EXACT ERROR PROBABILITY ANALYSIS OF MULTIMEDIA MULTICAST TRANSMISSION IN MIMO WIRELESS NETWORKS USING ORTHOGONAL SPACE-TIME BLOCK CODES

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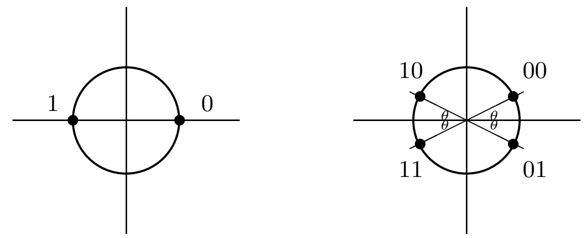
ABSTRACT

The exact probabilities of error in the case of multimedia multicast transmission in multiple-input multiple-output (MIMO) mobile wireless networks using orthogonal space-time block codes (OSTBC) are analyzed. The cases of nonuniform 4-PSK and 8-PSK constellations are considered. Comparisons of the derived expressions with numerical probabilities of error demonstrate the validity of our analysis.

1. INTRODUCTION

In wireless communication networks, it is often necessary for a user to broadcast or multicast messages. The broadcast transmission is a transmission intended for all the users in the network. The multicast transmission is a transmission intended for more than one user but not necessarily for all the users. We categorize the receivers in multicast transmission systems into two groups based on their received SNR. Let us refer to the receivers with a lower SNR as *less-capable* receivers, and to the receivers with a higher SNR as *more-capable* receivers. It is desirable to design the network such that it provides some basic information to the less-capable receivers while providing additional information to the more-capable receivers. The approach in [1] achieves this goal for single-input single-output (SISO) systems by the use of nonuniform M-PSK. This is done by expanding a uniform constant envelope signal constellation with N points into a larger nonuniform constant-envelope signal constellation with M points. The enlarged constellation can allocate $\log_2(N)$ bits per symbol for conveying the basic message and $\log_2(M/N)$ bits per symbol for conveying the additional message. The advantages of using M-PSK rather than QAM for multicast transmission were briefly discussed in [1].

In [1], the case of $M = 2N$ is discussed in detail, where a nonuniform M-PSK constellation is derived from a uniform N-PSK constellation by splitting each point into two *half-points*. The two half-points are split away from the position of the original point by equal amount around the circle defined by the original uniform N-PSK constellation. Figure 1 illustrates the construction of a nonuniform 4-PSK constellation from a uniform BPSK constellation. The two half-points are located at the angles θ and $-\theta$ relative to the angular position of the original point. Hence, the constructed nonuniform 4-PSK constellation has its points at the angles θ, $-\theta$, $\pi-\theta$, and $\pi+\theta$ where $0 < \theta < \pi/N$. The angle θ controls the division of transmit energy between the basic and the additional messages. Following [1], in the present paper we assume $M = 2N$. Therefore, each point in the non-uniform constellation conveys one bit of additional information, in addition to the $\log(N)$ bits of basic information.



(a) (b)

Fig. 1. The construction of a nonuniform 4-PSK constellation form a uniform BPSK constellation. (a) Original uniform BPSK constellation. (b) Final nonuniform 4-PSK constellation.

In [1], the SISO case was studied. However, the use of MIMO technology (along with appropriate coding over the space and time) has been shown to be a very promising approach both to increase the communication system throughput and to counteract fading. In particular, OSTBCs [2] are very popular due to their high performance and low decoding complexity. Certain space-time codes have also been proposed for broadcast communications in [3].

Recently, many attempts to analyze the error probability of OSTBCs have been made for the coherent receiver where the channel state information is known at the decoder. An approximate expression was obtained for the probability of error in the BPSK constellation case [4]. In [5], an upper bound on the pairwise symbol error probability of OSTBCs was found. In [6], an approximate expression was derived for the bit error rate (BER) of OSTBCs that use the uniform M-PSK modulation. Exact BER formulae were obtained in [7] for the Alamouti code in the uniform M-PSK case. In [8], exact expressions for the symbol error probability were derived for OSTBCs whose input signal constellations are uniform M-PSK or M-QAM. In [9], exact symbol error probabilities were obtained for OSTBCs with arbitrary input signal constellations.

In the present paper, we extend the SISO nonuniform M-PSK coding of [1] to MIMO systems using OSTBCs. It has been shown in [10] that in OSTBCs, the signal constellation at the receiver is merely a scaled version of the original constellation at the transmitter. This scaling factor has been shown to be equal to $\|\mathbf{H}\|_F$ where $\mathbf{H}$ is the channel matrix. This interesting property means that relative angles between any distinct points of the transmitted signal constellation are preserved at the receiver, while relative distances are scaled with the same factor $\|\mathbf{H}\|_F$. This property is called the *constellation space invariance* of OSTBCs.

In this paper, we use this property to obtain the exact error probabilities for MIMO multicast transmission systems using OSTBCs with nonuniform M-PSK signals.

2. EXACT ERROR PROBABILITY ANALYSIS FOR 4-PSK SIGNALS IN RAYLEIGH FADING CHANNELS

In this section, we derive exact expressions for the error probabilities for multicast 4-PSK signals, which are encoded using OSTBCs and transmitted over a Rayleigh channel with additive white Gaussian noise (AWGN) with zero mean and a variance $\sigma^2 = N_0$ per complex dimension.

Suppose that such 4-PSK signals are employed to send information from a single transmitter to multiple receivers. We will consider some of the receivers as less-capable receivers (which are intended to successfully demodulate the basic message only) and the others as more-capable receivers (which are intended to successfully demodulate the basic message and the additional message as well).

In SISO systems with an AWGN channel, if we denote the received symbol energy at the ith receiver by $\mathcal{E}_i$ and use the notation $e_i \triangleq \sqrt{\mathcal{E}_i/N_0}$, then the probabilities of bit error at the ith receiver for the basic and additional messages can be written as [1]

$$\begin{aligned} P_e(i,\, b) &= Q[\sqrt{2}e_i \cos\theta] \\ P_e(i,\, a) &= Q[\sqrt{2}e_i \sin\theta] \end{aligned}$$

respectively, where $Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$.

Let n_t be the number of transmit antennas, n_{r_i} be the number of antennas at the ith receiver, and T be the block length (assuming block fading). The MIMO multicast system is described by

$$\mathbf{Y}_i = \mathbf{X}\,\mathbf{H}_i + \mathbf{N}_i \tag{1}$$

where $\mathbf{X}$ is the $T \times n_t$ transmitted multicast complex signal matrix, $\mathbf{H}_i$ is the $n_t \times n_{r_i}$ channel matrix for the ith receiver, $\mathbf{Y}_i$ is the $T \times n_{r_i}$ received signal matrix at the ith receiver, and $\mathbf{N}_i$ is the $T \times n_{r_i}$ matrix of the noise at the ith receiver.

Using the results of [9, 10], we can compute the probability of bit error (for both the basic and the additional messages) in MIMO systems with known channel coefficients at the ith receiver. For that purpose, we use the idea of [9] and compute the probability of error by using the above SISO expressions and divide the standard deviation of the noise by the aforementioned scaling factor $\|\mathbf{H}\|_F$ as in [9]. If we use the notation $Z_i \triangleq \|\mathbf{H}_i\|_F^2$, then the probabilities of error in detecting the basic and the additional messages at the ith receiver are

$$P_e(i,\, b|Z_i = z) = Q[\sqrt{2z}\, e\, \cos\theta] \tag{2}$$

$$P_e(i,\, a|Z_i = z) = Q[\sqrt{2z}\, e\, \sin\theta] \tag{3}$$

respectively, where $e \triangleq \sqrt{\frac{\mathcal{E}}{N_0}}$, and $\mathcal{E}$ is the energy of the transmitted multicast signal.

To obtain the average error probability for the basic message over all Rayleigh channel realizations, we can write

$$P_e(i,\, b) = \int_0^\infty P_e(i,\, b|Z_i = z)\, p_{Z_i}(z)\, dz. \tag{4}$$

In Rayleigh channels, Z_i has a (central) χ^2 distribution with $r_i = 2\, n_t\, n_{r_i}$ degrees of freedom. Hence, the probability density function (pdf) of Z_i is

$$p_{Z_i}(z) = \frac{z^{\frac{r_i}{2}-1}\, e^{-\frac{z}{2\rho_i^2}}}{\rho_i^{r_i}\, \Gamma(\frac{r_i}{2})\, 2^{\frac{r_i}{2}}} \tag{5}$$

where ρ_i is the standard deviation of each real and imaginary element of $\mathbf{H}_i$. Moreover, ρ_i represents the strength of the channel at the specified receiver (e.g. if $\rho_i > \rho_j$ then the channel for the ith receiver is said to be stronger than the channel for the jth receiver, or, in other words, the ith receiver is more-capable while the jth receiver is the less-capable). Substituting (2) and (5) into (4) gives

$$\begin{aligned} P_e(i,\, b) &= \int_0^\infty Q[\sqrt{2z}\, e\, \cos\theta]\, \frac{z^{\frac{r_i}{2}-1}\, e^{-\frac{z}{2\rho_i^2}}}{\rho_i^{r_i}\, \Gamma(\frac{r_i}{2})\, 2^{\frac{r_i}{2}}}\, dz \\ &= \int_0^\infty \Big(\int_{\sqrt{2z}\, e\, \cos\theta}^\infty \frac{1}{\sqrt{2\pi}}\, e^{-\frac{t^2}{2}}\, dt \Big) \\ &\quad \times \frac{z^{\frac{r_i}{2}-1}\, e^{-\frac{z}{2\rho_i^2}}}{\rho_i^{r_i}\, \Gamma(\frac{r_i}{2})\, 2^{\frac{r_i}{2}}}\, dz \\ &= \frac{1}{\sqrt{2\pi}\, \rho_i^{r_i}\, \Gamma(\frac{r_i}{2})\, 2^{\frac{r_i}{2}}} \\ &\quad \times \int_{z=0}^\infty \int_{t=\sqrt{2z}\, e\, \cos\theta}^\infty e^{-\frac{t^2}{2}}\, z^{\frac{r_i}{2}-1}\, e^{-\frac{z}{2\rho_i^2}}\, dt\, dz \\ &= \frac{1}{\sqrt{2\pi}\, \rho_i^{r_i}\, \Gamma(\frac{r_i}{2})\, 2^{\frac{r_i}{2}}} \\ &\quad \times \int_{t=0}^\infty \left(\int_{z=0}^{\frac{t^2}{2\, e^2 \cos^2\theta}} z^{\frac{r_i}{2}-1}\, e^{-\frac{z}{2\rho_i^2}}\, dz \right) e^{-\frac{t^2}{2}}\, dt. \end{aligned} \tag{6}$$

After integration, we have [11]

$$\begin{aligned} P_e(i,\, b) &= \frac{1}{2} - \sqrt{\frac{2e^2}{\pi}}\, \rho_i\, \frac{\Gamma(\frac{r_i+1}{2})}{\Gamma(\frac{r_i}{2})}\, \cos\theta \\ &\quad \times {}_2F_1(\frac{1}{2}, \frac{r_i+1}{2}; \frac{3}{2}; -2\, e^2\, \rho_i^2 \cos^2\theta) \end{aligned} \tag{7}$$

where ${}_2F_1(a, b; c; z)$ is the Gauss hypergeometric function.

The average probability of error for the additional message over all Rayleigh channel realizations can be obtained by replacing $\cos\theta$ in (7) with $\sin\theta$

$$\begin{aligned} P_e(i,\, a) &= \frac{1}{2} - \sqrt{\frac{2e^2}{\pi}}\, \rho_i\, \frac{\Gamma(\frac{r_i+1}{2})}{\Gamma(\frac{r_i}{2})}\, \sin\theta \\ &\quad \times {}_2F_1(\frac{1}{2}, \frac{r_i+1}{2}; \frac{3}{2}; -2\, e^2\, \rho_i^2 \sin^2\theta). \end{aligned} \tag{8}$$

3. EXACT ERROR PROBABILITY ANALYSIS FOR 8-PSK SIGNALS IN RAYLEIGH FADING CHANNELS

In this section, we derive exact expressions for the error probabilities for multicast 8-PSK signals (assuming that one bit is allocated for the additional message) following the same steps as in Section 2. It can be easily shown that in an AWGN SISO channel, the probability of bit error for the basic message at the ith receiver is

$$P_e(i,\, b) = \frac{1}{2}\, [Q(\sqrt{2}\, e_i\, \sin(\frac{\pi}{4} - \theta)) + Q(\sqrt{2}\, e_i\, \sin(\frac{\pi}{4} + \theta))]$$

and for the additional message is

$$P_e(i,\,a) = \frac{1}{2} \;-\; \frac{2}{\sqrt{\pi}} \int_0^{e_i\ \sin\theta} e^{-y^2} [\frac{1}{2} - Q(\sqrt{2}y\cot\theta)]\,dy \;-\; \frac{2}{\sqrt{\pi}} \int_0^{e_i\ \cos\theta} e^{-y^2} [\frac{1}{2} - Q(\sqrt{2}y\tan\theta)]\,dy.$$

Using the results of [9, 10] and letting $Z_i = \|\mathbf{H}_i\|_F^2$, the probabilities of error for the basic and the additional messages in the case of a known channel at the ith receiver are given by

$$P_e(i,\,b|Z_i = z) \;=\; \frac{1}{2}\left[Q(\sqrt{2z}\,e\,\sin(\frac{\pi}{4}-\theta)) + Q(\sqrt{2z}\,e\,\sin(\frac{\pi}{4}+\theta))\right] \qquad (9)$$

and

$$P_e(i,\,a|Z_i = z) \;=\; \frac{1}{2} - \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{z}\,e\ \sin\theta} e^{-y^2} [\frac{1}{2} - Q(\sqrt{2}y\cot\theta)]\,dy - \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{z}\,e\ \cos\theta} e^{-y^2} [\frac{1}{2} - Q(\sqrt{2}y\tan\theta)]\,dy \qquad (10)$$

respectively.

The average error probability for the basic message using nonuniform 8-PSK can be obtained by substituting (5) and (9) into (4)

$$P_e(i,\,b) = \frac{1}{2}\int_0^{\infty} \left[Q(\sqrt{2z}\,e\,\sin(\frac{\pi}{4}-\theta)) + Q(\sqrt{2z}\,e\,\sin(\frac{\pi}{4}+\theta))\right] \frac{z^{\frac{r_i}{2}-1}\,e^{-\frac{z}{2\rho_i^2}}}{\rho_i^{r_i}\,\Gamma(\frac{r_i}{2})\,2^{\frac{r_i}{2}}}\,dz.$$

Following the same steps that have been used to derive (6), we obtain

$$\begin{aligned} P_e(i,\,b) \;&=\; \frac{1}{2} - \sqrt{\frac{e^2}{2\pi}}\rho_i \frac{\Gamma(\frac{r_i+1}{2})}{\Gamma(\frac{r_i}{2})} \\ &\times \left[\sin(\frac{\pi}{4}+\theta)\ {}_2F_1\left(\frac{1}{2}, \frac{r_i+1}{2}; \frac{3}{2}; -2\,e^2\,\rho_i^2 \sin^2(\frac{\pi}{4}+\theta)\right) \right. \\ &\left. + \sin(\frac{\pi}{4}-\theta)\ {}_2F_1\left(\frac{1}{2}, \frac{r_i+1}{2}; \frac{3}{2}; -2\,e^2\,\rho_i^2 \sin^2(\frac{\pi}{4}-\theta)\right)\right]. \end{aligned}$$

The average error probability for the additional message using nonuniform 8-PSK may be obtained by substituting (5) and (10) into

$$P_e(i,\,a) = \int_0^{\infty} P_e(i,\,a|Z_i = z)\,p_{Z_i}(z)\,dz \qquad (11)$$

to get

$$\begin{aligned} P_e(i,\,a) \;&=\; \frac{1}{2} - \frac{2}{\sqrt{\pi}\rho_i^{r_i}\,\Gamma(\frac{r_i}{2})\,2^{\frac{r_i}{2}}} \\ &\times \int_0^{\infty} \left[\int_0^{\sqrt{z}\,e\ \sin\theta} e^{-y^2} [\frac{1}{2} - Q(\sqrt{2}y\cot\theta)]\,dy \right. \\ &\left. + \int_0^{\sqrt{z}\,e\ \cos\theta} e^{-y^2} [\frac{1}{2} - Q(\sqrt{2}y\tan\theta)]\,dy\right] z^{\frac{r_i}{2}-1}\,e^{-\frac{z}{2\rho_i^2}}\,dz \\ &=\; \frac{1}{2} - \sqrt{\frac{2}{\pi}}\,e\,\rho_i \frac{\Gamma(\frac{r_i+1}{2})}{\Gamma(\frac{r_i}{2})} \\ &\times \left[\sin\theta\ {}_2F_1\left(\frac{1}{2}, \frac{r_i+1}{2}; \frac{3}{2}; -2\,e^2\,\rho_i^2 \sin^2\theta\right) + \cos\theta\ {}_2F_1\left(\frac{1}{2}, \frac{r_i+1}{2}; \frac{3}{2}; -2\,e^2\,\rho_i^2 \cos^2\theta\right)\right] \\ &+ \frac{2}{\sqrt{\pi}\rho_i^{r_i}\,\Gamma(\frac{r_i}{2})\,2^{\frac{r_i}{2}}} \int_0^{\infty} \left[\int_0^{\sqrt{z}\,e\ \sin\theta} e^{-y^2} [Q(\sqrt{2}y\cot\theta)]\,dy \right. \\ &\left. + \int_0^{\sqrt{z}\,e\ \cos\theta} e^{-y^2} [Q(\sqrt{2}y\tan\theta)]\,dy\right] z_i^{\frac{r_i}{2}-1}\,e^{-\frac{z_i}{2\rho_i^2}}\,dz. \qquad (12) \end{aligned}$$

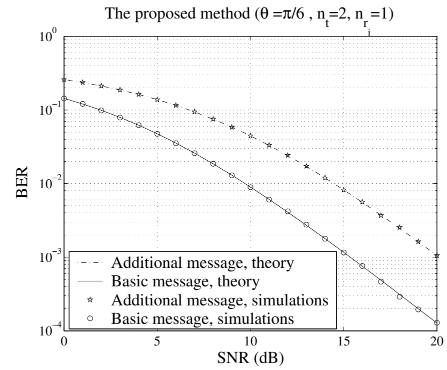


Fig. 2. Theoretical and simulated BERs versus SNR in the case of nonuniform 4-PSK signals.

The integral in (12) has no closed form but can be evaluated numerically.

4. SIMULATION RESULTS

Figure 2 compares the bit error rates (BERs) versus the SNR. These BERs are obtained via simulations and by theoretical evaluation using (7) and (8). It is assumed that we have two transmit antennas and one antenna at each of the receivers. The splitting angle θ is $\frac{\pi}{6}$. The simulation points are obtained for the scenario equivalent to that considered in theory, i.e., for the case of 4-PSK signals encoded using Alamouti's code [12] and decoded using coherent maximum likelihood (ML) decoder. This figure shows that the obtained theoretical and simulation results match perfectly.

In Figure 3, we plot the theoretically evaluated BERs of decoding the additional and basic messages for nonuniform 4-PSK and the theoretical BER for the original uniform BPSK versus the received SNR. We can notice some degradation in the performance of the multicast system in decoding the basic message compared to the BPSK-based broadcast system. This degradation is quite expected because of the distance change between constellation points in the nonuniform 4-PSK case relative to the BPSK case. It can be viewed as a price for sending additional messages in the multicast case.

The error probabilities of decoding the additional and basic messages in the nonuniform 8-PSK and uniform 4-PSK cases are shown in Figure 4. Similar to the previous message, we observe that, as expected, there is some degradation in the performance of the multicast system compared to the 4-PSK-based broadcast system.

In Figure 5, the error probabilities of decoding the additional

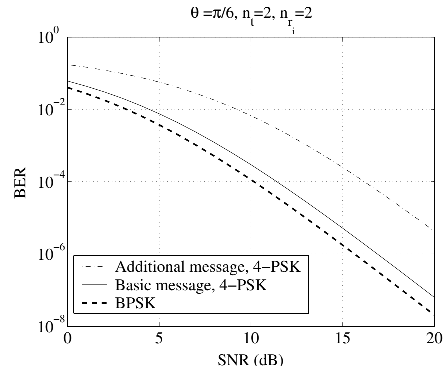


Fig. 3. Theoretical BERs versus SNR. The BPSK-based broadcast system is compared to the nonuniform 4-PSK-based multicast system.

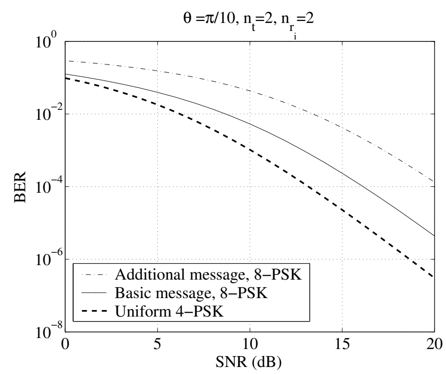


Fig. 4. Theoretical BERs versus SNR. The uniform 4-PSK-based broadcast system is compared to the nonuniform 8-PSK-based multicast system.

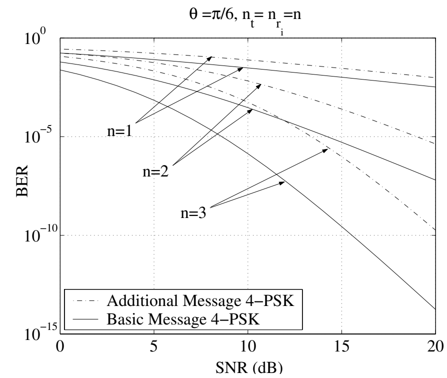


Fig. 5. Theoretical BERs versus SNR in the case of nonuniform 4-PSK-based multicast system and for different number of transmit and receive antennas.

and basic messages are plotted in the case of nonuniform 4-PSK with different number of antennas. This figure shows that, increasing the number of antennas from one to three, the probability of error can be improved greatly.

5. CONCLUSIONS

In this paper, we have analyzed the performance of the multicast transmission scheme using space-time block coded MIMO systems. The orthogonal space-time codes used can be easily generated and generalized to higher constellation size cases. Exact expressions for the probability of error in the cases of nonuniform 4-PSK and 8-PSK constellations have been derived.

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