ENERGY EFFICIENCY OF MIMO TRANSMISSIONS IN WIRELESS SENSOR NETWORKS WITH DIVERSITY AND MULTIPLEXING GAINS

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ABSTRACT

The energy efficiency of MIMO transmissions in wireless sensor networks is analyzed considering the trade-off between diversity and multiplexing gains. Various MIMOs are studied with non-cooperative, half-cooperative or cooperative realizations. Energies consumed in transmission, processing circuitry and cooperation are obtained, which show that the optimal energy efficiency requires both the diversity and the multiplexing gains be exploited.

1. INTRODUCTION

In wireless sensor networks, one of the primary objectives is to enhance energy efficiency. To study energy efficiency, the energy consumed in both wireless transmissions and processing circuitry has to be considered. When cooperative communications are used, we also need to consider the extra energy consumed in cooperation.

If considering transmission energy only, shorter transmission range may be always better. But if circuitry energy consumption is also considered, then the optimal transmission range is not necessarily small, but rather should be determined by the energy consumption per hop and the number of hops. As a example, in the typical protocol LEACH [1], local short-range transmissions are used before data fusion whereas long-range transmissions are exploited for transmitting the fused data to reduce the number of hops.

Long-range transmissions have dominating energy consumption. This is especially so if the transmissions experience deep fading, which is the case for the on-ground deployed sensors. In order to improve the energy efficiency of long-range transmissions, one of the useful techniques is diversity. If sensors have antenna arrays, then space-time processing can be directly used. Otherwise cooperative transmissions can be exploited, with which sensors perform transmission and receiving cooperatively. Since space-time block codes (STBC) have linear complexity [2], cooperative STBC are attractive and have been widely studied [3, 4].

If both the transmitting side and the receiving side have either physical or virtual arrays, the system is multiple-input multiple-output (MIMO), which can be used for both diversity gain and spatial multiplexing gain. For wireless sensor networks, so far only the diversity gain has been exploited. The other, i.e., spatial multiplexing gain, is relatively less studied. The multiplexing gain can be realized by BLAST systems [5]. However, traditionally BLAST is used to achieve high rate with relatively high transmission power. Nevertheless, high rate means short transmission time, which may thus enhance energy efficiency, even with high transmission power. This is especially important when circuitry energy consumption is considered.

With MIMO transmissions, there is a fundamental trade-off between the diversity gain and the multiplexing gain [5]. Though the results are obtained under high signal-to-noise-ratio (SNR), they can still be useful for analyzing the energy efficiency and for comparing various MIMO transmission schemes in wireless sensor networks.

In this paper, based on the trade-off between the diversity gain and the multiplexing gain, we analyze the energy efficiency of MIMO transmissions in wireless sensor networks. In particular, the energy efficiency of some typical MIMO transmission schemes are compared. These schemes can be realized as non-cooperative (with physical antenna arrays), half-cooperative (with physical antenna array in receiver) or cooperative MIMOs. Our results will show that with proper design, the energy efficiency of MIMO transmissions can be higher than that of the traditional single-input single-output (SISO) transmissions. The optimal energy efficiency is usually achieved when both the diversity gain and the multiplexing gain are used, which means it is suboptimal if only the diversity gain is considered.

This paper is organized as follows. In Section 2, we give the MIMO transmission schemes in wireless sensor networks. Then, in Section 3 we compare the two gains and their effect on energy efficiency of non-cooperative MIMO. In Section 4, we show the energy efficiency of the half-cooperative and cooperative MIMOs. Simulations are conducted in Section 5 and conclusions are presented in Section 6.

2. MIMO TRANSMISSION SCHEMES

2.1. Sensor networks with physical or virtual arrays

In a typical wireless sensor network, data collected by sensors are transmitted to a remote data collector through multi-hop relaying. If sensors are equipped with antenna arrays, then MIMO transmissions can be directly applied. Otherwise, virtual antenna array can be formed by a cluster of sensors with cooperative communication techniques, as shown in Fig. 1. In this section, we describe the general cooperative MIMO transmissions, which include the non-cooperative and the half-cooperative ones as special cases.

Let us define the sensors which form a cooperative array as a cluster. We can choose a primary head in the cluster, and define all other cooperative sensors as secondary heads. In a cooperative

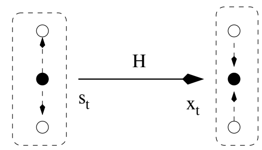


Fig. 1. Cooperative MIMO transmission.

transmitting array, the primary head holds the data to be transmitted. Before the long distance transmission, it first broadcasts its data to the secondary heads. Then at the next time slot, all the heads (both the primary and the secondary) perform cooperative transmissions. In contrast, in a cooperative receiving array, all the secondary heads forward their received signals to the primary head where the MIMO signal detection is performed. Such a transmission scheme can be efficiently implemented in the LEACH protocol with low overhead in synchronization and cooperation [4].

For the non-cooperative MIMO, the energy consumed includes the transmission energy and the circuitry energy. For the cooperative MIMO, the energy consumed as cooperation overhead needs to be considered. A special case is the half-cooperative MIMO where the receiver is the data collector which has physical antenna array and, more importantly, its energy efficiency is not a concern. For simplification, we assume that it is possible to achieve synchronization among the cooperating sensors with relatively low (and negligible) overhead [4].

2.2. MIMO signal model

Consider the MIMO system shown in Fig. 1. There are M_t transmitting antennas and M_r receiving antennas used to transmit and receive a sequence $\{b_t\}$. In general, certain space-time codes are applied before transmission so that the M_t antennas transmit $\mathbf{s}_t = [s_t(1), \cdots, s_t(M_t)]^T$, where $(\cdot)^T$ denotes transpose. We assume that $s_t(i)$ are i.i.d with zero mean and variance σ_s^2.

Let $\mathbf{H}$ be the $M_r \times M_t$ channel gain matrix whose elements are i.i.d complex circular symmetric Gaussian random variables with zero-mean and unit variance. The received signal by the M_r receiving antennas is $\mathbf{x}_t = [x_t(1), \cdots, x_t(M_r)]^T$, which equals

$$\mathbf{x}_t = \rho \mathbf{H} \mathbf{s}_t + \mathbf{v}_t, \tag{1}$$

where ρ is used to adjust transmission power, $\mathbf{v}_t$ is the corresponding AWGN, each of its elements has zero mean and variance σ_v^2. The received signal-to-noise ratio (SNR) at each antenna is $\text{SNR} = \rho^2 \sigma_s^2 M_t / \sigma_v^2$.

3. ENERGY EFFICIENCY OF NON-COOPERATIVE MIMO

3.1. Transmission energy efficiency

Define the diversity gain d_r and the multiplexing gain r as, respectively,

$$d_r = -\lim_{\text{SNR}\to\infty} \frac{\log P_e(\text{SNR})}{\log \text{SNR}}, \; r = \lim_{\text{SNR}\to\infty} \frac{R(\text{SNR})}{\log \text{SNR}}, \tag{2}$$

where P_e is the bit-error-rate (BER) and R is the transmission data rate. The optimal trade-off in [5] is obtained under sufficiently high SNR. For the STBC and BLAST, the trade-off curves are generally suboptimal. We list them as follows

$$d_r = \begin{cases} (1 - M_t - M_r + 2\lfloor r \rfloor) r + M_t M_r - \lfloor r \rfloor - \lfloor r \rfloor^2 \\ \qquad \text{(OPTIMAL)} \\ (\lfloor r \rfloor - M_r) r + (M_r - \lfloor r \rfloor)(M_r + \lfloor r \rfloor + 1)/2 \\ \qquad \text{(DBLAST)} \\ \max\{0,\ 1 - \frac{r}{M_r}\} \\ \qquad \text{(VBLAST)} \\ M_t M_r \max\{0,\ 1 - r\} \\ \qquad \text{(STBC)} \end{cases} \tag{3}$$

where $\lfloor r \rfloor$ denotes the maximum integer that is no larger than r. Note that for the OPTIMAL, we have $0 \leq r \leq \min\{M_t, M_r\}$. For the DBLAST and VBLAST, we have assumed $M_t = M_r$ for simplicity, and thus $0 \leq r \leq M_r$. The STBC is for $M_t = M_r = 2$ only, and thus $0 \leq r \leq 1$.

With the multiplexing gain r, the achievable data rate is

$$R = r \log \text{SNR}. \tag{4}$$

As a result, if the data rate R is independent of SNR, we have $r = 0$, i.e., there is no multiplexing gain.

Therefore, although the 2×2 STBC-MIMO with variable rate are optimized in [3], what was exploited is still the diversity gain, not the multiplexing gain, since the transmission rate is independent of SNR. This is the same for most existing cooperative STBC schemes developed for wireless sensor networks.

The diversity gain enhances the transmission energy efficiency by reducing the SNR required for certain BER, whereas the multiplexing gain enhances transmission energy efficiency by reducing the transmission time required for a data packet. The overall energy efficiency in terms of the two gains is mathematically involved. This problem is further complicated by the fact that d_r and r are defined under unknown coding and SNR$\to \infty$. However, similar to the examples in [5], we try an approximate analysis without considering the coding issue.

In order to analyze the transmission energy efficiency, we begin from a multiplexing gain r, and try to find the transmission energy required if a BER P_e should be met.

From r, we can obtain the diversity gain d_r through (3). Then, according to the Chernoff bound of BER, we have

$$\text{SNR} \approx P_e^{-\frac{1}{d_r}}. \tag{5}$$

Note that strictly speaking, the right hand side of (5) should have a multiplication factor M_t. But due to the approximating nature of the bound, as in [5], this factor is omitted.

Considering the transmission distance d, the SNR is related to the average transmission power P_t through

$$P_t = C d^n \rho^2 \sigma_s^2 M_t = C d^n \text{SNR} \sigma_v^2, \tag{6}$$

where the factor Cd^n denotes the large scale path loss with exponent n. From (5) and (6), we have the transmission power

$$P_t = C d^n \sigma_v^2 P_e^{-\frac{1}{d_r}}. \tag{7}$$

On the other hand, from (4), the transmission data rate becomes

$$R = -\frac{r}{d_r} \log P_e. \tag{8}$$

Assume the total data to be transmitted is N. The transmission energy is thus

$$J_t = P_t \frac{N}{R} = \frac{C d^n \sigma_v^2 N}{-\log P_e} \frac{d_r P_e^{-\frac{1}{d_r}}}{r}. \quad (9)$$

From (3) and (9), the impact of the two gains on transmission energy efficiency can be evaluated numerically.

3.2. Joint consideration of transmission and circuitry energies

In order to consider both the transmission energy and the circuitry energy, we use first-order energy models as [1, 4] for simplicity. The transmission energy is a linear function of d^n and transmission time N/R. The circuit energy is a linear function of transmission time, which can be written as $E_c N/R$, where E_c is a constant factor assumed identical for both the transmitter and the receiver. Note that the more complex analysis model in [3] gives in fact no drastically more accurate results than [1].

Let us consider the SISO transmission first in order to setup the transmission energy model by the parameters of Section 3.1. The SISO is equivalent to having $d_r = 1$ and $r = 0$ (but the rate $R_s \neq 0$). From (5), (7), (9), the SISO energy consumption in transmission and circuitry is easily found to be

$$\begin{aligned} J_s &= \sigma_v^2 \mathrm{SNR}_s C d^n N/R_s + 2E_c N/R_s \\ &\triangleq E_t \mathrm{SNR}_s d^n N/R_s + 2E_c N/R_s, \end{aligned} \quad (10)$$

where SNR_s denotes the SNR required in SISO, and $E_t \triangleq \sigma_v^2 C$. Note that in our case, $E_t\mathrm{SNR}_s$ is equivalent to the constant factor E_t in [1]. This separation of SNR_s from the constant factor is for the convenience of analyzing the MIMO transmission energy in the following.

For MIMO transmissions, based on (9), the transmission energy and circuitry energy consumption is

$$J_{tc} = K_{tc} E_t \frac{d_r d^n}{r} P_e^{-\frac{1}{d_r}} + K_{tc} E_c (M_t + M_r) \frac{d_r}{r}, \quad (11)$$

where $K_{tc} = N/(-\log P_e)$. Note that we have assumed that the circuit energy constant E_c does not change for various data rates (or symbol constellations) for simplicity.

Some special cases of $J_{tc}/K_{tc} \times 10^9$ are shown in Fig. 2 with $M_t = M_r = 2$, $P_e = 0.001$, $n = 2$ and $d = 10$ meters. $E_t\mathrm{SNR}_s = 100pJ$ and $E_c = 50nJ$, which are obtained from [1]. For comparison purpose, we have also shown the energy of SISO transmission (a point marked with □ since it exists only for $r = 0$). From the SISO BER $P_e = (1 - \sqrt{\mathrm{SNR}/(1+\mathrm{SNR})})/2$, we can calculate the SNR_s for certain P_e. In this case, $\mathrm{SNR}_s = 249$. In addition, we choose the data rate $R_s = 0.05$ for all $r = 0$ cases. This is a reasonable choice considering that R_s is fixed under various SNR. This value gives a smooth change in the figure toward $r = 0$. More important, the energy difference at $r = 0$ for SISO and other schemes meets well the difference of their diversity gains [4].

From Fig. 2, we see that even when d is small, MIMO transmissions can still be more energy efficient than SISO. However, when only diversity gain is considered, i.e., $r = 0$, then SISO becomes more energy efficient, which is the same as the conclusions obtained elsewhere [3, 4]. But in our case, the multiplexing gain can effectively reduce the transmission time, and hence enhance energy efficiency.

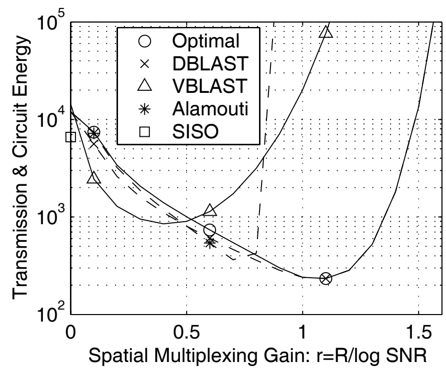


Fig. 2. Non-cooperative MIMO energy efficiency ($J_{tc}/K_{tc} \times 10^9$).

When d becomes larger, then the transmission energy dominates the energy consumption, and MIMO schemes can have even higher energy efficiency than SISO. The minimum transmission energies for MIMOs all come with non-zero r and non-zero d_r.

4. ENERGY EFFICIENCY OF COOPERATIVE MIMO

When either cooperative or half-cooperative MIMOs are used, then in addition to the transmission and circuitry energy consumption, we need to consider the energy consumption due to the cooperation overhead. Similarly to [4], the major cooperation procedure can be briefly outlined as follows.

Step 1: When the primary head needs to transmit a data packet to the next hop, it first chooses $M_t - 1$ secondary heads to assist it to do the long distance transmission. The overhead of this step is small, and can be skipped. Let the long transmission distance be d, and the local transmission distance among the heads be d_ℓ. Although local distances may be different between different heads, we assume they are same for simplicity.

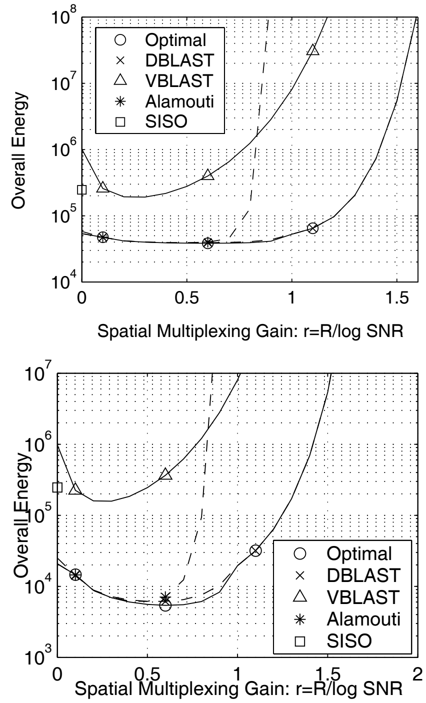
Step 2: The primary head broadcasts to all the secondary heads its data (N bits). Some additional information needs also to be embedded in order for the primary head to tell the second heads their roles in cooperative transmissions. But such an overhead can be neglected. Instead, the major overhead is the broadcasting of the N data bits, during which the total energy consumption is (*c.f.*,(10))

$$J_1 = \frac{N}{R_s N_{\ell_1}} E_t \mathrm{SNR}_s d_\ell^n + \frac{N}{R_s N_{\ell_1}} E_c + (M_t - 1) \frac{N}{R_s N_{\ell_1}} E_c. \quad (12)$$

We have made some assumptions in (12). First, the transmission from the primary head to the secondary heads is SISO, and thus has the same data rate R_s. In addition, the symbol alphabet may be different from that of the long distance SISO transmission, and larger alphabet size can be used to reduce transmission time. This is described by the parameter $N_{\ell_1} \geq 1$. Finally, the required SNR is still SNR_s as in the long distance SISO transmission for simplicity. Note that since SISOs all have diversity 1, their SNRs are approximately equivalent to each other in terms of Chernoff bound.

Step 3: The primary and secondary heads perform cooperative transmission with the same energy consumption as J_{tc} in (11).

Step 4: In the receiving cluster, the $M_r - 1$ secondary heads first quantize their samples and then transmit them as new symbol sequences to the primary head, where MIMO receiving is per-



(a) (b)

Fig. 3. (a) Cooperative MIMO energy ($J_a/K_{tc} \times 10^9$). (b) Half-cooperative MIMO energy ($J_h/K_{tc} \times 10^9$).

formed to recover the original N bits. The energy consumption in this step is

$$J_2 = (M_r - 1)\frac{N}{R_s} N_{\ell_2} E_t \mathrm{SNR}_s d_\ell^n + 2(M_r - 1)\frac{N}{R_s} N_{\ell_2} E_c. \quad (13)$$

Similarly as (12), we use R_s and SNR_s. N_{ℓ_2} is a combination result of quantization and symbol mapping.

Summing all the energy, we have the overall energy consumption of the cooperative MIMO transmissions,

$$J_a = J_1 + J_{tc} + J_2. \quad (14)$$

Based on this, we can optimize the cooperative MIMO energy efficiency as $\arg\min_{\{M_t, M_r, r, P_e\}} J_a$.

On the other hand, for half-cooperative MIMO, the overall energy consumption in concern is

$$J_h = J_1 + J_{tc}. \quad (15)$$

For illustration, we use $N_{\ell_1} = 2$, $N_{\ell_2} = 5$, $d_\ell = 10$, and $d = 100$. Other parameters are the same as those for drawing Fig. 2. We sketch the energies (normalized with K_{tc}) in Fig. 3. Compared with Fig. 2, the cooperation overhead increases the overall energy consumption and flattens the energy efficiency of MIMO transmissions. However, MIMO can still be more energy efficient than SISO. Comparing Fig. 3(a) and (b), we find that the receiver overhead J_2 has more significant reduction of energy efficiency because of the larger N_{ℓ_2} used in Fig. 3(a).

5. SIMULATIONS

In this section, we simulate the STBC with $M_t = M_r = 2$ to verify the trade-off between the multiplexing gain and the diversity gain, as well as their impact on transmission energy efficiency. The various r are obtained based on the assumption that the QAM symbol constellation has a size SNR^r for certain P_e [5]. For each QAM symbol constellation, we find the SNR required for P_e by Monte-Carlo simulations, then r can be calculated. In addition, the SNR is used to calculate the transmission energy J_t (6),(9).

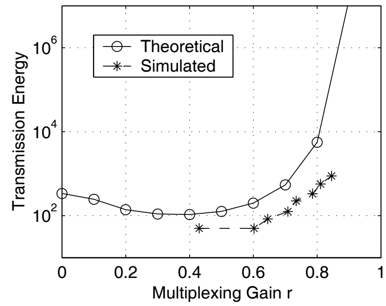


Fig. 4. Compare the simulated transmission energy consumption with the theoretical values J_t.

The simulation results are shown in Fig. 4. We compare the simulated values with those obtained theoretically in (9). As can be seen, the two sets of values fit well. The difference between them may be due to the fact that the theory values are obtained under high SNR with Chernoff upper bound, which usually gives higher SNR (and thus higher energy consumption).

6. CONCLUSIONS

In this paper, we analyze the MIMO transmission energy efficiency in wireless sensor networks when both the diversity gain and the multiplexing gain are considered. The trade-off between these two gains is also reflected in the energy consumptions. In order to obtain optimal energy efficiency, both diversity and multiplexing gains have to be exploited.

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