# CYCLOSTATIONARITY BASED DOA ESTIMATION FOR WIDEBAND SOURCES WITH A CONJUGATE MINIMUM-REDUNDANCY LINEAR ARRAY

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**ABSTRACT**

Cyclostationarity based Direction of Arrival (DOA) estimation is of interest due to its immunity to interference and noise. Recently the use of cyclic methods with a conjugate Minimum-Redundancy Linear Array (MRLA) and an appropriate matrix augmentation technique was exploited by Gelli and Izzo to improve the performance of Cyclic MUSIC for narrowband signals. In this paper, we propose a cyclostationarity based wideband signal DOA estimation method with a conjugate MRLA. Exploiting the similarity of our problem to DOA estimation for coherent signals, we propose to utilize Forward/Backward Spatial Smoothing (FBSS) technique in our method instead of using the matrix augmentation technique. It is shown that our new method, in addition to generalizing the previous method to wideband signals, is able to detect more signals with improved performance than the previous method.

## 1. INTRODUCTION

Cyclic MUSIC proposed by Gardner [1] exploits cyclostationary property possessed by most man-made communication signals in Direction of Arrival (DOA) estimation for narrowband signals. Due to its effectiveness to combat interference and noise, much activities followed [1] and different cyclostationarity based DOA estimation methods were proposed. On the other hand, for the conventional correlation case, Minimum-Redundancy Linear Array (MRLA) [2] together with appropriate matrix augmentation technique have been shown to provide better performance than the simple Uniform Linear array (ULA) configuration [3]. This method can be easily applied to the cyclic correlation case. However, in some cases, signals may only exhibit conjugate cyclostationarity. Gelli and Izzo [4] showed that in this case, the MRLA configuration should be different and named it as a conjugate MRLA. Together with an appropriate matrix augmentation technique, this method provides better performance than Cyclic MUSIC for narrowband signals.

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In this paper, we propose a cyclostationarity based DOA estimation method with the same conjugate MRLA used in [4]. But by exploiting and averaging cyclic conjugate correlation over different time delays, our method is applicable to wideband signals. Furthermore, by exploiting a similarity of our problem to DOA estimation for coherent signals, we propose to utilize the Forward/Backward Spatial Smoothing (FBSS) technique [5] in our method instead of using the matrix augmentation technique. Using a conjugate MRLA with $P$ antennas, which is sum coarray equivalent [6] to a ULA with $N$ antennas ($P \leq N$), [4] is able to detect DOAs of at most $N-1$ narrowband signals with a same cycle frequency of interest. Our method, however, is able to detect DOAs of at most $\lfloor 2/3 * (2N-1) \rfloor$ wideband signals with a same cycle frequency of interest, where $\lfloor n \rfloor$ denotes the largest integer less than or equal to $n$. Simulation results show that the performance of DOA estimation is also enhanced using our method.

## 2. ARRAY CONFIGURATION AND ASSUMPTIONS

Consider a linear array $\mathcal{A}$ consisting of $P$ antennas. Let $d_p$ represent the distance of the $p$th antenna from the reference element, i.e., the first antenna, for $p = 1, \cdots, P$. Accordingly, we have $d_1 = 0$ and $d_1 < d_2 < \cdots < d_P$. Furthermore it is assumed that this linear array is derived from a ULA $\mathcal{U}_N$ containing $N$ elements with intersensor spacing $\Delta$ by removing some of its elements. Thus $d_p$ is an integer multiple of $\Delta$.

Two arrays are said to be sum coarray equivalent if $C(\mathcal{A}) = C(\mathcal{B})$, where $C(\mathcal{A})$ denotes the sum coarray [6] of the array $\mathcal{A}$, which is defined as the set

$$C(\mathcal{A}) = \{y \mid y = d_p + d_q, \quad p, q = 1, 2, \cdots, P\} \qquad (1)$$

Note that $C(\mathcal{A})$ is only dependent on the set of $d_p + d_q$. Since the sum of different pairs of $d_p$ and $d_q$ could be the same for a ULA, ULA is considered to be redundant. A conjugate MRLA is a linear array with careful selection of $d_p$ such that among all its sum equivalent coarrays, $P$ is minimized. This problem is referred to as a postage stamp problem [7]. Here we only list in Table 1 some results for $4 \leq P \leq 10$,

| $P$ | $\{d_p\}/\Delta$ | $N$ |
|---|---|---|
| 4 | $\{0,1,3,4\}$ | 5 |
| 5 | $\{0,1,3,5,6\}$ | 7 |
| 6 | $\{0,1,3,5,7,8\}$ | 9 |
| 7 | $\{0,1,2,5,8,9,10\}$ | 11 |
| 8 | $\{0,1,2,5,8,11,12,13\}$ | 14 |
| 9 | $\{0,1,2,5,8,11,14,15,16\}$ | 17 |
| 10 | $\{0,1,3,4,9,11,16,17,19,20\}$ | 21 |

**Table 1.** Conjugate MRLA Configurations

$\{d_p\}/\Delta$ and $N$, representing the number of antennas, the configuration of the conjugate MRLA, and the number of antennas of the sum equivalent ULA coarray, respectively.

The following assumptions are made throughout this paper:

A1) Signals of Interest (SOI) $s_i(t)$, for $i = 1,\cdots,I$ exhibits conjugate cyclostationarity at cycle frequency $\alpha$.

A2) $s_i(t)$ is impinging on a Conjugate MRLA from direction $\theta_i$, for $i = 1,\cdots,I$. The MRLA has $P$ elements and its sum equivalent ULA coarray has $N$ elements.

A3) $s_i(t)$ are mutually cyclically uncorrelated.

A4) Interference and noise either have different cycle frequencies from $\alpha$ or do not exhibit cyclostationarity.

A5) Interference and noise are not cyclically correlated with SOI.

## 3. EXISTING METHOD

Now let us take a brief look at the DOA estimation method in [4] for narrowband signals using the conjugate MRLA discussed above. Since all the SOI are assumed to be narrowband, the signal received at the $p$th antenna with distance $d_p$ from the first antenna can be written as

$$x_p(t) = \sum_{i=1}^{I} s_i(t) e^{j2\pi f_0 \frac{d_p}{c}\sin\theta_i} + n_p(t) \tag{2}$$

where $f_0$ is the carrier frequency, $c$ is the propagation speed, and $n_p(t)$ includes interference and noise. Here the time delay with respect to the first antenna $\frac{d_p}{c}\sin\theta_i$ is factored out and treated as a phase shift. The cyclic conjugate cross correlation of $x_p(t)$ and $x_q(t)$ is calculated as

$$r^{\alpha}_{x_p x_q^*}(\tau) = \langle x_p(t+\frac{\tau}{2}) x_q(t-\frac{\tau}{2}) e^{-j2\pi\alpha t}\rangle \tag{3}$$

where $\langle\cdot\rangle$ denotes time averaging. Substituting (2) into (3), we obtain

$$r^{\alpha}_{x_p x_q^*}(\tau) = \sum_{i=1}^{I} r^{\alpha}_{s_i s_i^*}(\tau) e^{j2\pi f_0 \frac{d_p+d_q}{\Delta}\frac{\Delta\sin\theta_i}{c}} \tag{4}$$

Refer to Table 1, for a $P$ element conjugate MRLA, $\frac{d_p+d_q}{\Delta}$ combined by different $p$ and $q$, for $p,q = 1,2,\cdots,P$, could take values of $0,1,\cdots,2N-2$. Thus the following Cyclic Conjugate Correlation Vector (CCCV) could be obtained

$$[y(1), y(2),\cdots, y(2N-1)]^T \tag{5}$$

where

$$y(n) = \sum_{i=1}^{I} r^{\alpha}_{s_i s_i^*}(\tau) e^{j2\pi f_0 (n-1)\frac{\Delta\sin\theta_i}{c}} \tag{6}$$

for $n = \frac{d_p+d_q}{\Delta} + 1 = 1,2,\cdots,2N-1$.

Now a symmetric augmented matrix $\mathbf{Y}$ could be constructed with $[y(i), y(i+1),\cdots, y(i+N-1)]$ as its $i$th row, for $i = 1,2,\cdots,N$. It could be shown that the steering vector $[1, e^{j2\pi f_0\Delta\sin\theta_i/c},\cdots, e^{j2\pi f_0(N-1)\Delta\sin\theta_i/c}]^T$ is lying in the signal subspace or orthogonal to the noise subspace of $\mathbf{Y}$. Thus DOA estimation could be performed similar to the MUSIC algorithm. (See detail in [4])

## 4. PROPOSED DOA ESTIMATION METHOD

The existing method assumes that the signals are narrowband and the received signal at the $p$th antenna can be written in (2). However, in our algorithm, we assume that the signals might be wideband, thus no phase could be factored out and the signal induced at the $p$th antenna is

$$x_p(t) = \sum_{i=1}^{I} s_i(t + \frac{d_p}{c}\sin\theta_i) + n_p(t) \tag{7}$$

The first step of our method is to construct an Averaged Cyclic Conjugate Correlation Vector (ACCCV) similar to CCCV. Substituting (7) into (3), we obtain the cyclic conjugate cross correlation of $x_p(t)$ and $x_q(t)$ as

$$\begin{aligned} r^{\alpha}_{x_p x_q^*}(\tau) &= \langle \sum_{i=1}^{I} s_i(t + \frac{d_p}{c}\sin\theta_i + \frac{\tau}{2}) \\ &\quad \cdot \sum_{i=1}^{I} s_i(t + \frac{d_q}{c}\sin\theta_i - \frac{\tau}{2}) e^{-j2\pi\alpha t}\rangle \\ &= \sum_{i=1}^{I} \langle s_i(t + \frac{d_p}{c}\sin\theta_i + \frac{\tau}{2}) \\ &\quad \cdot s_i(t + \frac{d_q}{c}\sin\theta_i - \frac{\tau}{2}) e^{-j2\pi\alpha t}\rangle \\ &= \sum_{i=1}^{I} r^{\alpha}_{s_i s_i^*}(\tau + \frac{d_p - d_q}{c}\sin\theta_i) e^{j2\pi\frac{\alpha}{2}\frac{d_p+d_q}{\Delta}\frac{\Delta\sin\theta_i}{c}} \end{aligned} \tag{8}$$

Note that in the first equality of (8), $n_p(t)$ and $n_q(t)$ are ignored due to the assumptions A4-A5, the second equality is due to A3, and the third equality applies the shift property of cyclic conjugate correlation, i.e., if $y(t)=x(t+T)$, then $r^{\alpha}_{yy^*}(\tau)=r^{\alpha}_{xx^*}(\tau)e^{j2\pi\alpha T}$.

First let us look at the factor $r^{\alpha}_{s_is_i^*}(\tau+\frac{d_p-d_q}{c}\sin\theta_i)$ in (8). Since our intention is to get a form similar to (4) for the narrowband case, it is desirable that this factor be independent of $d_p$ or $d_q$. We propose to evaluate (8) at different time delays $\tau$ and average them to obtain an averaged cyclic conjugate cross correlation denoted by $\langle r^{\alpha}_{x_px_q^*}\rangle_\tau$, where $\langle\cdot\rangle_\tau$ denotes averaging over $\tau$. $\langle r^{\alpha}_{x_px_q^*}\rangle_\tau$ could then be written as

$$\langle r^{\alpha}_{x_px_q^*}\rangle_\tau=\sum_{i=1}^{I}\langle r^{\alpha}_{s_is_i^*}\rangle_\tau e^{j2\pi\frac{\alpha}{2}\frac{d_p+d_q}{\Delta}\frac{\Delta\sin\theta_i}{c}} \tag{9}$$

Now similar to [4], we construct a vector

$$\mathbf{r}=[r(1),r(2),\cdots,r(2N-1)]^T \tag{10}$$

where

$$r(n)=\sum_{i=1}^{I}\langle r^{\alpha}_{s_is_i^*}\rangle_\tau e^{j2\pi\frac{\alpha}{2}(n-1)\frac{\Delta\sin\theta_i}{c}} \tag{11}$$

for $n=\frac{d_p+d_q}{\Delta}+1=1,2,\cdots,2N-1$. We name this vector $\mathbf{r}$ as Averaged Cyclic Conjugate Correlation Vector (ACCCV).

Since (10) and (5) are of the same form, to estimate $\theta_i$, we could also form an augmented matrix and follow the same steps as in [4], we will then be able to detect DOAs of $N-1$ wideband cyclostationary signals with cycle frequency $\alpha$. But in this paper, we propose to use another method to increase the number of detectable signals and to improve the performance of DOA estimation.

Define

$$\mathbf{a}_N(f,\theta_i)=\left[1,e^{j2\pi f\frac{\Delta\sin\theta_i}{c}},\cdots,e^{j2\pi f(N-1)\frac{\Delta\sin\theta_i}{c}}\right]^T \tag{12}$$

$$\mathbf{A}_N(f)=[\mathbf{a}_N(f,\theta_1),\cdots,\mathbf{a}_N(f,\theta_I)] \tag{13}$$

$$\mathbf{r}_s=\left[\langle r^{\alpha}_{s_1s_1^*}\rangle_\tau,\cdots,\langle r^{\alpha}_{s_Is_I^*}\rangle_\tau\right]^T \tag{14}$$

where $\mathbf{a}_N(f,\theta_i)$ is the steering vector evaluated at frequency $f$ and direction $\theta_i$ with $N$ elements, $\mathbf{A}_N(f)$ is the steering matrix containing the steering vectors, and $\mathbf{r}_s$ is the vector containing the averaged cyclic conjugate correlation of the sources. Then using (12)-(14), the ACCCV of (10), could be written as

$$\mathbf{r}=\mathbf{A}_{2N-1}(\frac{\alpha}{2})\mathbf{r}_s \tag{15}$$

Note that the steering vector is of size $2N-1$, and is evaluated at frequency $\alpha/2$.

Obviously $\mathbf{r}_s\mathbf{r}_s^H$ is of rank one. Thus, application of MUSIC to $\mathbf{r}\mathbf{r}^H$ will not yield correct estimate of $\theta_i$. This is analogous to the well-known problem in DOA estimation for narrowband coherent signals in which the coherence causes rank deficiency in the data correlation matrix. Therefore, the well known FBSS technique [5], which was originally developed to decorrelate the coherent narrowband signals for DOA estimation, can be utilized in our problem at hand.

Take the $l$th through $(l+M-1)$th elements of the ACCCV, $\mathbf{r}$ in (10), to construct the $l$th forward sub-vector

$$\mathbf{r}_l^f=[r(l),r(l+1),\cdots,r(l+M-1)]^T \tag{16}$$

Since the size of $\mathbf{r}$ is $2N-1$ and the size of the forward sub-vector (16) is $M$, the total number of such sub-vectors we can construct is $L=2N-M$. Using (12) to (14), $\mathbf{r}_l^f$ can be written as

$$\mathbf{r}_l^f=\mathbf{A}_M(\frac{\alpha}{2})\mathbf{D}^{l-1}\mathbf{r}_s \tag{17}$$

where $\mathbf{D}^{l-1}$ denotes the $(l-1)$th power of the $I$ by $I$ diagonal matrix

$$\mathbf{D}=diag\left[e^{j2\pi\frac{\alpha}{2}\frac{\Delta\sin\theta_1}{c}},\cdots,e^{j2\pi\frac{\alpha}{2}\frac{\Delta\sin\theta_I}{c}}\right] \tag{18}$$

Similarly, take the conjugate of $(2N-l)$th through $(2N-M-l+1)$th elements of the ACCCV to construct the $l$th backward sub-vector

$$\mathbf{r}_l^b=[r^*(2N-l),r^*(2N-l-1),\cdots,r^*(2N-l-M+1)]^T \tag{19}$$

The size of this backward sub-vector is also $M$, therefore the number of backward sub-vectors we can construct is also $L=2N-M$. Again, using (12) to (14), $\mathbf{r}_l^b$ can be written as

$$\mathbf{r}_l^b=\mathbf{A}_M(\frac{\alpha}{2})\mathbf{D}^{l-(2N-1)}\mathbf{r}_s^* \tag{20}$$

Now similar to the FBSS technique, we define a spatially smoothed "correlation" matrix as

$$\begin{aligned}\overline{\mathbf{R}}&=\frac{1}{2}\left(\frac{1}{L}\sum_{l=1}^{L}\mathbf{r}_l^f\left[\mathbf{r}_l^f\right]^H+\frac{1}{L}\sum_{l=1}^{L}\mathbf{r}_l^b\left[\mathbf{r}_l^b\right]^H\right)\\&=\mathbf{A}_M(\frac{\alpha}{2})\overline{\mathbf{R}}_s\mathbf{A}_M^H(\frac{\alpha}{2})\end{aligned} \tag{21}$$

where

$$\begin{aligned}\overline{\mathbf{R}}_s=&\frac{1}{2L}\sum_{l=1}^{L}\mathbf{D}^{l-1}\mathbf{r}_s\mathbf{r}_s^H\left[\mathbf{D}^{l-1}\right]^H+\\&\frac{1}{2L}\sum_{l=1}^{L}\mathbf{D}^{l-(2N-1)}\mathbf{r}_s^*\mathbf{r}_s^T\left[\mathbf{D}^{l-(2N-1)}\right]^H\end{aligned} \tag{22}$$

For $\overline{\mathbf{R}}_s$ to be full rank, we must have $2L\geq I$ (refer to [5] for more details). And to be able to detect $I$ signals, the number of elements of the forward and backward sub-vectors should be greater than $I$, or $M\geq I+1$. Since the total number of elements of $\mathbf{r}$ is $2N-1=L+M-1$, we have $2N-1\geq I/2+I+1-1$, or $I\leq 2/3*(2N-1)$. Thus Our method can detect DOAs of at most $\lfloor 2/3*(2N-1)\rfloor$ wideband cyclostationary signals with a same cycle frequency.

## 5. SIMULATION RESULTS

In this section, three computer simulations are performed to show the effectiveness of our proposed method. The signals used in these simulations are all wideband BPSK signals with baud rate 10 MHz, carrier frequency 20 MHz. The Signal to Noise Ratio (SNR) is assumed to be 15 dB. A conjugate MRLA of size $P = 4$ is used.

*Example 1:* In this example, we illustrate that the method in [4] can be extended to wideband signals by using ACCCV instead of CCCV. Two wideband BPSK signals are impinging on the array from directions of $20°$ and $50°$. The upper part of Fig.1 shows the result using the matrix augmentation technique on CCCV as in [4]. The lower part of Fig.1 shows the result using the matrix augmentation technique on ACCCV. We can see that our method works well for the wideband signals, while the method in [4] fails to detect the DOAs.

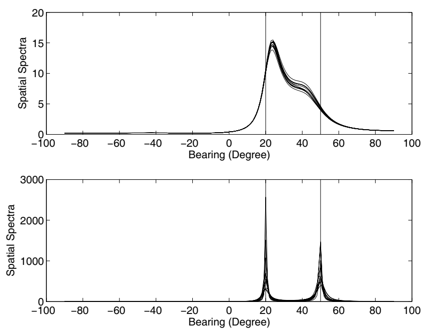


**Fig. 1.** Estimated spatial spectra: applying matrix augmentation technique on CCCV (upper) and ACCCV (lower)

*Example 2:* In this example, we illustrate that by applying the FBSS technique on ACCCV, the performance of DOA estimation is further improved, compared with that applying the matrix augmentation technique, in terms of the ability to separate two closely impinging DOAs. Two wideband BPSK signals are impinging closely from $20°$ and $25°$. As shown in Fig.2, the method using ACCCV and the FBSS technique separates the two DOAs from $20°$ and $25°$, but the other method using ACCCV and the matrix augmentation technique fails to detect the two DOAs.

*Example 3:* In this example, we illustrate that by applying the FBSS technique on ACCCV using a conjugate MRLA, DOAs of more sources than the virtual number of antennas $N$ can be detected. [4] stated that at most $N - 1$ DOAs of narrowband signals can be detected with a conjugate MRLA of size $P$ (Refer to Table 1). In this case $P$ is 4, the corresponding $N$ is 5, thus at most 4 DOAs can be detected. While our method can detect at most $\lfloor 2/3 * (2N - 1) \rfloor = 6$ DOAs of wideband signals. Now let us take 5 wideband signals from directions of $-60°$, $-30°$, $0°$, $25°$ and $55°$ for example. Fig.3 shows that our method detects these DOAs successfully.

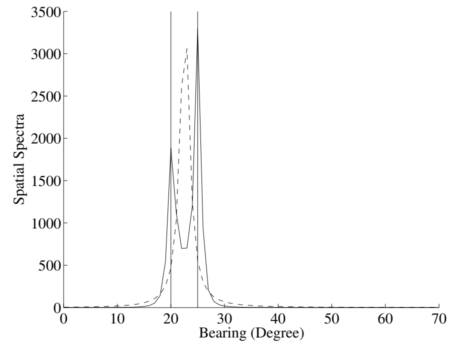


**Fig. 2.** Separation of two closely impinging DOAs ($20°$ and $25°$): $--$ applying matrix augmentation technique on ACCCV; — applying FBSS technique on ACCCV

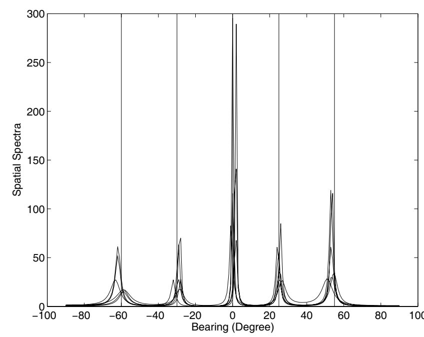


**Fig. 3.** 5 DOAs of wideband cyclostationary signals are detected using a conjugate MRLA of size 4 by our proposed method: apply FBSS technique on ACCCV

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