BIT ERROR RATE PERFORMANCE OF DECISION FEEDBACK DETECTION FOR SPACE–TIME BLOCK CODED SYSTEMS OVER TIME-SELECTIVE FADING CHANNELS

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ABSTRACT

In wireless channels, time-selective fading effects arise mainly due to Doppler shift and carrier frequency offset. In time-selective fading channels, Alamouti-based decoding scheme has an error floor caused by interference due to time-selectivity. This paper proposes decision feedback detector for Alamouti scheme to mitigate the effects of a time-selective fading channels. Moreover, we presents the evaluation of the average bit error rate (BER) performance of the proposed scheme over time-selective fading channels.

1. INTRODUCTION

As wireless communication systems look to make the transition from voice communication to interactive Internet data, achieving higher data rates become both increasingly desirable and challenging. The advantages of using multiple antennas at both the transmit and receive ends of a wireless communications link have recently been noted. Recently, space-time coding (STC) was proposed as an alternative and attractive solution for high-capacity data transmission in wireless systems [1]. In particular, Alamouti [2] discovered a remarkable space-time block coding (STBC) scheme for transmission with two transmit antennas achieves full diversity gains using a linear maximum-likelihood (ML) decoder. Alamouti's STBC has been adopted in several wireless standards such as IS-136, WCDMA, and CDMA-2000.

Most existing STC schemes have been developed for flat fading channels. Different from [2], we consider here more realistic time-selective but frequency-flat fading channels. In wireless mobile communications, time selectivity is mainly caused by Doppler shifts and carrier frequency offsets, which are jointly independent. Time-selective fading channels can be modeled either deterministic models or random processes. Typically, deterministic channel models require estimates of more parameters than random models, which makes to sensitivity due to over-parameterization. Information theoretic results have been shown that the first-order Gauss-Markov random processes provides a accurate model for time-selective fading channels [3].

The impact of time-selective fading channels on the performance of the transmit-diversity technique proposed by Alamouti and the problem of channel tracking for the Alamouti scheme were investigated in [4]. Alamouti-based decoding scheme has an error floor caused by interference due to time-selectivity. In order to remove this error floor problem [4] proposed a decision feedback detection scheme for Alamouti-based STBC transmission over time-selective fading channels, but the exact evaluation of the average bit error rate (BER) performance is unknown. In this paper, we derive an exact average BER analysis for decision feedback detection scheme over time-selective fading channels. Numerical examples are presented in order to illustrate the effect and robustness to time-selectivity.

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2. SYSTEM MODEL AND ALAMOUTI SCHEME

Consider a wireless system equipped with two transmit antennas and one receive antenna as shown in Fig. 1. Information symbols $s(n)$ are transmitted using Alamouti's space-time block encoder. Different from previous work [2] where the channels are assumed flat fading, we consider time-selective but frequency-flat fading channels. Denote by $h_i(n),\ i=1,2$, the time-selective fading channel from the ith transmit antenna to the receive antenna. At the receive antenna, the two successive received samples $y(2n)$ and $y(2n+1)$ are given by

$$\begin{aligned} y(2n) &= h_1(2n)s(2n)+h_2(2n)s(2n+1)+w(2n) \\ y(2n+1) &= -h_1(2n+1)s^*(2n+1) \\ &\quad +h_2(2n+1)s^*(2n)+w(2n+1). \end{aligned} \tag{1}$$

In this paper, we assume the followings about the system model in (1).

- A1 The information sequences $s(n)$ is zero-mean and white with variance E_s.
- A2 The additive noise $w(n)$ is zero-mean circularly symmetric complex Gaussian (ZMCSCG) with variance σ_w^2.
- A3 The channel $h_i(n)$ is ZMCSCG with unit-variance.
- A4 The information sequences, channels, and noise are jointly independent.
- A5 There is sufficient transmit antenna spacing, therefore, there is no spatial correlation.
- A6 The receiver knows the channel state information (CSI) perfectly.

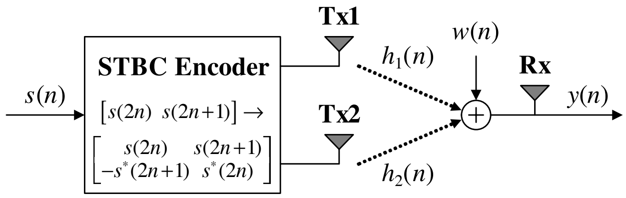

Fig. 1. Space-time block coded transmission diagram.

From the above assumption A1 and A2, we know that the averaged received SNR per symbol is $\mathrm{SNR}_{\mathrm{av}} = E_s/\sigma_w^2$.

Among various channel model, the information theoretic results in [3] have shown that the first-order Gauss-Markov process provides a accurate model for time-selective fading channels and, therefore, will be adopted henceforth. The dynamics of the channel state $h_i(n)$ are modeled by

$$h_i(n) = \alpha h_i(n-1) + v_i(n) \tag{2}$$

where the $v_i(n)$ is the white complex Gaussian with zero-mean and covariance $\sigma_v^2/2$ per dimension and is statistically independent of $h_i(n-1)$. Parameter $\alpha \in [0,1]$ is the fading correlation coefficient that characterizes the degree of time variations. In wireless mobile communications, channel time-varying characteristics arise mainly due to Doppler shifts arising from relative motion between the transmitter and the receiver, and the carrier frequency offsets due to the transmitter-receiver oscillators' mismatch. Denote by f_o the carrier frequency offset and by T_s the symbol duration. We can factorize $h_i(n)$ into

$$h_i(n) = \bar{h}_i(n) e^{j2\pi f_o T_s n}. \tag{3}$$

where $\bar{h}_i(n)$ and $e^{j2\pi f_o T_s n}$ account for the Doppler and the carrier frequency offset effects, respectively. From assumption A3, we know that

$$\sigma_v^2 = 1 - |\alpha|^2, \;\; \alpha = E[h_i(n)h_i^*(n-1)]. \tag{4}$$

The fading correlation coefficient is determined by the channel model, the maximum Doppler frequency f_d and symbol duration T_s. We consider three channel models: classic, uniform, and two-path models, in which the fading correlation coefficients are given by

$$\alpha = \begin{cases} J_0(2\pi f_d T_s) e^{j2\pi f_o T_s}, & \text{classic model} \\ \mathrm{sinc}(f_d T_s) e^{j2\pi f_o T_s}, & \text{uniform model} \\ \cos(2\pi f_d T_s) e^{j2\pi f_o T_s}, & \text{two-path model} \end{cases} \tag{5}$$

where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind.

The receiver observations $y(2n)$ and $y(2n+1)$ corresponding to the two symbol periods are given by

$$\mathbf{y}(n) = \mathbf{H}(n)\mathbf{s}(n) + \mathbf{w}(n) \tag{6}$$

where, $\mathbf{y}(n) = [y(2n)\ y^*(2n+1)]^T$; $\mathbf{s}(n) = [s(2n)\ s(2n+1)]^T$; $\mathbf{w}(n) = [w(2n)\ w(2n+1)]^T$; and the channel matrix

$$\mathbf{H}(n) = \begin{bmatrix} h_1(2n) & h_2(2n) \\ h_2^*(2n+1) & -h_1^*(2n+1) \end{bmatrix}. \tag{7}$$

Because of the white Gaussian noise, the joint maximum-likelihood (ML) detector choose the pair of symbol $\mathbf{s}(n)$ to minimize

$$\| \mathbf{y}(n) - \mathbf{H}(n)\mathbf{s}(n) \|^2 . \tag{8}$$

To decode $\mathbf{s}(n)$, the space-time block decoder is designed by forming the two consecutive output sample vector, $\mathbf{z}(n) = [z(2n)\ z(2n+1)]^T$, as

$$\mathbf{z}(n) = \mathbf{H}^H(n)\mathbf{y}(n). \tag{9}$$

Based on the definition (7), it follows by

$$\mathbf{R}(n) = \mathbf{H}^H(n)\mathbf{H}(n) = \begin{bmatrix} \rho_1(n) & \epsilon(n) \\ \epsilon^*(n) & \rho_2(n) \end{bmatrix} \tag{10}$$

where

$$\begin{aligned} \rho_1(n) &= |h_1(2n)|^2 + |h_2(2n+1)|^2, \\ \rho_2(n) &= |h_1(2n+1)|^2 + |h_2(2n)|^2, \text{ and} \\ \epsilon(n) &= h_1^*(2n)h_2(2n) - h_1^*(2n{+}1)h_2(2n{+}1). \end{aligned} \tag{11}$$

From (9), we know that

$$\begin{aligned} \mathbf{z}(n) &= \begin{bmatrix} \rho_1(n) & 0 \\ 0 & \rho_2(n) \end{bmatrix} \mathbf{s}(n) \\ &+ \begin{bmatrix} 0 & \epsilon(n) \\ \epsilon^*(n) & 0 \end{bmatrix} \mathbf{s}(n) + \mathbf{H}^H(n)\mathbf{w}(n). \end{aligned} \tag{12}$$

The first part in (12) contains the maximum ratio combined signals from the two transmit antennas whereas the second part contains inter-symbol-interference (ISI) on the off-diagonal elements caused by time-selective channels [4].

BER performance analysis of the detector (12) is possible for a given constellation under perfect channel knowledge. We first obtain from (12),

$$\begin{aligned} z(2n) &= \rho_1(n)s(2n){+}\epsilon(n)s(2n+1){+} \\ &\quad h_1^*(2n)w(2n){+}h_2(2n+1)w^*(2n+1). \end{aligned} \tag{13}$$

Treating the interference as noise and after some mathematical manipulation about $h_1(2n)$ and $h_2(2n+1)$, we compute the instantaneous SNR $\gamma(n)$ as following

$$\gamma(n) = \frac{\frac{1}{2}\rho_1^2(n)E_s}{[\rho_1(n)\sigma_v^2 {+} \{\kappa(n){-}\rho_1(n)\}\sigma_v^4]E_s {+} \rho_1(n)\sigma_w^2} \tag{14}$$

where E_s denote the symbol energy of $s(n)$ and $\kappa(n) = |h_1(2n)|^2 \cdot |h_2(2n)|^2$. A similar equation can be obtained for $s(2n+1)$. Supposing that QPSK modulation is used, the BER $P_b(n)$ can be expressed as following equation. We neglecting the fourth order term σ_v^4 (≈ 0). When $E_s \gg \sigma_w^2$ (high SNR), we observe that $P_b(n)$ does not decrease with E_s but approach an error floor given by

$$P_b(n) = Q\left(\sqrt{2\gamma(n)}\right) \approx Q\left(\sqrt{\frac{\rho_1(n)}{\sigma_v^2}}\right). \tag{15}$$

In time-selective fading channels, Alamouti-based decoding scheme has an error floor caused by interference as shown in (12). In order to remove this error floor, we do not model the interference in (12) as noise, but treat it as ISI and propose the decision feedback detector, to decode $\mathbf{s}(n)$ from $\mathbf{y}(n)$, at the cost of smaller receiver complexity.

3. PROPOSED DECISION FEEDBACK DETECTOR

From (10), we know that $\mathbf{R}(n)$ is Hermitian, therefore, it has a unique Cholesky factorization of the form $\mathbf{R}(n) = \mathbf{G}^H(n)\mathbf{G}(n)$, where $\mathbf{G}(n)$ is lower triangular with real diagonal element. With $\mathbf{H}(n)$ defined by (7), we can verify easily that

$$\mathbf{G}(n) = \frac{1}{\sqrt{\rho_2(n)}} \begin{bmatrix} \rho_0(n) & 0 \\ \epsilon^*(n) & \rho_2(n) \end{bmatrix} \tag{16}$$

where

$$\rho_0(n) = |h_1(2n)h_1^*(2n+1) + h_2(2n)h_2^*(2n+1)|. \tag{17}$$

Multiplying both vectors in (8) by the unitary matrix $\left[\mathbf{H}(n)\mathbf{G}^{-1}(n)\right]^H$, we find that the ML detector can be equivalent choose $\mathbf{s}$ to minimize

$$\| \mathbf{x}(n) - \mathbf{G}(n)\mathbf{s}(n) \|^2 . \tag{18}$$

Substituting (6), we find that the output $\mathbf{x}(n)$ is related to $\mathbf{s}(n)$ by

$$\mathbf{x}(n) = [\mathbf{H}(n)\mathbf{G}^{-1}(n)]^H \mathbf{y}(n) = \mathbf{G}(n)\mathbf{s}(n) + \mathbf{n}(n) \tag{19}$$

where the white Gaussian noise $\mathbf{n}(n)$ has the same statistics as $\mathbf{w}(n)$. The decision feedback detector uses a decision about $s(2n)$ to help make a decision about $s(2n+1)$. Because the channel model $\mathbf{G}(n)$ is lower triangular, there is no interference from $s(2n+1)$ to $x(2n)$, and thus a suboptimal decision $\hat{s}(2n)$ can be found by quantizing $x(2n)$ as following

$$x(2n) = \frac{\rho_0(n)}{\sqrt{\rho_2(n)}}\, s(2n) + n(2n). \tag{20}$$

Then, assuming this decision is correct, the contribution from $s(2n)$ in $x(2n+1)$ can be recreated and subtracted off, allowing the receiver to determine the decision $\hat{s}(2n+1)$ by quantizing the resulting difference $x'(2n+1)$ as following

$$\begin{aligned} x'(2n+1) &= x(2n+1) - \frac{\epsilon^*(n)}{\sqrt{\rho_2(n)}}\, \hat{s}(2n) \\ &= \sqrt{\rho_2(n)} s(2n+1) + n(2n+1). \end{aligned} \tag{21}$$

From (21), we know that $x'(2n+1)$ has no interference component, therefore, the receiver detect the symbol by quantizing the resulting $x'(2n+1)$

Let us analyze the BER performance of the proposed decision feedback detector. The BER is thus again of the form $P_1(n) = Q(\sqrt{2\gamma_1(n)})$ and $P_2(n) = Q(\sqrt{2\gamma_2(n)})$ for QPSK modulation, where $\gamma_1(n)$ and $\gamma_2(n)$ are the effective instantaneous SNR for (23) and (24), respectively:

$$\gamma_1(n) = \frac{\rho_0^2(n)E_s}{2\rho_2(n)\sigma_w^2}, \quad \gamma_2(n) = \frac{\rho_2(n)E_s}{2\sigma_w^2}. \tag{22}$$

Firstly, let us compute $\overline{P}_2$ for $x'(2n+1)$. The instantaneous SNR per symbol $\gamma_2(n)$ is given by (22) and averaged SNR per symbol is $E[\gamma_2] = E_s/\sigma_w^2 = \mathrm{SNR_{av}}$. Since $h_1(2n+1)$ and $h_2(2n)$ are independent and Gaussian, therefore, γ_2 has a cental χ^2-distribution with four degrees of freedom [6]. Therefore, we can get averaged BER $\overline{P}_2$ [7, eq. (3.36)]

$$\begin{aligned} \overline{P}_2 &= \int_0^\infty Q\left(\sqrt{2\gamma_2(n)}\right) p(\gamma_2) d\gamma_2 \\ &= \frac{1}{4}\left[1 - \frac{1}{\sqrt{1+\frac{2}{\mathrm{SNR_{av}}}}}\right]^2 \left[2 + \frac{1}{\sqrt{1+\frac{2}{\mathrm{SNR_{av}}}}}\right]. \end{aligned} \tag{23}$$

Secondly, let us obtain the average BER $\overline{P}_1$. $x(2n)$ and instantaneous SNR $\gamma_1(n)$ are given by (20) and (22), respectively. The channel responses $h_1(2n)$ and $h_1(2n+1)$ or $h_2(2n)$ and $h_2(2n+1)$ are mutually correlated complex Gaussian random variables, and can be expressed using a standard transform of random variables as following [6]

$$e_1 = \frac{h_1(2n) - \alpha h_1(2n+1)}{\sqrt{1-|\alpha|^2}}, \; e_2 = \frac{h_2(2n+1) - \alpha^* h_2(2n)}{\sqrt{1-|\alpha|^2}} \tag{24}$$

where e_1 and e_2 are independent and identically distributed with the same pdf as $h_1(2n+1)$ and $h_2(2n)$. Moreover, e_1 and $h_1(2n+1)$ are uncorrelated and e_2 and $h_2(2n)$ are uncorrelated. Using (24), instantaneous SNR γ_1 can be rewritten as

$$\begin{aligned} \gamma_1 &= \left| \alpha\sqrt{\rho_2(n)} + \sqrt{1-\alpha^2} \cdot \frac{h_1^*(2n+1)e_1 + h_2(2n)e_2^*}{\sqrt{\rho_2(n)}} \right|^2 \\ &\cdot \frac{\mathrm{SNR_{av}}}{2}. \end{aligned} \tag{25}$$

To simplify this formula, we replace $h_1^*(2n+1)e_1 + h_2(2n)e_2^*$ with $b_1 + jb_2$, The distribution of $b_1 + jb_2$ reduces to the distribution of $h_1(2n)$, therefore, (25) can be rewritten as following

$$\begin{aligned} \gamma_1 = &\left[\alpha\sqrt{\rho_2(n)\frac{\mathrm{SNR_{av}}}{2}} + \sqrt{(1-|\alpha|^2)\frac{\mathrm{SNR_{av}}}{2}} \cdot b_1\right]^2 + \\ &\left[\sqrt{(1-|\alpha|^2)\frac{\mathrm{SNR_{av}}}{2}} \cdot b_2\right]^2 = (A+B_1)^2 + B_2^2. \end{aligned} \tag{26}$$

Therefore, given A, instantaneous SNR for signal $x(2n)$, γ_1 has a noncentral χ^2-distribution with two degrees of freedom as following [6]

$$\begin{aligned} p_{\gamma_1|A}(\gamma_1|a) &= \frac{4}{(1-|\alpha|^2)\mathrm{SNR_{av}}} \cdot \exp\left[\frac{-4(a^2+\gamma_1)}{(1-|\alpha|^2)\mathrm{SNR_{av}}}\right] \\ &\cdot I_0\left(\frac{8a\sqrt{\gamma_1}}{(1-|\alpha|^2)\mathrm{SNR_{av}}}\right) \end{aligned} \tag{27}$$

where $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind, i.e., $I_0(x) = J_0(jx)$. And A in (26) has a Rayleigh distributed with four degrees of freedom,

$$p_A(a) = \frac{8a}{|\alpha|^2\mathrm{SNR_{av}}} \cdot \exp\left[\frac{-2a^2}{\alpha^2\mathrm{SNR}_{av}}\right]. \tag{28}$$

Therefore, pdf of γ_1 can be obtained using (27) and (28) as following [8, (6.631)]

$$\begin{aligned} p(\gamma_1) &= \int_0^\infty p_{\gamma_1|A}(\gamma_1|a) \cdot p_A(a) da = \frac{2(1-|\alpha|^2)}{\mathrm{SNR_{av}}} \cdot \\ &\exp\left[\frac{-2\gamma_1}{\mathrm{SNR_{av}}}\right] \cdot \left[1 + \frac{2|\alpha|^2 \cdot \gamma_1}{(1-|\alpha|^2)\mathrm{SNR_{av}}}\right] \end{aligned} \tag{29}$$

Averaging $Q\left(\sqrt{2\gamma_2(n)}\right)$ over this pdf, we can obtain the exact BER $\overline{P}_1$ of $x_1(2n)$ [7, eq. (3.36)], [8, (6.283)]

$$\begin{aligned} \overline{P}_1 = &\frac{(1-|\alpha|^2)}{2}\left[1 - \frac{1}{\sqrt{1+\frac{2}{\mathrm{SNR_{av}}}}}\right] + \\ &\frac{|\alpha|^2}{4}\left[1 - \frac{1}{\sqrt{1+\frac{2}{\mathrm{SNR_{av}}}}}\right]^2 \left[2 + \frac{1}{\sqrt{1+\frac{2}{\mathrm{SNR_{av}}}}}\right]. \end{aligned} \tag{30}$$

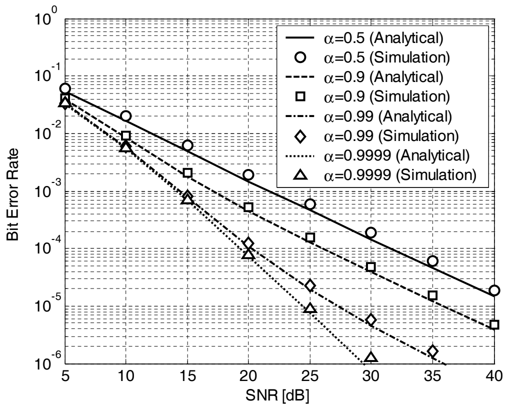

Fig. 2. Performance of the decision feedback detector for different values of α.

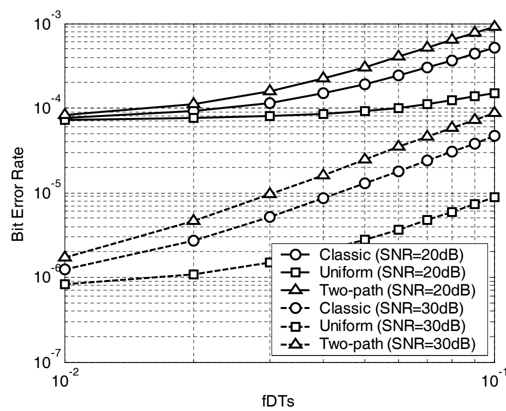

Fig. 3. Comparison of analytical and simulated BER of decision feedback detector with different level of correlation.

Let us express the exact BER of the proposed decision feedback detector. Combining, we can obtain the BER $\overline{P}_b$ as $\overline{P}_b = \frac{1}{2}(\overline{P}_1 + \overline{P}_2)$

$$\overline{P}_b = \left(\frac{1-|\alpha|^2}{4}\right)\left[1-\frac{1}{\sqrt{1+\frac{2}{\mathrm{SNR_{av}}}}}\right] + \left(\frac{1+|\alpha|^2}{8}\right)\cdot \left[1-\frac{1}{\sqrt{1+\frac{2}{\mathrm{SNR_{av}}}}}\right]^2\left[2+\frac{1}{\sqrt{1+\frac{2}{\mathrm{SNR_{av}}}}}\right]. \quad (31)$$

4. SIMULATION RESULTS

In this section, we use computer simulations to evaluate the performance of the analytical and simulated results. We use (3) to generate $h_i(n)$ and assume that QPSK modulation is considered. The generation of $\bar{h}_i(n)$ follows the three channel models as described in Section 2 with the parameters f_d and T_s corresponding to a carrier frequency of 1.9 GHz, a mobile speed of 250 km/h, and a transmission rate of 144 kb/s. It does not exist the carrier frequency offsets. It was shown that the decision feedback detector is robust to carrier frequency offset [4]. QPSK modulation is considered. In this case, fading correlation coefficient α corresponds to 0.9999.

Fig. 2 shows the BER performance of the decision feedback detector for different values of α. The analytical BER performance is given by (31). For simulated results, we average over 5000 channel and noise realizations for each SNR points. The analytical results fit the simulated results correctly. We can see the BER performance depends on the fading correlation coefficient α. Fig. 3 shows the BER versus f_dT_s when perfect channel estimation is assumed in three channel models as described in Section 2. As shown in this figure, the decision feedback detector in the uniform channel gives the best BER performance and has more robustness to channel correlation than the others.

5. CONCLUSION

We investigate the impact of fading correlation on the performance of decision feedback detection scheme for Alamouti transmit diversity scheme in time-selective fading channels and derive an exact average BER formula over time-selective fading channels. Thorough analytical results, we evaluate the performance over time-selective fading channels are described by the fading correlation coefficient α. Some numerical examples are presented in order to verify the results.

6. REFERENCES

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