DATA-ADAPTIVE ARRAY INTERPOLATION FOR DOA ESTIMATION IN CORRELATED SIGNAL ENVIRONMENTS

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ABSTRACT

Many popular direction-of-arrival (DOA) estimators rely on the fact that the array response vector of the array is Vandermonde, for example, that of a uniform linear array (ULA). Array interpolation is a preprocessing technique to transform the array response vector of a planar array of arbitrary geometry to that of a ULA over an angular sector. While good approximation within the target sector is attained with the various existing array interpolation approaches, the response of the interpolated array in the out-of-sector region is at best partially controlled. Accordingly, out-of-sector signals, especially those highly correlated with the in-sector signals, can degrade significantly the performance of DOA estimators that rely on the Vandermonde form to work correctly. Recently, we proposed an improved array interpolation approach that takes into account the array response over the full azimuth. In this paper, we develop the idea further and present a simple data-adaptive array interpolation scheme that can provide significantly better accuracy in the DOA estimates. We present examples to demonstrate the effectiveness of our proposal.

1. INTRODUCTION

Since the inception of array signal processing, uniform linear arrays (ULAs) have received by far the most attention. This is due in large part to their uniform spatial sampling which results in their array response vectors having a Vandermonde form. This form is central to the derivation of many important DOA estimators such as root-MUSIC [1], MUSIC with spatial smoothing [2], and root-WSF [3]. ULAs are, however, not always used in practice. Practical considerations and the requirement for 360° of coverage in the azimuthal plane in many applications such as radar, sonar and wireless communications can restrict the choice of array geometry.

To provide 360° of coverage, one has to deploy planar arrays such as uniform circular arrays (UCAs). However, the array response vector of a planar array is, in general, not Vandermonde. A number of techniques have been proposed to transform this vector to Vandermonde form. One such class of techniques is *array interpolation*, first proposed by Bronez [4], and later under different formulations by Friedlander [5], Pesavento *et al*. [6], and Lau *et al.* [7]. In array interpolation, the array response vector of the planar array is mapped using a transformation matrix to that of a ULA over an angular sector in the azimuth, called the *in-sector*. Thus the approach involves sector-by-sector processing to cover the full azimuth [5].

Recently, we showed that DOA estimation with the Friedlander and Pesavento formulations can behave poorly when the received signals are highly correlated and are not all confined to the in-sector [7]. This is due to the fact that, just outside the in-sector, the gain of the interpolated array is still significant while the phase response has deviated significantly from being Vandermonde. DOA estimators that rely on the Vandermonde form may thus not be able to deal properly with received signals in this region. In [7], we proposed a new formulation to address this limitation. In particular, the new formulation deals explicitly with the entire out-of-sector region by setting a target response for this region in addition to the target response for the in-sector region.

Other related developments include the recent works of Hyberg *et al*. [8] and Bühren *et al.* [9]. Both methods aim to reduce the bias in the DOA estimates which is an artifact of the Friedlander formulation. Whilst both methods will reduce bias, but like Friedlander, they also ignore the out-of-sector response. In the more recent work of Bühren *et al*. [10], the interesting idea of generalizing the interpolated array to a non-physical array with a Vandermonde steering vector is pursued. Differential geometry is employed to achieve well-conditioned transformation matrices and larger in-sectors for the same transformation errors. However, the out-of-sector response is again ignored. Moreover, the given example shows that a very large sector size can reduce the reliability of DOA estimates (i.e. presence of outliers) near the sector edges. A work closer in spirit to the present work is [11], which deals with the problem of out-of-sector signals by modeling them as colored noise. A beamspace transformation which preserves the Cramér-Rao bounds (CRBs) for the parameters of interest is applied to the spatially colored noise problem. The motivation is mainly in reducing the dimension of the problem and very good performance can be achieved via the data-adaptive approach. However, their algorithm relies heavily on the Capon beamformer which is known to break down in correlated signal environments as considered in this paper.

In this paper, we show that while the new formulation [7] can give better results than the Friedlander and Pesavento formulations, the attempt to control the response over the entire azimuth comes at a cost of relatively high approximation errors in the steering vector. These larger errors, in turn, increase the DOA estimation bias. One way to reduce the bias is to optimize the design of the data-independent weighting function in the least squares formulation [7]. For example, the weighting for the stopband may be relaxed to give "just enough" attenuation in the array response, so that the approximation error can be reduced elsewhere. Nevertheless, the requirement of achieving good DOA estimation performance over the entire azimuth for all signal scenarios is stringent and fundamentally limited by the underlying physics. In order to obtain further improvements, we allow the weighting function to be *data-adaptive*. Our idea is to reduce the approximation error where it matters, i.e. at the signal DOAs. A simple way to achieve this is to use the conventional beamformer power response as the weighting function. Even though the conventional response has a relatively large beamwidth as compared to the Capon beamformer response or MUSIC spectrum, its simplicity makes it robust to array modeling errors and signal correlation. A direct consequence to the use of a data-adaptive weighting function is that the transformation matrix must be constantly updated for each of the angular sectors. Fortunately, the weighted least squares (WLS) formulation is computationally simple to solve. A slowly varying environment and the inherent robustness introduced by the large beamwidth can also reduce the updating requirement. In the numerical examples, we use root-MUSIC [5] with spatial smoothing [2] to demonstrate the superiority of our proposed modification in DOA estimation of in-sector signals.

2. SIGNAL AND ARRAY MODELS

Consider a planar array with N elements. The nth component of the array response vector $\mathbf{a}(\theta)$, $n = 1, \ldots, N$, to a narrowband signal of wavelength λ arriving from azimuth angle $\theta \in [-\pi, \pi]$ is given by

$$a_n(\theta) = G_n(\theta) \exp\left[jk\left(x_n \cos\theta + y_n \sin\theta\right)\right], \qquad (1)$$

where $k = 2\pi/\lambda$, and $G_n(\theta)$ and (x_n, y_n) are the complex gain pattern and location of the nth element, respectively. The azimuth angle θ is measured from the positive x-axis in the anti-clockwise direction. Suppose the interpolated ULA has M elements and is aligned along the y-axis. The mth component of its array response vector $\mathbf{b}(\theta)$, $m = 1, \ldots, M$, is given by

$$b_m(\theta) = \exp\left[jkd\left(m-1\right)\sin\theta\right], \qquad (2)$$

where d is the inter-element spacing of the ULA.

Suppose the planar array receives L narrowband signals, $s_1(t), \ldots, s_L(t)$, each arriving from a distinct direction $\theta_1, \ldots, \theta_L$. The array output vector is given by

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \qquad (3)$$

where $\mathbf{A} = \left[\mathbf{a}(\theta_1) \ \cdots \ \mathbf{a}(\theta_L)\right]$, $\mathbf{s}(t) = \left[s_1(t) \ \cdots \ s_L(t)\right]^T$, $\mathbf{n}(t) = \left[n_1(t) \ \cdots \ n_N(t)\right]^T$, $n_n(t)$ is the noise output of the nth sensor (assumed white, circular complex Gaussian and i.i.d. across the elements), and $\mathbf{n}(t)$ and $\mathbf{s}(t)$ are assumed to be stationary, zero mean, and uncorrelated with each other. The linear transformation on the output of the array is given by

$$\mathbf{y}(t) = \mathbf{T}\mathbf{x}(t), \qquad (4)$$

where $\mathbf{T}$ is the $M \times N$ transformation matrix. Note that $\mathbf{T}$ will color the sensor noise at the interpolated array output. Hence, pre-whitening is required before we apply any DOA algorithms.

In the sequel, we shall assume, for convenience, that $G_n(\theta) = 1$ for $n = 1, \ldots, N$.

3. PROPOSED APPROACH

3.1. Problem Formulation

The principal idea of the Lau formulation [7] is to approximate the array response vector of the interpolated array to that of a ULA within the in-sector and concurrently control its response over the entire out-of-sector region. The WLS formulation is given as follows

$$\min_{\mathbf{T}} \int_{-\pi}^{\pi} W(\theta) \left\| \mathbf{T}\mathbf{a}(\theta) - s(\theta-\theta_c)\mathbf{b}(\theta-\theta_c)\right\|^2 d\theta, \qquad (\mathcal{P}1)$$

where $W(\theta)$ is the weighting function, $s(\theta)$ shapes the ULA response and θ_c is the center of the in-sector region defined by $\Delta\theta = \left[-\theta_0 + \theta_c, \theta_0 + \theta_c\right]$. The WLS solution to $(\mathcal{P}1)$ is well known. The design of the shaping function $s(\theta)$ is arbitrary, provided the requirements given in [7] are met. We proposed in [7] the raised cosine function. It should be noted that the idea of controlling the out-of-sector response can also be formulated in other ways, e.g. as a minimax problem with constraints on the in-sector error. The weighted least squares formulation $(\mathcal{P}1)$ is considered here because of its simplicity. The guidelines for the design of the interpolated array are also given in [7].

3.2. Fixed Weighting

In [7], $W(\theta)$ was set to 1. This choice gives a reasonable error performance over the entire azimuth (see Fig. 1). The data-independence of $W(\theta)$ allows $\mathbf{T}$ to be calculated only once for all scenarios. Moreover, only a renumbering of elements in $\mathbf{T}$ is required for different sectors if symmetry is preserved in the choice of in-sector with respect to the orientations of the UCA and interpolated ULA. While this approach is convenient, it also limits the performance of DOA estimation.

3.3. Data-Adaptive Weighting

Here, we propose a data-adaptive weighting function $W(\theta, \mathbf{R_x})$ which is a function of the sample covariance matrix $\mathbf{R_x} = \frac{1}{K}\sum_{p=1}^{K} \mathbf{x}[p]\mathbf{x}^H[p]$ (for $\mathbf{x}[p] = \mathbf{x}(pT_s)$, K snapshots, and T_s sampling period). Although this appears to be a minor modification, we shall demonstrate that by incorporating some knowledge of the signal scenario into the formulation, a large performance improvement can be realized. Although other factors in the formulation, e.g. in-sector sizes and their orientations, can also be adaptive to obtain further improvements, here we focus only on the use of $W(\theta, \mathbf{R_x})$. In this respect, the modified formulation is only partially data-adaptive. Due to an improved approximation error (at the signals of interest), in-sector sizes can be larger, thus reducing the number of sectors required. This is provided the sector edges are not too close to the endfire of the interpolated ULA. Otherwise, resolution capability can degrade significantly. Alternatively, smaller UCAs (of fewer elements) may be used for a given performance.

In this paper, we utilize the adaptive weighting function

$$W(\theta, \mathbf{R_x}) = \mathbf{a}^H(\theta)\mathbf{R_x}\mathbf{a}(\theta), \qquad (5)$$

which is the conventional beamformer's power response. The motivations for this choice include:

1. Simplicity. Not only in computational cost, but the entire procedure is easily automated.
2. Robustness. It does not breakdown in correlated signal environments and can better tolerate modeling errors, as discussed in the Introduction.
3. Poor angular resolution of the conventional beamformer is not critical, as the purpose of (5) is to *focus* the WLS fit to the regions of interest and reduce the approximation error of these regions compared to when $W(\theta) = 1$.

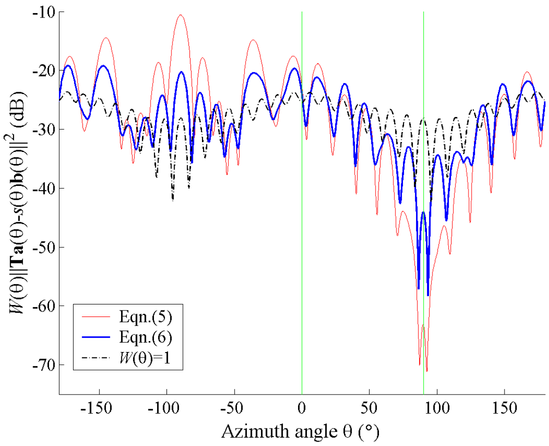

Fig. 1. Error performance of the Lau formulation with different weighting functions for UCA with $N = 30$, $r = 1.913\lambda$, and interpolated ULA with $M = 11$, 120°-sector. Vertical solid lines denote DOAs of signals.

Nevertheless, (5) has a number of drawbacks:

1. Recalculation of **T** is necessary for every sector and every new signal scenario.
2. The weighting function (5) is ideal for a scenario where signals have similar SNRs, and when high SNR signals are of interest. However, in a scenario where the dynamic range of SNRs of the signals of interest is large, the transformation errors at the low SNR signals can be relatively large.

One approach to address the second drawback is to use the more general expression

$$W(\theta, \mathbf{R_x}) = \left(\mathbf{a}^H(\theta)\mathbf{R_x}\mathbf{a}(\theta)\right)^{\alpha}, \quad (6)$$

where the choice of $\alpha \in (0,1]$ should reflect the dynamic range of the signals of interest in a given scenario. If the dynamic range is large, then a small α would counteract to give low SNR signals higher weighting. Observe that (5) corresponds to $\alpha = 1$ while the non-adaptive weighting $W(\theta) = 1$ corresponds to $\alpha = 0$. In Fig. 1, we plot the curves for (5) and (6) ($\alpha = 0.5$) for the case of two coherent signals (with correlation coefficient $e^{j\pi/3}$) at 0° and 90°, with SNRs of 0 dB and 20 dB, respectively. It is observed that even though the error at the stronger signal is poorer for (6), the error is improved (albeit slightly) for the weaker signal.

4. NUMERICAL EXAMPLES

It is evident from the numerical examples in [6], [7] that the Friedlander formulation can behave poorly in a correlated signal environment. In [7], it has been shown that the Pesavento formulation experiences a similar problem when signals fall into the uncontrolled rolloff region (see Fig. 3 of [7]). To be fair, the numerical examples in [7] did not incorporate the proposed adaptive optimization grid selection. This is because all other array interpolation formulations of Bronez, Friedlander and Lau were non-adaptive to signal environment and thus do not involve recalculations of **T**. Nevertheless, the adaptive grid selection does not influence the fixed don't care region, which must exist for finite rolloff. While the choice of a narrow rolloff region can reduce this problem (by minimizing the probability of signals falling into this region), it also degrades the error performance of the in-sector, particularly at the sector edges. We illustrate this in Fig. 2 where the power response of the interpolated ULA is shown for a case of two 10 dB signals at 25° and 50°. Optimization grid points are chosen according to conventional beamforming pattern [6][1]. The out-of-sector sidelobe (magnitude) constraint is set to 15 dB and the minimax criterion [6] is used. We observe the large in-sector error at 25° due to the narrow rolloff region, despite having the dense grid. Moreover, adaptive grid selection also involves solving the second-order cone formulation for each sector and each signal scenario. This is in contrast to our computationally simpler WLS formulation. For these reasons, and due to space constraint, we will not consider the Friedlander and Pesavento formulations any further.

In this section, we evaluate and compare the performance of the Lau formulation with different weighting functions: the non-adaptive weighting function $W(\theta) = 1$ [7] and the proposed adaptive versions of (5) and (6) ($\alpha = 0.5$). The DOA estimator is root-MUSIC with forward-backward spatial smoothing (FBSS) [2] and sector-by-sector processing [5]. We make two small refinements to the sector-by-sector processing procedure in [5] which are found to perform very well in numerous simulation runs and are also useful for all other formulations.

1. In the original procedure, at each in-sector, we discard all estimates outside the sector. This can be problematical for signals falling very close to the sector edges. We allow a small region in the vicinity of the edges (e.g. ±1°), to capture such signals.
2. Occasionally, DOA estimates can occur in clusters beyond the resolution capability of the estimator for finite sample. For instance, refinement 1 can result in multiple estimates of one signal. A simple solution is to retain only one estimate for a cluster of estimates, e.g. those within 1° of one another.

Two studies were conducted for a scenario of two coherent signals with correlation coefficient $e^{j\pi/3}$. The DOA of the first signal is fixed at 0° and the second signal is varied from 10° to 180°, in steps of 10°. The UCA has radius $r = 1.913\lambda$, $N = 30$ elements, and is oriented with an element at 0°. The interpolated ULA has $M = 11$ elements and an in-sector size of 120°. We use the raised cosine shaping function of [7]. The sample covariance matrix is estimated from $K = 200$ snapshots and the RMSE results are taken over 200 trials. For FBSS, two subarrays are used. The corresponding unconditional CRBs [12] are also calculated.

In the first study, we consider for the three weighting functions, the performance of root-MUSIC with FBSS. The SNR of both signals are fixed at 20 dB. The RMSE performances with the three weighting functions are given in Fig. 3. As expected, the focusing effect of (5) and (6) vastly improve the performance as compared to the non-adaptive case. Also, (5) and (6) are very close in performance, although (5) is generally better. This is because the power response (5) forces heavier weightings than the magnitude response (6), relative to their respective sidelobes. Also note that the RMSE of the second signal for $W(\theta) = 1$ is close to the CRB for some data points. These can be understood from Fig. 1, where the error curve fluctuates over the azimuth. Where error is smaller, the RMSE (due to bias) will be smaller.

In the second study (see Fig. 4), we repeat the first study, but with the SNR of the signal at 0° reduced to 0 dB. As can be seen, the performance of (6) is generally superior to that of (5) for the first signal. This is due to the large dynamic range of the signal SNR, where (5) imposes a large weight in the vicinity of

[1] Thus there is a link between the method of [6] and our methods.

the second signal relative to that of the first, thus sacrificing the first signal's performance more than (6). This improvement is achieved with only a minor tradeoff in the performance of the second signal. It is also obvious that the first (or weaker) signal's DOA estimate does not approach the CRB for any of the three weighting functions. It is found that the RMSE of the first signal is dominated by the variance, which is attributed to the inherently poor performance of subspace-based DOA estimators at low SNR, particularly for highly correlated signals.

5. CONCLUSIONS

Array interpolation can greatly simplify array signal processing for arbitrary planar arrays. Recently, we proposed a new formulation that effectively mitigates problems with the existing formulations where coherent out-of-sector signals interfere with the DOA estimation of the in-sector signals. It does so by taking into account the array response over the entire azimuth. In this paper, we propose a data-adaptive version, which significantly improves the DOA estimation performance of the original, non-adaptive counterpart. This is confirmed with numerical studies.

6. REFERENCES

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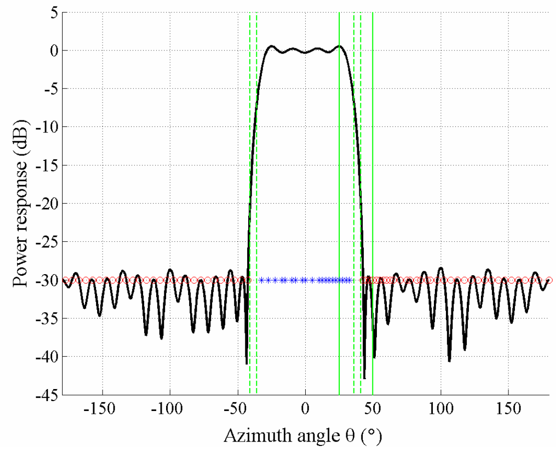

Fig. 2. Power response of the interpolated ULA for the Pesavento formulation for $N = 30$, $M = 11$, $r = 1.913\lambda$, 72°-sector, and Edges of 5° rolloff regions (vertical dashed lines); DOAs (vertical solid lines); Optimization grid points for in-sector (*) and out of sector (o).

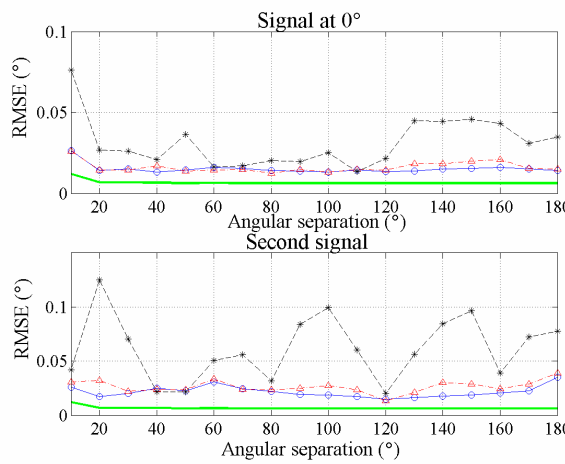

Fig. 3. RMSE of Root-MUSIC for first study. Eqn.(5) (-o-); Eqn(6) (-△-); $W(\theta) = 1$ (-*-); CRB (—).

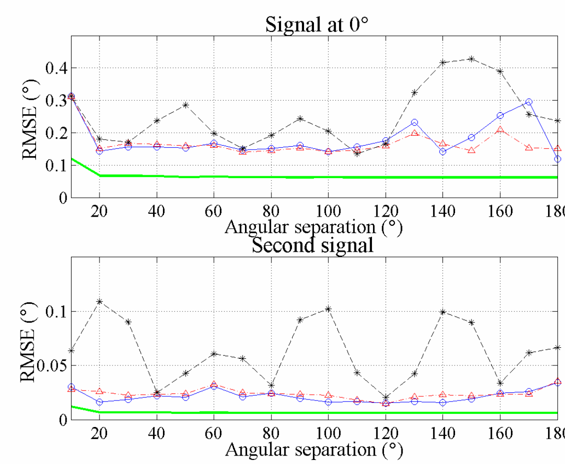

Fig. 4. RMSE of Root-MUSIC for second study. Eqn.(5) (-o-); Eqn(6) (-△-); $W(\theta) = 1$ (-*-); CRB (—).