# ON THE IMPACT OF MULTI-ANTENNA RF TRANSCEIVERS' AMPLITUDE AND PHASE MISMATCHES ON TRANSMIT MRC

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**ABSTRACT**

Transmit Maximum-Ratio Combining (transmit MRC) is a popular antenna diversity technique that provides both spatial diversity and array gain in downlink Multiple-Input Single-Output (MISO) links. These gains, however, critically depend on the availability of the downlink Channel State Information (CSI). In time-division duplexing systems, channel reciprocity has been commonly put forth to justify the convenient use of the CSI already acquired from the uplink, in the calculation of the transmit-MRC weights. Recent work has questioned this practice, based on the non-reciprocity of multi-antenna RF transceivers, due to significant amplitude and phase mismatches across the antennas. Furthermore, expensive digital calibration solutions have been proposed to enforce the reciprocity of the multi-antenna RF transceivers. Both the impact of multi-antenna amplitude and phase mismatches and the performance of the proposed calibration approaches have only been assessed via simulations. In this contribution, we propose an alternative statistical analysis of the impact of these mismatches on transmit MRC. This analysis allows a faster and more reliable characterization as well as provides insight into the relative importance of these mismatches. Consequently, sufficient matching requirements can be extracted for the multi-antenna RF transceivers, for which simpler and cheaper calibration solutions can be devised.

## 1. INTRODUCTION

Transmit Maximum-Ratio Combining (transmit MRC) is a simple yet powerful antenna diversity technique that provides both spatial diversity and array gain [1]. It is particularly attractive in Multiple-Input Single-Output (MISO) downlink scenarios, where the multiple-antenna basestation would optimally weigh the transmit data stream across its antennas, such that channel filtering leads to maximum-SNR coherent reception at the single-antenna user terminal. Assuming $M_T$ uncorrelated transmit antennas at the basestation, it is well-known that transmit MRC achieves full spatial diversity as well as $M_T$-fold SNR gain [1] [2, p. 95]. However, the calculation of the transmit MRC weights requires knowledge of the downlink Channel State Information (CSI).

For Time-Division Duplexing (TDD) systems, state-of-the-art contributions commonly assume channel reciprocity as long as the round-trip delay is shorter than the coherence time of the channel. Consequently, the CSI estimated during the uplink is used for the calculation of the transmit MRC weights in the downlink. Even though the propagation channel is reciprocal, recent work [6, 7, 8, 9] has highlighted that it is certainly not the case for the RF transceivers, which may exhibit significant amplitude and phase mismatches between the uplink and the downlink as well as across the basestation antennas. These mismatches essentially compromise the correct calculation of the transmit MRC weights and may lead to severe performance degradation.

In order to mitigate the multi-antenna transceivers' mismatches problem, several digital calibration techniques have been proposed that follow one of two approaches. The first approach essentially measures, via additional RF calibration hardware, the actual multi-antenna transmit and receive front-ends mismatches and compensates for them digitally [6, 8, 10]. The second approach consists of a blind adaptive calibration algorithm [7]. Both the impact of the mismatches and the performance of the proposed calibration techniques have been assessed only via simulations. In this contribution, we propose a statistical analysis of the impact of multi-antenna transceivers amplitude and phase mismatches for transmit MRC. This analytical approach allows both simpler and more reliable evaluation of the impact of each of the mismatches as well as insight into their relative importance. Based on that, sufficient multi-antenna RF front-ends matching requirements can be extracted, which would balance performance degradation and calibration hardware complexity.

The rest of the paper is organized as follows: Section 2 introduces the data and multi-antenna RF transceivers' amplitude and phase mismatches models. Based on that, we analytically evaluate the impact of the mismatches on the performance of transmit MRC in Section 3. In Section 4, simulation results are provided that validate the proposed analysis, for flat-fading channels. Finally, we draw some conclusions in Section 5. *Notations:* In all the following, normal letters designate scalar quantities, boldface lower-case letters indicate column vectors and boldface capitals represent matrices. Finally, $||\mathbf{m}||_2$ stands for the 2-norm of $\mathbf{m}$.

## 2. SYSTEM MODEL

### 2.1. Data model

The transmit-MRC wireless communication system, under consideration, is depicted in Figure 1. It consists of a basestation equipped with $M_T$ antennas and a single-antenna user terminal. At sampling instant $k$, the input symbol stream $s(k)$ is multiplied by the transmit precoder $\mathbf{w}$ prior to transmission through the $M_T$

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transmit front-ends. At the user terminal, the output of the receive front-end is denoted $r(k)$. Further equalization would lead to the detection of the symbol stream $\hat{s}(k)$. However, we are only interested in $r(k)$. The single-tap equalization would neither alter the SNR nor the results of the proposed analysis. We consider flat-

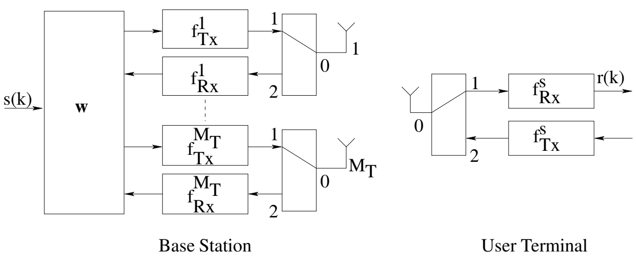


**Fig. 1**. *The transmit-MRC MISO system including the RF transceiver responses*

fading channels. Nevertheless, the analysis and the results would also apply to frequency-selective channels, provided that multicarrier modulation is used to convert them into multiple flat-fading subcarriers. The system model for flat-fading channels reads:

$$r(k) = f^{s}_{Rx} \cdot \left[h^1 f^1_{Tx} \; \cdots \; h^{M_T} f^{M_T}_{Tx}\right] \cdot \mathbf{w} \cdot s(k) + f^{s}_{Rx} n(k) \quad (1)$$

where $n(k)$ is the receiver noise at the user terminal, at discrete-time index $k$. $f^s_{Tx}$ and $f^s_{Rx}$ represent the complex baseband equivalent responses of the transmit and receive front-end at the user terminal, respectively. The composite channel coefficient $h^i f^i_{Tx}$ stands for the concatenation of the baseband equivalent propagation channel $h^i$ and the baseband equivalent response of the transmit front-end $f^i_{Tx}$, corresponding to the basestation's $i^{th}$ antenna. Finally, $f^i_{Rx}$ similarly denotes the receive front-end response at the basestation's $i^{th}$ antenna. In all the following, the discrete-time index $k$ is dropped for notational brevity.

The idealized transmit precoder $\mathbf{w}$, which overlooks the RF front-ends contributions, is based on the uplink CSI, $\mathbf{CSI}_{uplink} = f^s_{Tx}\left[h^1 f^1_{Rx} \; \cdots \; h^{M_T} f^{M_T}_{Rx}\right]$. It is defined by $\mathbf{w} = \mathbf{CSI}^H_{uplink}/||\mathbf{CSI}_{uplink}||_2$. Consequently, the received signal of (1) can be explicitly re-written as:

$$r = \underbrace{\frac{f^s_{Rx} f^{s^*}_{Tx}}{|f^s_{Tx}|}}_{\text{user terminal related}} \cdot \frac{\Sigma_{i=1}^{M_T} |h^i|^2 f^i_{Tx} f^{i^*}_{Rx}}{\sqrt{\Sigma_{i=1}^{M_T} |h^i f^i_{Rx}|^2}} \cdot s + f^s_{Rx} n \quad (2)$$

The corresponding Signal-to-Noise Ratio (SNR) is given by:

$$SNR = \frac{\left|\Sigma_{i=1}^{M_T} |h^i|^2 f^i_{Tx} f^{i^*}_{Rx}\right|^2}{\Sigma_{i=1}^{M_T} |h^i f^i_{Rx}|^2} \cdot \frac{E_s}{\sigma_n^2} \quad (3)$$

where $E_s/\sigma_n^2$ is the average transmit power over the receiver noise power. It would also correspond to the average receive SNR for a Single-Input Single-Output (SISO) system with the same average transmit power. Clearly, the user terminal related coefficient in (2) does not alter the performance of transmit MRC. Consequently, the user terminal front-end will be omitted in the subsequent analysis. On the other hand, (3) shows that the amplitude and phase mismatches in the multi-antenna basestation transceivers disturb the response of the idealized transmit MRC. This ideal response is given by $SNR_{ideal} = \Sigma_{i=1}^{M_T} |h^i|^2 E_s/\sigma_n^2$ [2].

### 2.2. Multi-antenna transceivers' amplitude and phase mismatches model

An ideal front-end has a baseband equivalent response of unit-amplitude and zero-phase. Around this ideal response, we model the mismatches in the responses of the transmit and receive front-ends using the simple linear model, which represents such complex gains as $f = |f|e^{j arg(f)}$ where:

- the amplitude $|f|$ is a real Gaussian variable of unit-mean and variance $\sigma^2$.
- the angle $arg(f)$ is uniformly distributed in the range $[-\Phi, \Phi]$.

The parameters $\sigma^2$ and $\Phi$ reflect how well the branches of the multi-antenna transmit/receive front-end are matched. These parameters may be different for the transmit and receive paths. While the Gaussian model is commonly used to model RF amplitude errors, it is assumed that the variance $\sigma^2$ is small (up to around 40%), such that the occurence of negative realizations is negligible.

## 3. ANALYSIS OF THE IMPACT OF MULTI-ANTENNA RF FRONT-ENDS' AMPLITUDE AND PHASE MISMATCHES

To gain insight into their respective contributions to the degradation of the system performance, we evaluate the impact of each mismatch separately. The performance degradation is measured in terms of the SNR loss with respect to the ideal transmit-MRC response, $R = SNR/SNR_{ideal}$. $R$ is in linear units.

### 3.1. Transmit amplitude mismatch only

This scenario arises when the $M_T$ receive front-ends are ideal; $\{f^i_{Rx}\}_{1 \le i \le M_T} = 1$, and the $M_T$ transmit front-end phases are equal to zero; $\{arg(f^i_{Tx})\}_{1 \le i \le M_T} = 0$. The resulting SNR loss, $R$ is given by:

$$R = \left(\frac{\Sigma_{i=1}^{M_T} |h^i|^2 |f^i_{Tx}|}{\Sigma_{i=1}^{M_T} |h^i|^2}\right)^2 \quad (4)$$

Based on our model, the various transmit amplitude mismatches $\{|f^i_{Tx}|\}_i$ are i.i.d Gaussian variables of unit-mean and variance $\sigma^2_{||}$ i.e $|f^i_{Tx}| \sim \mathcal{N}(1, \sigma^2_{||})$. This model, however, would artificially lead to an increase of the average transmit power by a factor $(1+\sigma^2_{||})$, which is basically the mean of $|f^i_{Tx}|^2$. Consequently, the transmit amplitude mismatches must be normalized to ensure that the average transmit power is $E_s$. Thus, the transmit amplitude mismatch should rather be modelled as $|f_{Tx}| \sim \mathcal{N}\left(1/\sqrt{1+\sigma^2_{||}}, \sigma^2_{||}/(1+\sigma^2_{||})\right)$.

Being a sum of scaled versions of independent Gaussian variables, the numerator of (4) is also Gaussian distributed as $\mathcal{N}\left((\Sigma_{i=1}^{M_T}|h^i|^2)/\sqrt{1+\sigma^2_{||}}, (\Sigma_{i=1}^{M_T}|h^i|^4)\sigma^2_{||}/(1+\sigma^2_{||})\right)$. The division by the denominator, $\Sigma_{i=1}^{M_T}|h^i|^2$, leads to a ratio that is Gaussian distributed:

$$\sqrt{R} \sim \mathcal{N}\left(\mu = \frac{1}{\sqrt{1+\sigma^2_{||}}}, \sigma^2 = \frac{\sigma^2_{||}}{1+\sigma^2_{||}} \frac{\Sigma_{i=1}^{M_T}|h^i|^4}{(\Sigma_{i=1}^{M_T}|h^i|^2)^2}\right) \quad (5)$$

Finally, as the square of a non-central Gaussian variable, $R$ follows a non-central Chi-square distribution with mean $E\{R\} = \mu^2 + \sigma^2$ and variance $Var\{R\} = 4\mu^2\sigma^2 + 2\sigma^4$:

$$R \sim \mathcal{X}_{1,1}\left(\mu^2+\sigma^2, 4\mu^2\sigma^2 + 2\sigma^4\right) \quad (6)$$

### 3.2. Transmit phase mismatch only

This scenario occurs when the $M_T$ receive front-end are ideal; $\{f^i_{\text{Rx}}\}_{1\le i\le M_T} = 1$, and the $M_T$ transmit front-end amplitudes are equal to one; $\{|f^i_{\text{Tx}}|\}_{1\le i\le M_T} = 1$. The corresponding SNR loss, $R$, reads:

$$R = \left| \frac{\Sigma_{i=1}^{M_T} |h^i|^2 e^{jarg(f^i_{\text{Tx}})}}{\Sigma_{i=1}^{M_T} |h^i|^2} \right|^2 \tag{7}$$

To identify of the statistics of $R$, we substitute $e^{jarg(f^i_{\text{Tx}})} = cos[arg(f^i_{\text{Tx}})] + jsin[arg(f^i_{\text{Tx}})]$ and develop (7) into:

$$R = 1 + \frac{2\Sigma_{i\neq j}|h^i h^j|^2}{(\Sigma_{i=1}^{M_T}|h^i|^2)^2}(Y_{i,j} - 1) \tag{8}$$

where $Y_{i,j} = cos[arg(f^i_{\text{Tx}}) - arg(f^j_{\text{Tx}})]$. We further introduce the set of random variables $\{Z_i = cos[arg(f^i_{\text{Tx}})]\}_{1\le i\le M_T}$. This choice is motivated by the fact that the joint distribution of $\{Z_i\}_i$, contrarily to that of $\{Y_{i,j}\}_{i,j}$, is easily related to that of $\{arg(f^i_{\text{Tx}})\}_i$, as follows:

$$f_{\{Z_i\}_i}(\{z_i\}_i) = \frac{1}{\Phi^{M_T}} \frac{1}{\Pi_{i=1}^{M_T}\sqrt{1-z_i^2}},\ \{z_i\}_i \in [cos\Phi, 1] \tag{9}$$

Denoting sign$[arg(f^i_{\text{Tx}})]$ by sign$^i$, the desired variable $Y_{i,j}$ can be re-written in terms of $\{Z_i\}_i$:

$$Y_{i,j} = \begin{cases} Z_i Z_j + \sqrt{1-Z_i^2}\sqrt{1-Z_j^2}, & \text{sign}^i = \text{sign}^j \\ Z_i Z_j - \sqrt{1-Z_i^2}\sqrt{1-Z_j^2}, & \text{sign}^i \neq \text{sign}^j \end{cases} \tag{10}$$

Exploiting (9) and (10), the expected value of the SNR loss, $R$, in (8), was found to be:

$$E\{R\} = 1 + \frac{(\Sigma_{i=1}^{M_T}|h^i|^2)^2 - \Sigma_{i=1}^{M_T}|h^i|^4}{(\Sigma_{i=1}^{M_T}|h^i|^2)^2}\left[\frac{sin^2\Phi}{\Phi^2} - 1\right] \tag{11}$$

Similarly, (9) and (10) are used to determine the variance of $R$. Due to the lack of space, we only provide the expression for the case of $M_T = 4$, which is the value used in our simulations:

$$Var\{R\} = 4\left[\frac{1}{2}\left(\frac{sin^2 2\Phi}{(2\Phi)^2} + 1\right) - \frac{sin^4\Phi}{\Phi^4}\right]\frac{\Sigma_{i\neq j}|h^i|^4|h^j|^4}{(\Sigma_{i=1}^4|h^i|^2)^4}$$
$$4\left[\frac{(\Phi + sin\Phi cos\Phi)\, sin^2\Phi}{2\Phi^3} - \frac{sin^4\Phi}{\Phi^4}\right]\cdot$$
$$\frac{\left(\Sigma_{i\neq j}|h^i h^j|^2\right)^2 - \Sigma_{i\neq j}|h^i h^j|^4 - 6|h^1h^2h^3h^4|^2}{(\Sigma_{i=1}^4|h^i|^2)^4} \tag{12}$$

Nevertheless, a similar approach can be used to evaluate $R$'s variance for an arbitrary $M_T$, provided all the cross-correlation terms, $E\{Y_{i,j}Y_{k,l}\}_{(i\neq k)\&(k\neq l)}$, in the covariance are accounted for.

### 3.3. Receive amplitude mismatch only

This scenario depicts the case when the $M_T$ transmit RF chains are ideal; $\{f^i_{\text{Tx}}\}_{1\le i\le M_T} = 1$, and the $M_T$ receive front-ends's phases are equal to zero; $\{arg(f^i_{\text{Rx}})\}_{1\le i\le M_T} = 0$. The SNR loss, $R$, is now expressed as:

$$R = \frac{1}{\Sigma_{i=1}^{M_T}|h^i|^2}\left(\Sigma_{i=1}^{M_T}|h^i|\frac{|h^i||f^i_{\text{Rx}}|}{\sqrt{\Sigma_{i=1}^{M_T}|h^i f^i_{\text{Rx}}|^2}}\right)^2 \tag{13}$$

We note that each $x_i = |h^i||f^i_{\text{Rx}}|$ is Gaussian distributed as $\mathcal{N}(\mu_i = |h^i|, var_i = \sigma^2_{||}|h^i|^2)$. Furthermore, $\{x_i\}_{1\le i\le M_T}$ are statistically independent. Thus, their joint pdf is simply given by $p(\{x_i\}_i) = e^{-\Sigma_{i=1}^{M_T}(x_i-\mu_i)^2/2var_i}/\sqrt{2\pi\Pi_{i=1}^{M_T}var_i}$. The mean as well as the variance of $R$ can then be determined by evaluating two $M_T$-tuple infinite integrals over $\{x_i\}_{1\le i\le M_T}$. To ensure both the convergence and ease of the numerical integration, we convert them to finite ones. This is achieved by making the simple but key observation that the $M_T$-dimensional vector $Y = [x_1 \cdots x_{M_T}]^T/\sqrt{\Sigma_{i=1}^{M_T}x_i^2}$ lies on an $M_T$-dimensional unit hypersphere. Consequently, it can be represented using the $M_T$-dimensional spherical coordinates $(r, \phi_1, \cdots, \phi_{M_T-1})$, whose pdf can be simply related to that of $\{x_i\}_i$, as follows [5]:

$$\begin{cases} p(r, \{\phi_k\}_k) = r^{M_T-1}\Pi_{i=2}^{M_T} sin^{M_T-i}\phi_{M_T-i+1}\cdot p(\{x_i\}_i) \\ r \in [0, +\infty[\quad \phi_1 \in [0, 2\pi[\quad \{\phi_i\}_{2\le i\le M_T-1} \in [0,\pi] \end{cases} \tag{14}$$

Since $Y$ lies on an $M_T$-dimensional unit sphere, it is independent of the radius $r$ and is only parametrized by the angles $\{\phi_k\}_k$. Therefore, we only need the joint distribution of $\{\phi_k\}_k$, which is obtained by the integration of (14) with respect to $r$. The desired pdf was found to be:

$$p(\{\phi_i\}_i) = \frac{c}{16a^{7/2}}\Big[2\sqrt{a}(b^2+4a) + b(6a+b^2)e^{\frac{b^2}{4a}}\sqrt{\pi}(1 - erf[\frac{-b}{2\sqrt{a}}])\Big] \tag{15}$$

where $a$, $b$ and $c$ are given by:

$$\begin{aligned} a &= \frac{cos^2\phi_{M_T-1}}{2var_{M_T}} + \Sigma_{i=1}^{M_T-1}\frac{\Pi_{k=1}^{M_T-1}sin^2\phi_k cos\phi_{i-1}}{2var_i} \\ b &= \frac{\mu_{M_T}cos\phi_{M_T-1}}{var_{M_T}} + \Sigma_{i=1}^{M_T-1}\frac{\mu_i\Pi_{k=1}^{M_T-1}sin\phi_k cos\phi_{i-1}}{var_i} \\ c &= \frac{\Pi_{i=2}^{M_T}sin^{M_T-i}\phi_{M_T-i+1}}{(\sqrt{2\pi})^{M_T}\Pi_{i=1}^{M_T}\sqrt{var_i}}\cdot e^{-\Sigma_{i=1}^{M_T}\frac{\mu_i^2}{2var_i}} \end{aligned}$$

Finally, re-formulating the SNR loss $R$ of (13), in terms of the $M_T$-dimensional spherical coordinates:

$$R = \frac{(\mu_{M_T}cos\phi_{M_T-1} + \Sigma_{i=1}^{M_T-1}\mu_i\Pi_{k=1}^{M_T-1}sin\phi_k cos\phi_{i-1})^2}{\Sigma_{i=1}^{M_T}\mu_i^2}$$

It is clear that the pdf of (15) will enable us to calculate the expected value as well as the variance of $R$, by evaluating $(M_T-1)$-tuple finite integrals with respect to $\{\phi_i\}_{1\le i\le M_T-1}$.

### 3.4. Receive phase mismatch only

This scenario corresponds to the case where the $M_T$ transmit front-ends are ideal; $\{f^i_{\text{Tx}}\}_{1\le i\le M_T} = 1$, and the $M_T$ receive front-ends' amplitudes are equal to one; $\{|f^i_{\text{Rx}}|\}_{1\le i\le M_T} = 1$. The SNR loss, $R$, is given by:

$$R = \left|\frac{\Sigma_{i=1}^{M_T}|h^i|^2 e^{-jarg(f^i_{\text{Rx}})}}{\Sigma_{i=1}^{M_T}|h^i|^2}\right|^2 \tag{16}$$

Recalling that the phase mismatches $arg(f_{\text{Tx}})$ and $arg(f_{\text{Rx}})$ follow the same distribution, which is symmetrical around zero, (16) and (7) are basically equivalent. More importantly, all the results on the characterization of the statistics of the SNR loss, $R$, obtained for transmit phase mismatch hold here as well, provided that the value of $\Phi$ is adjusted to that of the receive front-ends.

## 4. VALIDATION OF THE PROPOSED ANALYSIS VIA SIMULATIONS

In this section, the earlier-derived expressions for the mean and variance of the SNR loss $R$, corresponding to the different multi-antenna RF front-ends' mismatches, are first verified via simulations for an $4 \times 1$ flat-fading MISO system. The relative importance of these mismatches is then discussed. Figure 2, 3 and 4 illustrate, for a single realization of the flat-fading channel, the perfect agreement between the analytically-evaluated means and variances and their simulated counterparts for transmit amplitude mismatch, transmit phase mismatch and receive amplitude mismatch respectively. These plots also provide the results averaged over 100 channel realizations. These Figures further confirm that

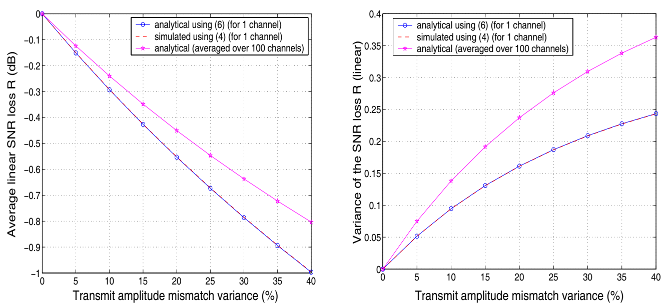


**Fig. 2**. *The average (in dB) and the variance (in linear units) of the SNR loss $R$ in the presence of transmit amplitude mismatches*

these mismatches can severely degrade the transmit MRC performance and may even annihilate most of its potentional SNR/array gain. However, the different effects are not equally detrimental. Indeed, phase mismatch appears to be the most detrimental multi-antenna RF front-ends' mismatch. This is understandable since it is phase mismatches that induce destructive combining at the receiving user terminal. The amplitude mismatches lead to a lesser degradation since they only appear as real weights that disturb the maximum-ratio property of the combining. Furthermore, receive amplitude mismatch is shown to be less harmful than its transmit counterpart. This is because it is attenuated by the normalization in the calculation of the transmit MRC weights, as shown in (13).

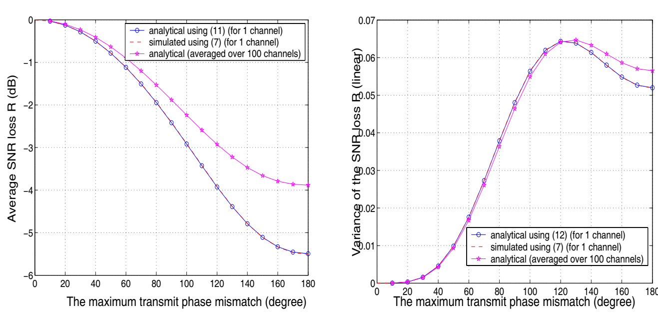


**Fig. 3**. *The average (in dB) and the variance (in linear units) of the SNR loss $R$ in the presence of transmit/receive phase mismatches*

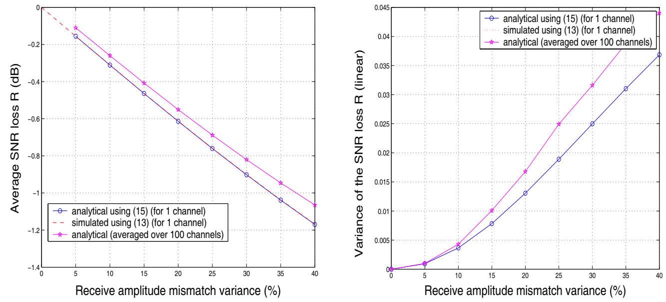


**Fig. 4**. *The average (in dB) and the variance (in linear units) of the SNR loss $R$ in the presence of receive amplitude mismatches*

## 5. CONCLUSIONS

In this paper, we have proposed a novel statistical analysis of the impact of multi-antenna RF front-end amplitude and phase mismatches on transmit MRC. The obtained numerical results, for flat-fading channels, suggest that these effects can completely annihilate the SNR gain promised by transmit MRC. More importantly, phase mismatch is the most detrimental effect. Consequently, an alternative would be to rather go for Equal Gain Combining (EGC) and only implement phase calibration. The latter calibration may turn out to be simpler and cheaper, as it requires less additional calibration circuity. We are currently investigating this last proposal.

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