INSTANTANEOUS DOA ESTIMATION FOR MOVING WIDEBAND CYCLOSTATIONARY SOURCES UNDER MULTIPATH

H. Howard Fan and Huiqin Yan*

Department of ECECS, University of Cincinnati
Cincinnati, OH 45221-0030, USA

ABSTRACT

Cyclostationarity has been exploited in Direction of Arrival (DOA) estimation due to its signal selectivity and immunity against noise and interference. For coherent signals, e.g., those arising from a multipath propagation environment, the resulting rank deficiency of the signal cyclic correlation matrix hampers proper DOA estimation. Spatial Smoothing (SS) is an effective tool to cope with this proplem. However, SS reduces effective array span, thus also reduces the DOA resolution as well as the number of detectable sources. In this paper, we propose an instantaneous DOA estimation method for moving wideband cyclostationary sources under a multipath environment. Based on the movement of the sources, our method is able to decorrelate wideband coherent signals caused by multipath without SS. Effectiveness of our method is demonstrated by simulations.

1. INTRODUCTION

Many types of man-made communication signals exhibit cyclostationarity [1]. Gardner first exploited this property and developed a signal selective Direction of Arrival (DOA) estimation method, Cyclic MUSIC [2]. However Cyclic MUSIC was originally developed for narrowband signals. In addition, it would fail under a multipath environment where multiple and coherent signals are induced. Some methods have been proposed later to cope with these two problems separately. To cope with wideband signals, a Spectral Correlation Signal Subspace Fitting (SC-SSF) algorithm [3] and a Wideband Cyclic MUSIC algorithm [4] have been developed. [3] calculates the cyclic autocorrelation of the signals received at each sensor and exploits the phase differences between them, which contain the information of steering vectors regardless of the signal bandwidth. [4] calculates the cyclic cross spectral densities, from which steering vectors can be easily obtained even for the wideband case. However, both these two methods are not able to handle multipath propagation, or coherent signals.

*Corresponding author. E-mail: h.fan@uc.edu. This work is supported by the Office of Naval Research under Grant N00014-04-1-0059.

On the other hand, some methods have been proposed to cope with coherent signals, such as a new model based DOA estimation method [5] and Hankel Approximation Method (HAM) [6]. [5] assumes that the sources are moving, causing random phase shifts, which are uncorrelated for different paths. Thus these paths could be separated. [6] applies a preprocessing scheme referred to as Spatial Smoothing (SS) [7]. The SS method divides the entire array into several overlapped subarrays to handle DOA estimation in the presence of coherent signals. However both these two methods deal with narrowband signals only. In addition, SS results in a smaller effective array aperture, thus less DOAs could be detected and the angle resolution is reduced. Recently, we proposed a method which is able to cope with both wideband and coherent signals in [8], but it still uses the SS scheme.

In this paper, we propose a new method that is able to perform instantaneous DOA estimation for moving wideband cyclostationary sources under a multipath environment. We apply the idea of Averaged Cyclic MUSIC in [8] to handle the wideband case. However, unlike [8], this new method does not apply the SS scheme to cope with coherent signals. Instead, it uses the fact that the sources are moving at approximately constant speeds, which causes compression or expansion of the signals in the time domain, thus leading to decorrelation of the coherent signals caused by multipath propagation. Alternatively, the moving sources cause a shift in the cycle frequency, and the shift is not likely to be the same for different paths. Thus due to its ability to perform signal selective DOA estimation, our new cyclostationarity based approach can readily separate different paths associated with each source. Effectiveness of the new method is demonstrated by simulations.

2. EXISTING CYCLIC MUSIC ALGORITHM

Consider a Uniform Linear Array (ULA) of size N with intersensor spacing d, which receives I signals of interest (SOI) with cycle frequency α from directions θ_i, $i = 1, \cdots,$ I. The incident waves are assumed to be plane waves from far field sources with propagation speed c. Other signals that either have different cycle frequencies or do not exhibit cyclostationarity are considered as interference, which together with noise have no effect on DOA estimation and are thus ignored due to the properties of cyclostationarity [2]. If we choose the first sensor as a reference element, then the signal received at the nth sensor will be

$$x_n(t) = \sum_{i=1}^{I} s_i(t + (n-1)\Delta_i) \tag{1}$$

where $\Delta_i = d \sin\theta_i / c$ is the time delay. For the narrowband case, the time delay could be factored out and treated as a phase shift, i.e.

$$x_n(t) = \sum_{i=1}^{I} s_i(t) e^{j2\pi f_0 (n-1)\Delta_i} \tag{2}$$

where f_0 is the carrier frequency. If we define the source vector as $\mathbf{s}(t) = [s_1(t), \cdots, s_I(t)]^T$, and the received signal vector as $\mathbf{x}(t) = [x_1(t), \cdots, x_N(t)]^T$, where $[\cdot]^T$ denotes transpose, then we can write $\mathbf{x}(t)$ in a matrix form as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) \tag{3}$$

where $\mathbf{A} = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_I)]$ contains the steering vectors $\mathbf{a}(\theta_i)$ defined as

$$\mathbf{a}(\theta_i) = [1, e^{j2\pi f_0 \frac{d\sin\theta_i}{c}}, \cdots, e^{j2\pi f_0 (N-1)\frac{d\sin\theta_i}{c}}]^T \tag{4}$$

for $i = 1, \cdots, I$. Cyclic MUSIC calculates the cyclic correlation matrix of $\mathbf{x}(t)$ as

$$\begin{aligned} \mathbf{R}^{\alpha}_{\mathbf{xx}}(\tau) &= \langle \mathbf{x}(t + \frac{\tau}{2}) \mathbf{x}^H (t - \frac{\tau}{2}) e^{-j2\pi\alpha t} \rangle \\ &= \mathbf{A}\mathbf{R}^{\alpha}_{\mathbf{ss}}(\tau)\mathbf{A}^H \end{aligned} \tag{5}$$

where $\langle\cdot\rangle$ denotes time average, $[\cdot]^H$ denotes Hermitian transpose, and $\mathbf{R}^{\alpha}_{\mathbf{ss}}(\tau)$ is the cyclic correlation matrix of $\mathbf{s}(t)$. Then Cyclic MUSIC performs DOA estimation by applying Singular Value Decomposition (SVD) to (5) (See details in [2]). Since (3) is only true for narrowband signals, Cyclic MUSIC does not work for wideband signals.

3. PROPOSED METHOD

3.1. Algorithm

In this section, we assume that the SOI are wideband cyclostationary signals with cycle frequency α. We further consider, due to multipath, that there are K_i coherent signals induced from the ith source. The kth multipath of the ith source is from direction θ_{ik}. Then the signal received at the nth antenna will be

$$x_n(t) = \sum_{i=1}^{I} \sum_{k=1}^{K_i} \beta_{ik} s_i(t - \tau_{ik} + (n-1)\Delta_{ik}) \tag{6}$$

where β_{ik} is the attenuation factor and τ_{ik} is the reference delay for the kth path of the ith source.

To cope with wideband signals, we proposed an Averaged Cyclic MUSIC algorithm in our recent paper [8]. It averages the cyclic correlation over different lags τ and obtains an averaged cyclic correlation matrix $\langle \mathbf{R}^{\alpha}_{\mathbf{xx}} \rangle_\tau$, such that it can be written in the same form as in (5). Here $\langle\cdot\rangle_\tau$ denotes averaging over τ. Therefore, DOA estimation can be performed in the same way as in Cyclic MUSIC, but now is applicable to wideband signals. We will use this idea in our new algorithms too. However, unlike [8] which utilize SS to decorrelate the coherent signals induced by multipath propagation, we propose a new method based on the source movement.

Now assume that the sources are moving and let us exploit this source movement to overcome the multipath problem without SS. We recognize that β_{ik} and θ_{ik} change very little over one time frame of, say, J samples with J ranges from hundreds to thousands. Take an extreme example: An airplane moving at a speed of Mach 2, or approximately 700 m/s, and within a range of 1 km, will traverse 0.04 angular degrees during a 1 ms interval. This change is indeed negligible. Therefore θ_{ik} can be regarded as constant over this 1 ms time frame, and be updated every 1 ms time interval. At the same time, assuming a wideband signal of 2 MHz bandwidth and a sampling frequency of 10 MHz. Then during this 1 ms interval there will be 10,000 samples, more than enough for computation for DOA estimation. Since θ_{ik} changes so little over one time frame, the signal attenuations, β_{ik}, do not change much either. Therefore it is reasonable to treat both θ_{ik} and β_{ik} as constant in one time frame.

Now let us take a look at the change of the reference delay τ_{ik}. Consider an example scenario shown in Fig.1. If a source is moving at a constant velocity v, it will cause an additional delay of $(v\, t \cos \mathrm{a}_1)/c$ and $(v\, t \cos \mathrm{a}_2)/c$ for the direct path and the reflected path, respectively. For narrow-

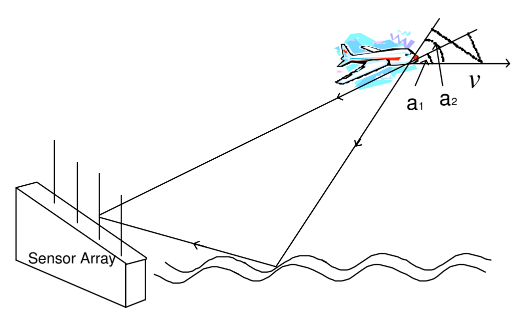

Fig.1. Example for moving source, two paths considered

band signals, [5] factored τ_{ik} out as it is the signal phase, and showed that although the change of τ_{ik} may be small, the resulted phase change could be more than 180 degrees. For wideband signals, however, no phase can be factored out, and τ_{ik} must be treated where it is as a function of t. We propose to model τ_{ik} by a linear function of t under the assumption that the sources are moving at a constant speed, i.e.,

$$\tau_{ik}(t) = \tau_{ik,0} + \gamma_{ik} t \tag{7}$$

Thus, from (7), it is seen that for the example in Fig.1, γ_{ik} equals to $(v \cos \mathrm{a}_1)/c$ and $(v \cos \mathrm{a}_2)/c$ for the direct path and the reflected path, respectively, which are most likely to be different.

To see the effect of such movement on the cyclic correlation, let us first consider a single source with a single path for simplicity. We therefore suppress the subscripts i and k of (6) and (7). Then substituting (7) into (6), we obtain

$$x_n(t) = \beta s((1-\gamma)t - \tau_0 + (n-1)\Delta) \tag{8}$$

Then the cyclic correlation $r^{\alpha}_{x_p x_n}(\tau)$ could be written as

$$\begin{aligned} & r^{\alpha}_{x_p x_n}(\tau) \\ &= \langle x_p(t+\frac{\tau}{2}) x_n^*(t-\frac{\tau}{2}) e^{-j2\pi\alpha t} \rangle \\ &= \langle \beta s((1-\gamma)(t+\frac{\tau}{2}) - \tau_0 + (p-1)\Delta) \\ &\quad \cdot \beta s^*((1-\gamma)(t-\frac{\tau}{2}) - \tau_0 + (n-1)\Delta) \\ &\quad \cdot e^{-j2\pi\alpha t} \rangle \end{aligned} \tag{9}$$

where $[\cdot]^*$ denotes complex conjugate. By letting $t' = (1-\gamma)t$ and $\alpha' = \alpha/(1-\gamma)$, (9) becomes

$$\begin{aligned} & r^{\alpha}_{x_p x_n}(\tau) \\ &= \langle \beta^2 s(t' + (1-\gamma)\frac{\tau}{2} - \tau_0 + (p-1)\Delta) \\ &\quad \cdot s^*(t' - (1-\gamma)\frac{\tau}{2} - \tau_0 + (n-1)\Delta) \\ &\quad \cdot e^{-j2\pi\alpha' t'} \rangle \\ &= e^{j\pi\alpha'(p-1)\Delta} \\ &\quad \cdot \left[\beta^2 r^{\alpha'}_{ss}\left((1-\gamma)\tau + (p-n)\Delta\right) e^{-j2\pi\alpha'\tau_0} \right] \\ &\quad \cdot e^{j\pi\alpha'(n-1)\Delta} \end{aligned} \tag{10}$$

where the shift property of cyclic correlation is applied, i.e., if $y(t) = x(t+T)$, then $r^{\alpha}_{yy}(\tau) = r^{\alpha}_{xx}(\tau) e^{j2\pi\alpha T}$. In order to make the factor in the middle of (10) independent on p or n, we average (10) over τ and obtain

$$\begin{aligned} \langle r^{\alpha}_{x_p x_n} \rangle_\tau &= e^{j\pi\alpha'(p-1)\Delta} \left[\beta^2 \langle r^{\alpha'}_{ss} \rangle_\tau e^{-j2\pi\alpha'\tau_0} \right] \\ &\quad \cdot e^{j\pi\alpha'(n-1)\Delta} \end{aligned} \tag{11}$$

From (8), we note that the factor $(1-\gamma)$ corresponds to compression (or expansion) of the signal in the time domain. Coherent signals after compression (or expansion) at different factors are not coherent any more, or they are decorrelated. Alternatively, from (10) we can see that the source movement causes the cycle frequency to shift from α to $\alpha' = \alpha/(1-\gamma)$. We name α' effective cycle frequency. As long as the effective cycle frequencies of different paths associated with the same source are different, the paths could be separated due to the properties of cyclostationarity [2].

Now if we assume that there are I sources with the same effective cycle frequency α' for a particular path for each source, and the sources are mutually cyclically uncorrelated, (11) is extended to

$$\begin{aligned} \langle r^{\alpha}_{x_p x_n} \rangle_\tau &= \sum_{i=1}^{I} e^{j\pi\alpha'(p-1)\Delta_i} \left[\beta_i^2 \langle r^{\alpha'}_{s_i s_i} \rangle_\tau e^{-j2\pi\alpha'\tau_{i,0}} \right] \\ &\quad \cdot e^{j\pi\alpha'(n-1)\Delta_i} \end{aligned} \tag{12}$$

Here the signals from other paths with an effective cycle frequency different from α' are treated as interference due to the reason explained above, and are thus ignored. The case of two different propagation paths from the same source giving rise to the same α',i.e., $\gamma_{ik} = \gamma_{il}$, for $k \neq l$, has probability zero in a realistic application. Thus the subscript k can be suppressed. Then $\langle \mathbf{R}^{\alpha}_{\mathbf{xx}} \rangle_\tau$ whose (p,n)-th element is given in (12) can be written as

$$\langle \mathbf{R}^{\alpha}_{\mathbf{xx}} \rangle_\tau = \mathbf{A}(\frac{\alpha'}{2}) \mathbf{M} \mathbf{A}^T(\frac{\alpha'}{2}) \tag{13}$$

where

$$\mathbf{A}(f) = [\mathbf{a}(f,\theta_1), \cdots, \mathbf{a}(f,\theta_I)] \tag{14}$$

$$\mathbf{a}(f,\theta) = [1, e^{j2\pi f \frac{d \sin\theta}{c}}, \cdots, e^{j2\pi f (N-1) \frac{d \sin\theta}{c}}]^T \tag{15}$$

and $\mathbf{M}$ is a diagonal matrix of size I with the i-th diagonal element being $\beta_i^2 \langle r^{\alpha'}_{s_i s_i} \rangle_\tau e^{-j2\pi\alpha'\tau_{i,0}}$. Note that (13) is in the same form as (5), and $\mathbf{M}$ is full rank. Thus the remaining steps of DOA estimation are the same as those of Cyclic MUSIC except now the steering vector is evaluated at $\alpha'/2$.

3.2. Discussion of Applications

Refer to Fig.1 again. Note that γ_{ik} for the direct path reaches its maximum when the source is moving along the DOA of the direct path, i.e., $\gamma_{ik} = v/c$. For surface and air operations with electromagnetic waves, even for a very high speed $v = 700\, m/s$, the large $c = 3 * 10^8\ m/s$ makes the γ_{ik} only at $2.3 * 10^{-6}$. This is too small for the change of cycle frequency to be observable. Thus for this type of applications our new method is not applicable, and the SS method [7] is still needed to overcome the multipath problem.

However, for the underwater acoustic environment, although v can only reach no more than $20\, m/s$ for surface vessels, c is approximately $1000\, m/s$, making γ_{ik} at the level of 0.02. This change in the cycle frequency is large enough to be observable. Thus different paths associated with the same source may become separable by our cyclic correlation method. In summary, our new method works only under these two conditions: 1) γ_{ik} is large enough; 2) γ_{ik} is different for different paths of the same source.

4. SIMULATION RESULTS

Simulation 1: In this simulation, we assume that there is only one source with two paths. The source generates a wideband BPSK digital communication signal with cycle frequency α. The Signal to Noise Ratio (SNR) is assumed to be $10\,dB$. Five sensors are used to receive a direct path from a DOA of $30°$ and a reflected path from $45°$. Here the simulation is for applications of the underwater acoustic environment, and we assume the delay change rates γ as $10/c$ and $20/c$ for the direct path and the reflected path, respectively, where $c = 1000\,m/s$. Then the effective cycle frequency for these two paths are $\alpha'_1 = \alpha/(1-0.01)$ and $\alpha'_2 = \alpha/(1-0.02)$ for the direct path and the reflected path, respectively. The results are shown in Fig.2. It can be seen that when we choose the cycle frequency of interest as α'_1, only the direct path is detected. And when we choose the cycle frequency of interest as α'_2, only the reflected path is detected.

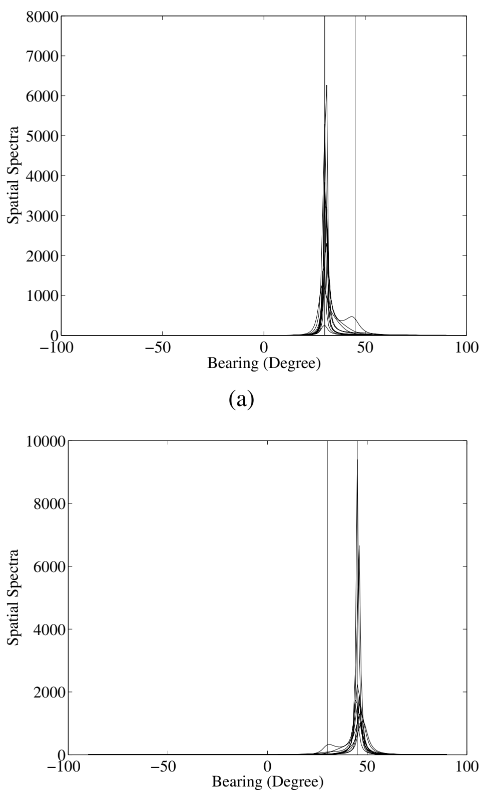

Fig.2. Spatial spectra when choosing different cycle frequencies (a) direct path from $30°$ is detected, (b) reflected path from $45°$ is detected.

Simulation 2: In this simulation, we assume that there are two sources each with two paths. The sources generate wideband BPSK digital communication signals with cycle frequency α. SNR is assumed to be $10\,dB$. Seven sensors are used to receive the direct paths from directions of $30°$ and $-30°$ for the two sources, respectively. Their effective cycle frequency is assumed to be $\alpha'_1 = \alpha/(1-0.01)$. The reflected paths are from $-45°$ and $35°$, and their effective cycle frequencies are assumed to be different from α'_1. We choose the cycle frequency of interest as α'_1 and the results are shown in Fig.3. It can be seen that only the direct paths of the two sources with an effective cycle frequency α'_1 are detected, and the reflected paths are suppressed.

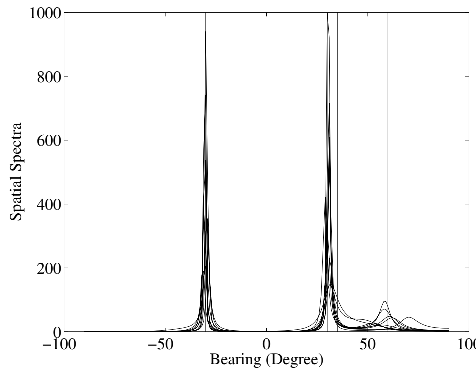

Fig.3. Direct paths of the two sources from $-30°$ and $30°$ are detected.

5. REFERENCES

[1] W. A. Gardner, *Introduction to Random Processes with Applications to Signals and Systems.* New York: Macmillan Pub. Co., 1986.

[2] W. A. Gardner, "Simplification of MUSIC and ESPRIT by exploitation of cyclostationarity," *Proc. of IEEE,* vol. 76, pp. 845-847, July 1988.

[3] G. Xu and T. Kailath, "Direction-of-arrival estimation via exploitation of cyclostationarity-A combination of temporal and spatial processing," *IEEE Trans. Signal Processing,* vol. 40, pp. 1775-1786, July 1992.

[4] K.D. Mauck, "Wideband cyclic MUSIC," *Proc. of IEEE International Conference on Acoust., Speech, and Signal Processing,* vol. 4, pp. 288-291, April 1993.

[5] Liang Jin, Minli Yao and Qinye Yin, "A new model for the DOA estimation of the coherent signals," *Proc. of IEEE International Symposium on Circuits and Systems,* vol. 4, June 1998.

[6] Y-T Lee and J-H Lee, "Direction-Finding methods for cyclostationary signals in the presence of coherent sources," *IEEE Trans. Antennas Propagat.,* vol. 49, December 2001.

[7] T.-J. Shan, M. Wax, and T. Kailath, "On spatial smoothing for direction-of-arrival estimation of coherent signals," *IEEE Trans. Acoust., Speech, and Signal Processing,* vol. 33, pp. 806-811, August 1985.

[8] Huiqin Yan and H. Fan, "DOA estimation for wideband cyclostationary signals under multipath environment," *Proc. of IEEE International Conference on Acoust., Speech, and Signal Processing,* vol.2, pp.77-80, May 2004.