x},\mathbf{y})$. According to (17)(21), the first and second derivatives of $\tilde{f}_4(\mathbf{M})$ and $f(\mathbf{M})$ are equal in $\mathbf{E}$=$\mathbf{0}$. Consequently, the second order Taylor expansion in $\mathbf{E}$=$\mathbf{0}$ of $f(\mathbf{M})$ and $\tilde{f}_4(\mathbf{M})$ are equal:

$$\begin{aligned}
f(\mathbf{M}) &= \tilde{f}(\mathbf{M}) + o(\|\mathbf{E}\|^2), \qquad (23)\\
\tilde{f}(\mathbf{M}) &= \varepsilon^{H}\mathbf{Q}(\mathbf{M})\varepsilon,\\
\mathbf{Q}(\mathbf{M}) &= \mathbf{T}_1^{H}\breve{\mathbf{Q}}(\mathbf{M})\mathbf{T}_1 +\\
&\quad g\left(\mathbf{T}_2^{T}\breve{\mathbf{Q}}(\mathbf{M})^{T}\mathbf{1}\right) + g\left(\mathbf{T}_2^{H}\breve{\mathbf{Q}}(\mathbf{M})\mathbf{1}\right)
\end{aligned}$$

where $\tilde{f}(\mathbf{M})$ is the second order contribution of $\tilde{f}_4(\mathbf{M})$ in ε, $\mathbf{1}$=[1 $\mathbf{0}^T$], $\varepsilon^H g(\mathbf{w})\,\varepsilon = \mathbf{w}^T\mathbf{e}_T^{\otimes 2}$ with $\mathbf{e}_T$=$[\mathbf{e}^T\ \mathbf{e}^{*T}]^T$. Permutation matrices $\mathbf{T}_1$ and $\mathbf{T}_2$, made up of ones and zeros check:

$$\varepsilon^{\otimes 2} = \mathbf{T}_1\,\varepsilon\ + \mathbf{T}_2\,\mathbf{e}_T^{\otimes 2}. \tag{24}$$

Using (23) and replacing the function $\tilde{f}(\mathbf{M})$ by $f(\mathbf{M})$ in expressions (15)(16), the derivatives of $J_{\mathbf{E}}(\theta_m)$ are given by:

$$\dot{J}_{\mathbf{E}}(\theta_m) = \varepsilon^{H}\dot{\mathbf{Q}}_m\varepsilon + o(\|\mathbf{E}\|^2), \tag{25}$$

$$\ddot{J}_{\mathbf{E}}(\theta_m) = \varepsilon^{H}\ddot{\mathbf{Q}}_m\varepsilon + o(\|\mathbf{E}\|^2), \tag{26}$$

where

$$\dot{\mathbf{Q}}_m = \sum_{i=1}^{4}\mathbf{Q}\left(\mathbf{M}^{i}_{1122,m}\right) - \mathbf{Q}\left(\mathbf{M}^{i}_{1221,m}\right), \tag{27}$$

$$\begin{aligned}
\ddot{\mathbf{Q}}_m &= 2\sum_{i=1}^{4}\sum_{j=i+1}^{4}\mathbf{Q}\left(\mathbf{M}^{ij}_{1122,m}\right) - \mathbf{Q}\left(\mathbf{M}^{ij}_{1221,m}\right)\\
&\quad + \sum_{i=1}^{4}\mathbf{Q}\left(\mathbf{M}^{ii}_{1122,m}\right) - \mathbf{Q}\left(\mathbf{M}^{ii}_{1221,m}\right).
\end{aligned}$$

The DOA estimation error $\Delta\theta_m$ (13) becomes:

$$\Delta\theta_m \approx -\frac{\varepsilon^{H}\dot{\mathbf{Q}}_m\varepsilon}{\varepsilon^{H}\ddot{\mathbf{Q}}_m\varepsilon}, \text{ with } \varepsilon = \left[1\ vec(\mathbf{E})^{T}\ \ vec(\mathbf{E})^{H}\right]^{T}. \tag{28}$$

4. BIAS AND RMS ERROR OF MSP

Papers [7][8] provide an approximate expression of the moments of a ratio of Hermitians forms, similar to (28) when $||\mathbf{E}||<<||\mathbf{B}||$. According to (28), the bias of $\Delta\theta_m$ is:

$$E\left[\Delta\theta_m\right] \approx -\frac{trace\left(\dot{\mathbf{Q}}_m\mathbf{R}_{\varepsilon}\right)}{trace\left(\ddot{\mathbf{Q}}_m\mathbf{R}_{\varepsilon}\right)}, \tag{29}$$

where $\mathbf{R}_{\varepsilon}$=$E\left[\varepsilon\varepsilon^{H}\right]$ and $trace\,(\mathbf{A})$ is the trace of the matrix $\mathbf{A}$. Noting that $(\varepsilon^H\mathbf{Q}\,\varepsilon)^2$=$\left(\varepsilon^{\otimes 2}\right)^{H}\left(\mathbf{Q}^{\otimes 2}\right)\left(\varepsilon^{\otimes 2}\right)$, the RMS of $\Delta\theta_m$ is:

$$RMS_m \approx \sqrt{\frac{trace\left(\dot{\mathbf{Q}}_m^{\otimes 2}\mathbf{R}_{\varepsilon}^{(4)}\right)}{trace\left(\ddot{\mathbf{Q}}_m^{\otimes 2}\mathbf{R}_{\varepsilon}^{(4)}\right)}}, \tag{30}$$

where $\mathbf{R}_{\varepsilon}^{(4)}$=$E\left[\varepsilon^{\otimes 2}\varepsilon^{\otimes 2H}\right]$. In the case of a circular Gaussian distribution of ε, the relation between $\mathbf{R}_{\varepsilon}$ and $\mathbf{R}_{\varepsilon}^{(4)}$ is [7][8]:

$$\begin{aligned}
\mathbf{R}_{\varepsilon}^{(4)} &= \begin{bmatrix} \mathbf{R}_{1,1} & \cdots & \mathbf{R}_{1,K}\\ \vdots & \ddots & \vdots\\ \mathbf{R}_{K,1} & \cdots & \mathbf{R}_{K,K}\end{bmatrix}, \qquad (31)\\
\mathbf{R}_{i,j} &= \mathbf{R}_{\varepsilon}\mathbf{R}_{\varepsilon}(i,j) + \mathbf{r}_{\varepsilon}(j)\,\mathbf{r}_{\varepsilon}(i)^{H} - \mathbf{1}_K\mathbf{1}_K^{H}\,\delta_i\delta_j,
\end{aligned}$$

where $\mathbf{R}_{\varepsilon}$=$[\mathbf{r}_{\varepsilon}(1)\cdots\mathbf{r}_{\varepsilon}(K)]$, $\mathbf{1}_K$ is the $K\times 1$ vector $[1\ 0\ \cdots\ 0]^T$, K=1+2NM, $\mathbf{R}_{\varepsilon}(i,j)$ is the ij^{th} element of $\mathbf{R}_{\varepsilon}$ and δ_i =1 for i=1 and 0 otherwise.

5. SIMULATIONS

Simulations are conducted in self-calibration context [10], with a mutual coupling matrix $\mathbf{Z}$ [1] depending on a single parameter η corresponding to the mutual-coupling coefficient between two neighbouring sensors. The case of a circular array of sensors of radius R=1λ is considered, λ being the wavelenght. Let's remember that the steering vector $\mathbf{b}(\theta,\eta)$ checks $\mathbf{b}(\theta,\eta) = \mathbf{Z}(\eta)\,\mathbf{a}(\theta)$, where $\mathbf{a}(\theta) = [a_1(\theta)\cdots a_N(\theta)]^T$, with

$$a_n(\theta) = exp(j2\pi(R/\lambda)\cos(\theta-\varphi_n)),$$

and $\varphi_n = 2\pi((n-1)/N)$. Therefore,

$$\mathbf{Z}(\eta) = \begin{bmatrix} 1 & \eta & 0 & \eta\\ \eta & 1 & \ddots & 0\\ 0 & \ddots & \ddots & \eta\\ \eta & 0 & \eta & 1\end{bmatrix},$$

and $\mathbf{u}_1(\theta) = \mathbf{a}(\theta)$, $\mathbf{u}_2(\theta)$=$[v_1(\theta)\cdots v_N(\theta)]^T$ with $v_i(\theta) = a_{i-1}(\theta) + a_{i+1}(\theta)$ for $1 < i < N$, $v_1(\theta) = a_2(\theta) + a_N(\theta)$ and $v_N(\theta) = a_2(\theta) + a_{N-1}(\theta)$.

Simulations are conducted over 10000 Monte Carlo realizations. We consider the case of M=2 sources of DOA θ_1=100 and θ_2=117 degrees, with N=5 sensors. The modeling error is Gaussian and circular so that $E[\mathbf{e}_i \mathbf{e}_j^H]$=$\delta_{i-j}\sigma^2 \mathbf{I}_N$ with σ=0.122. Figures 1 and 2 respectively compare the empirical RMS error and bias of the first source with theoretical performance given by equations (30) and (29). Figures are plotted as a function of η. Empirical and theoretical results are in good adequacy. As expected from equations (2)–(4), the adequacy of empirical and theoretical RMS error deteriorates as η decreases, due to an increase of modeling error contribution in (2).

6. CONCLUSION

This paper provides a closed form expression of the asymptotic performance of the MSP algorithm due to modeling errors. Simulations, conducted in a self-calibration context, show a very good agreement between empirical and theoretical results. Extension of these results to a multiple nuisance parameter case is an ongoing work.

7. REFERENCES

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Fig. 1. RMS of the DOA θ_1

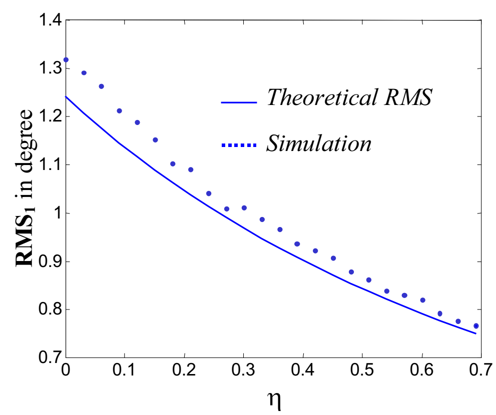

Fig. 2. Bias of the DOA θ_1