A NEW DIFFERENTIAL UNITARY SPACE-TIME MODULATION SCHEME WITH REDUCED RECEIVER COMPLEXITY

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ABSTRACT

The differential unitary space-time modulation (DUSTM) scheme provides full diversity without channel knowledge at either the transmitter or the receiver. However, the complexity of maximum-likelihood decoding is exponential in the number of transmit antennas and the data rate. For the case where the number of transmit antennas is even, we propose a new DUSTM scheme which preserves full diversity while reducing the decoding complexity. Moreover, our scheme simplifies the constellation design. Theoretical analysis and simulation results show that for some typical scenarios, the proposed scheme achieves better bit-error-rate performance than the original DUSTM scheme.

1. INTRODUCTION

Differential modulation for multiple transmit antennas has received considerable attention as they obviate the need for channel estimation at the receiver. A differential form of the Alamouti's transmit diversity scheme [1] was proposed in [2] for two transmit antennas. Applying the space-time block codes (STBC) designed in [3], this scheme was later generalized in [4] to more than two antennas for real constellations. More recently, a differential STBC scheme based on amicable orthogonal designs was developed in [5].

Motivated by the information theoretical arguments in [6], another class of differential space-time modulation techniques employ unitary space-time constellations. In [7], constellations with group property were considered and optimal group codes were explicitly constructed for two transmit antennas. In order to simplify constellation design for multiple antennas, Hochwald and Sweldens [8] suggested a special class of unitary group constellations comprising diagonal signals which were designed to maximize diversity and coding gains. The resulting differential modulation scheme is referred to as cyclic codes.

The use of cyclic codes simplifies differential encoding process as well as the search for good codes. However, it suffers a loss in coding gain by imposing the special constellation structure. In addition, the computational complexity of maximum-likelihood (ML) decoding is exponential in the number of transmit antennas and the data rate. Although a class of non-group constellations was proposed in [9] to achieve larger coding gain than cyclic codes, the ML decoding complexity was not reduced.

This paper proposes a new DUSTM scheme that applies to the case where the number of transmit antennas M is even. Block-diagonal Alamouti-type matrices constructed from two $M/2 \times M/2$ unitary diagonal matrices, rather than $M \times M$ unitary diagonal matrices for cyclic codes, are used to perform differential encoding. The ML decoding of the two $M/2 \times M/2$ diagonal matrices can be decoupled, leading to a computationally simpler receiver. Furthermore, our scheme simplifies constellation design by reducing the number of parameters to be determined by half. The scheme still provides full transmit diversity and outperforms cyclic codes for some typical applications.

Notation: Upper (boldface lower) case letters are used for matrices (column vectors). Superscript ^{T} denotes transpose and * complex conjugate transpose. We reserve $\otimes$, $|\cdot|$ and $\|\cdot\|$ for the Kronecker product, the determinant and the Frobenius norm, respectively. I_M denotes the $M \times M$ identity matrix, and $\mathrm{diag}(\boldsymbol{x})$ stands for a diagonal matrix with the entries of the vector $\boldsymbol{x}$ on its main diagonal.

2. DIFFERENTIAL UNITARY SPACE-TIME MODULATION (DUSTM)

Consider a wireless communication link with M transmit and N receive antennas and flat fading channel. Fading coefficients between each pair of transmit and receive antennas are modeled as i.i.d. circularly symmetric complex Gaussian random variables with zero mean and unit variance. DUSTM is employed at the transmitter. Denoting by S_t the t-th $M \times M$ unitary transmitted signal matrix, the corresponding $N \times M$ received signal matrix can be expressed as

$$X_t = \sqrt{\rho} H_t S_t + W_t \tag{1}$$

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where H_t and W_t are the $N \times M$ matrices of channel coefficients and additive white Gaussian noise with zero mean and unit variance, respectively; ρ is the average signal-to-noise ratio (SNR) per receive antenna.

Suppose that a data sequence of integers $z_1, z_2, \ldots$ with $z_t \in \{0, \ldots, L-1\}$ is to be sent, then we need a constellation $\mathcal{V} = \{V_0, \ldots, V_{L-1}\}$ of $M \times M$ unitary matrices to build a one-to-one mapping between z_t and $\mathcal{V}$, say $\Psi : z_t \rightarrow V_{z_t}$. To achieve a data rate of R bits/channel use, the constellation size L will be $L = 2^{RM}$. The unitary transmitted signal matrix S_t is generated according to the following differential encoding rule

$$S_t = S_{t-1} V_{z_t}, \quad t = 1, 2, \ldots \tag{2}$$

with $S_0 = \mathrm{I}_M$ initializing the differential transmission. Under the assumption that fading coefficients are constant over $T = 2M$ symbol periods, two successive received signal matrices are related by

$$X_t = X_{t-1} V_{z_t} + \tilde{W}_t \tag{3}$$

where $\tilde{W}_t := W_t - W_{t-1} V_{z_t}$ is the equivalent noise matrix. Compared to W_t, the entries of $\tilde{W}_t$ have the same statistical property except for double variance. The ML estimate of z_t is given by

$$\hat{z}_t = \arg \min_{l=0,\ldots,L-1} \| X_t - X_{t-1} V_l \|. \tag{4}$$

With this ML receiver, the average pairwise error probability (PEP) of mistaking V_l for $V_{l'}$ has Chernoff upper bound

$$\Pr(V_l \rightarrow V_{l'}) \leq \prod_{m=1}^{M} \left[1 + \frac{\rho^2 \sigma_m^2 (V_l - V_{l'})}{4(1+2\rho)} \right]^{-N} \tag{5}$$

where $\sigma_m(V_l - V_{l'})$ is the m-th singular value of $V_l - V_{l'}$. If the constellation $\mathcal{V}$ enables full transmit diversity, i.e., $V_l - V_{l'}$ has full rank for any $l \neq l'$, then for large ρ, this bound behaves as

$$\Pr(V_l \rightarrow V_{l'}) \leq (\xi_{ll'})^{-MN} \left(\frac{\rho}{8} \right)^{-MN} \tag{6}$$

where $\xi_{ll'} = |(V_l - V_{l'})(V_l - V_{l'})^*|^{1/M}$. Following the definition in [10], the coding gain for this constellation can be quantified as $\xi = \min_{l \neq l'} \xi_{ll'}$, which should be maximized in designing a constellation.

In order to simplify the constellation design, a special class of group-structured constellation called cyclic codes was proposed in [8] where each signal has the form

$$V_l = \mathrm{diag}\left(e^{j2\pi k_1 l/L}, \ldots, e^{j2\pi k_M l/L} \right), l = 0, \ldots, L-1 \tag{7}$$

with the parameters $k_1, \ldots, k_M$ determined by

$$\{k_1, \ldots, k_M\} = \arg \max_{0 \leq k_1, \ldots, k_M \leq L-1} \min_{l=1,\ldots,L-1} \xi_{0l} \tag{8}$$

where $\xi_{0l} = 4 \left| \prod_{m=1}^{M} \sin(\pi k_m l/L) \right|^{2/M}$. Although the use of cyclic codes simplifies constellation design, the computational complexity of the ML receiver is still exponential in both the number of transmit antennas and the data rate. We describe in the next section a new DUSTM scheme which reduces the receiver complexity to some extent.

3. A NEW DUSTM SCHEME

3.1. Differential Modulation

We consider the scenario in which the number of transmit antennas M is even. The binary information to be sent can be combined into a data sequence of integers $d_1, d_2, \ldots$ with $d_t \in \{0, \ldots, K-1\}$ and $K = 2^{RM/2}$. Following the approach described in Section 2 to design cyclic codes , a constellation $\mathcal{G} = \{G_0, \ldots, G_{K-1}\}$ of $M/2 \times M/2$ diagonal unitary matrices is constructed. For $t = 1, 3, 5, \ldots$, we map d_t and d_{t+1} into G_{d_t} and $G_{d_{t+1}}$, respectively, and construct the block-diagonal space-time code matrix as

$$C_{(t+1)/2} := \frac{1}{\sqrt{2}} \begin{bmatrix} G_{d_t} & G_{d_{t+1}} \\ -G^*_{d_{t+1}} & G^*_{d_t} \end{bmatrix} \tag{9}$$

where the factor $\frac{1}{\sqrt{2}}$ is used to make the code matrix unitary. Since C_k contains RM bits of information, the effective transmission rate is also R bits/channel use. We observe that C_k looks like the Alamouti code matrix [1] with scalars replaced by diagonal matrices, and thus, we refer to matrices possessing this structure as *block-Alamouti* matrices.

The code matrix C_k is used to perform differential encoding between two successive transmitted matrices via

$$S_k = C_k S_{k-1}, \quad k = 1, 2, \ldots \tag{10}$$

and $S_0 = \mathrm{I}_M$ initializes the differential transmission. It is clear that S_k is unitary for any k. Each row of S_k represents the signals simultaneously transmitted from M transmit antennas at a time. Because matrix multiplication in (10) only involves block-Alamouti matrices, the differential encoding process has very low complexity. Note that when $M = 2$, (10) reduces to the differential encoding scheme proposed in [2].

3.2. Differential Detector

For notational simplicity, we focus on a system with a single receive antenna, although the derivation here extends in a straightforward way to multiple receive antennas. When S_k is transmitted, the corresponding $M \times 1$ received vector is given by

$$\boldsymbol{x}_k = \sqrt{\rho} S_k \boldsymbol{h} + \boldsymbol{w}_k \tag{11}$$

where $\boldsymbol{h}$ and $\boldsymbol{w_k}$ are the $M \times 1$ vectors of channel coefficients and additive noise, respectively. From (10) and (11), we have

$$\boldsymbol{x}_k = C_k \boldsymbol{x}_{k-1} + \boldsymbol{v}_k \tag{12}$$

where $\boldsymbol{v}_k := \boldsymbol{w}_k - C_k \boldsymbol{w}_{k-1}$ is the equivalent noise vector with zero mean and correlation matrix $\mathrm{R}(\boldsymbol{v}_k) = 2\mathrm{I}_M$.

Define the $M/2 \times 1$ vectors $\boldsymbol{x}_{k,1}$ and $\boldsymbol{x}_{k,2}$ which are determined by $\boldsymbol{x}_k^T = [\boldsymbol{x}_{k,1}^T, \boldsymbol{x}_{k,2}^T]^T$, and the diagonal matrices $X_{k,i} := \mathrm{diag}(\boldsymbol{x}_{k,i})$ for $i = 1, 2$. Similar definitions associated with the noise vector $\boldsymbol{v}_k$ are also made. Exploiting the block-Alamouti structure of the code matrix C_k, we can rewrite (12) as

$$\underbrace{\begin{bmatrix} X_{k,1} \\ X_{k,2}^* \end{bmatrix}}_{\bar{X}_k} = \frac{1}{\sqrt{2}} \underbrace{\begin{bmatrix} X_{k-1,1} & X_{k-1,2} \\ X_{k-1,2}^* & -X_{k-1,1}^* \end{bmatrix}}_{\mathcal{X}_{k-1}} \begin{bmatrix} G_{d_{2k-1}} \\ G_{d_{2k}} \end{bmatrix} + \underbrace{\begin{bmatrix} V_{k,1} \\ V_{k,2}^* \end{bmatrix}}_{\bar{V}_k}. \tag{13}$$

The block-Alamouti matrix $\mathcal{X}_{k-1}$ specified in (13) satisfies $\mathcal{X}_{k-1}^* \mathcal{X}_{k-1} = \mathrm{I}_2 \otimes \Lambda_{k-1}$, where the $M/2 \times M/2$ diagonal matrix $\Lambda_{k-1} := \sum_{i=1}^2 X_{k-1,i}^* X_{k-1,i}$ has positive diagonal entries. It can be easily verified that $\mathcal{Y}_{k-1} := \left(\mathrm{I}_2 \otimes \Lambda_{k-1}^{-1/2}\right) \mathcal{X}_{k-1}^*$ is a unitary matrix. Pre-multiplying $\bar{X}_k$ by $\mathcal{Y}_{k-1}$, we obtain

$$\begin{aligned} Z_k &:= \mathcal{Y}_{k-1} \bar{X}_k \\ &= \frac{1}{\sqrt{2}} \left(\mathrm{I}_2 \otimes \Lambda_{k-1}^{1/2}\right) \begin{bmatrix} G_{d_{2k-1}} \\ G_{d_{2k}} \end{bmatrix} + N_k \end{aligned} \tag{14}$$

where $N_k = \mathcal{Y}_{k-1} \bar{V}_k$ is the resulting noise term which has the same statistical distribution as $\bar{V}_k$. Note that both Z_k and N_k consist of two $M/2 \times M/2$ diagonal matrices. Denoting by $Z_{k,1}$ ($Z_{k,0}$) the top (bottom) $M/2 \times M/2$ diagonal matrix of Z_k, and analogously $N_{k,1}$ and $N_{k,0}$ based on N_k, we can recast (14) into two separate equations

$$Z_{k,i} = \frac{1}{\sqrt{2}} \Lambda_{k-1}^{1/2} G_{d_{2k-i}} + N_{k,i}, \quad i = 0, 1 \tag{15}$$

which show that the ML detection of $G_{d_{2k-1}}$ and $G_{d_{2k}}$, or equivalently d_{2k-1} and d_{2k}, can be decoupled. Specifically, the ML estimates of d_{2k-1} and d_{2k} are given by

$$\hat{d}_{2k-i} = \arg \min_{l=0,\ldots,K-1} \|Z_{k,i} - \frac{1}{\sqrt{2}} \Lambda_{k-1}^{1/2} G_l\|, \quad i = 0, 1 \tag{16}$$

which amounts to a search among $K = 2^{RM/2}$ possible matrices to detect $RM/2$ bits of information.

Therefore, to recover RM bits of information, the ML receiver for the proposed DUSTM scheme needs to search among a total of $2 \times 2^{RM/2}$ matrices, instead of 2^{RM} matrices for cyclic codes. For example, when $M = 4$ and $R = 2$, the receiver computational complexity of our scheme reduces by approximately a factor of eight as compared with that of cyclic codes. The decrease in receiver complexity is more significant as M and/or R grow. Furthermore, it can be seen from (8) that our scheme also simplifies constellation design, not only because the constellation size is reduced, but also because the number of parameters to be determined drops from M to $M/2$.

4. PERFORMANCE ANALYSIS

In this section, we evaluate the performance of the new DUSTM scheme by analyzing its diversity and coding gains. The probability that the ML receiver in (16) erroneously decodes G_l as $G_{l'}$ conditioned on the previous received block $\boldsymbol{x}_{k-1}$ has the Chernoff upper bound

$$\Pr(G_l \to G_{l'} | \boldsymbol{x}_{k-1}) \leq \exp\left(\frac{-\mathrm{d}^2(G_l - G_{l'})}{8}\right) \tag{17}$$

where $\mathrm{d}^2(G_l - G_{l'}) = \|\frac{1}{\sqrt{2}} \Lambda_{k-1}^{1/2} (G_l - G_{l'})\|^2$. Assuming high SNR at the receiver and exploiting the fact that S_k is unitary for any k, Λ_{k-1} can be well approximated as

$$\Lambda_{k-1} \simeq \rho \left(H_1^* H_1 + H_2^* H_2\right) \tag{18}$$

where $H_i = \mathrm{diag}(\boldsymbol{h}_i)$ with $\boldsymbol{h}_i$ being the $M/2 \times 1$ vectors determined by $\boldsymbol{h}^T = [\boldsymbol{h}_1^T, \boldsymbol{h}_2^T]^T$. After some manipulations, the expression for $\mathrm{d}^2(G_l - G_{l'})$ can be simplified as

$$\mathrm{d}^2(G_l - G_{l'}) = \frac{\rho}{2} \boldsymbol{h}^* \Phi_{ll'} \boldsymbol{h} \tag{19}$$

where $\Phi_{ll'} = \mathrm{I}_2 \otimes [(G_l - G_{l'})^* (G_l - G_{l'})]$. Substituting (19) into (17) and averaging it with respect to the statistical distribution of the channel vector $\boldsymbol{h}$, we obtain the average pairwise error probability of the proposed DUSTM system

$$\Pr(G_l \to G_{l'}) \leq \prod_{m=1}^{M/2} \left[1 + \frac{\rho}{16} \sigma_m^2 (G_l - G_{l'})\right]^{-2} \tag{20}$$

where $\sigma_m(G_l - G_{l'})$ is the m-th singular value of $G_l - G_{l'}$.

Applying the method described in Section 2 to design the signal set $\mathcal{G}$ ensures that $G_l - G_{l'}$ has full rank for any $l \neq l'$. Therefore, the bound (20) can be rewritten as

$$\Pr(G_l \to G_{l'}) \leq \left(\frac{1}{2} \zeta_{ll'}\right)^{-M} \left(\frac{\rho}{8}\right)^{-M} \tag{21}$$

where $\zeta_{ll'} = |(G_l - G_{l'})^* (G_l - G_{l'})|^{2/M}$. We deduce from (21) that our scheme achieves the maximum transmit diversity order of M, and a coding gain of

$$\begin{aligned} \zeta &:= \frac{1}{2} \min_{l \neq l'} \zeta_{ll'} \\ &= \frac{1}{2} \min_{l \neq l'} |(G_l - G_{l'})^* (G_l - G_{l'})|^{2/M}. \end{aligned} \tag{22}$$

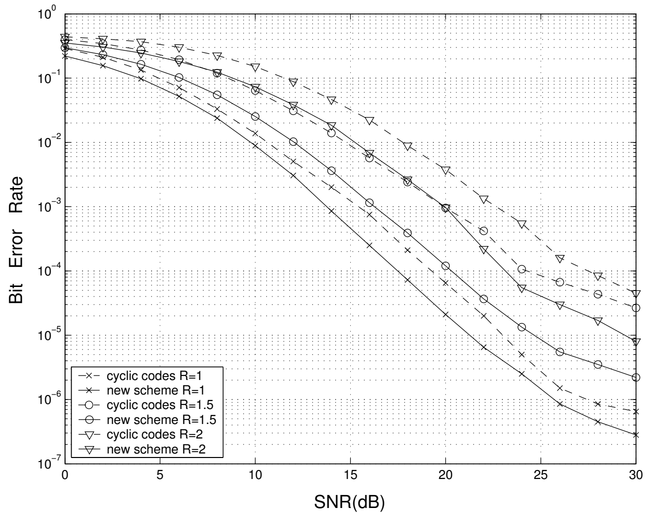

Fig. 1. BER performance of the cyclic codes and the proposed scheme.

Remarkably, we have found that for some typical applications, our scheme has larger coding gains than cyclic codes. Consider M=4 and R=2 as an example, the coding gain of our scheme is 0.2928 as compared to 0.1950 for cyclic codes.

5. SIMULATION RESULTS

Simulations are performed to test the performance of the proposed DUSTM scheme and compare it with cyclic codes. We consider a system with four transmit antennas and a single receive antenna. The channels are assumed to change continuously with normalized Doppler shift $f_d T_s = 0.0025$. We use the model introduced in [11] to generate these time-varying channels.

Fig.1 shows the bit-error-rate performance of the proposed scheme and cyclic codes for different data rates. For $R = 1.5$ and 2 bits/channel use, the proposed scheme has larger coding gains than cyclic codes. As expected, we observe significant performance improvement achieved by our scheme in these scenorios. On the other hand, although our scheme has a slightly smaller coding gain than cyclic codes for $R = 1$, it is somewhat surprising to observe that the former also has better performance. This may be explained by the fact that the constellation size of our scheme is much smaller than that of cyclic codes.

6. CONCLUSIONS

We have proposed a new differential unitary space-time modulation scheme, which simplifies constellation design and reduces the computational complexity of the ML receiver. The proposed scheme applies to the case where the number of transmit antennas is even. We have shown that the scheme achieves full transmit diversity and has larger coding gains than cyclic codes for some typical applications.

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