DECISION FUSION IN A WIRELESS SENSOR NETWORK WITH A RANDOM NUMBER OF SENSORS

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ABSTRACT

For a wireless sensor network (WSN) with a random number of sensors, a decision fusion rule that uses the total number of detections reported by local sensors for hypothesis testing, is proposed. It is assumed that the number of sensors follows a Poisson distribution and the locations of sensors follow a uniform distribution within the region of interest (ROI). Both analytical and simulation results for the system level detection performance are provided. This fusion rule can achieve a very good system level detection performance even at very low signal to noise ratio (SNR), if the average number of sensors is sufficiently large. In addition, the problem of choosing an optimum local sensor level threshold is investigated for various system parameters.

1. INTRODUCTION

One of the most important tasks a WSN needs to perform is target detection, typically in a distributed manner. There are already numerous papers in the literature on the conventional distributed detection problem [1, 2, 3, 4].

However, most of these results are based on the assumption that the local sensors' performances are known. For a dynamic target and passive sensors, it is very hard to estimate local sensors' performances via experiments because these performances are time-varying as the target moves through the wireless sensor field. Even if the local sensors can somehow estimate their detection performances in real time, it will be very expensive to transmit them to the fusion center, especially for a WSN with very limited system resources. On the other hand, usually a WSN consists of a large number of low-cost and low-energy sensors, which are densely deployed in the environment. Taking advantage of these unique characteristics of WSNs, in our previous paper [5], we proposed a fusion rule that uses the total number of detections ("1"s) transmitted from local sensors as the statistic.

In [5], we assumed that the total number of sensors in the ROI is known. However, in many applications, the sensors are deployed randomly in and around the ROI, and oftentimes some of them are malfunctioning or out of battery.

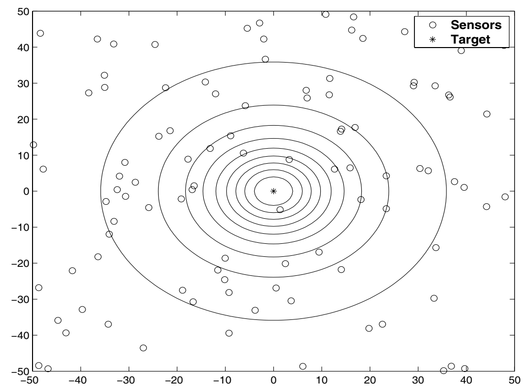

Fig. 1. The signal power contour of a target located in a sensor field.

Therefore, in practice the total number of sensors that work properly in a ROI is a random variable. In this paper, the performance of the fusion rule proposed in [5] will be analyzed with this extra uncertainty about the total number of sensors.

2. MODELING AND DECISION FUSION RULE

2.1. Problem Formulation

As shown in Fig. 1, a total of N sensors are randomly deployed in the ROI, which is a square with area b^2. N is a random variable that follows a Poisson distribution:

$$p(N) = \frac{\lambda^N e^{-\lambda}}{N!} \qquad for \quad N = 0, \cdots, \infty \tag{1}$$

The locations of sensors are i.i.d. and follow a uniform distribution in the ROI.

Noises at local sensors are i.i.d and follow the standard Gaussian distribution with zero mean and unit variance. For a local sensor i, the binary hypothesis testing problem is:

$$\begin{aligned} H_1 : \quad s_i &= a_i + n_i \\ H_0 : \quad s_i &= n_i \end{aligned} \tag{2}$$

where s_i is the received signal, and a_i is the signal amplitude. We adopt the same isotropic signal power attenuation model as that presented in [5]:

$$a_i^2 = \frac{P_0}{1 + \alpha d_i^n} \tag{3}$$

where P_0 is the signal power emitted by the target at distance zero, d_i is the distance between the target and local sensor i. n is the signal decay exponent and takes values between 2 and 3. α is an adjustable constant. Because the noise has unit variance, it is evident that the SNR at distance zero is

$$SNR_0 = 10 \log_{10} P_0 \tag{4}$$

Assuming that all the local sensors use the same threshold τ to make a decision, we have the local sensor level false alarm rate and probability of detection:

$$\begin{aligned} p_{fa} &= \int_\tau^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \\ &= Q(\tau) \end{aligned} \tag{5}$$

$$p_{d_i} = Q\left(\tau - a_i\right) \tag{6}$$

where $Q(\cdot)$ is the complementary distribution function of the standard Gaussian.

We assume that the ROI is very large and the signal power decays very fast. Hence, only within a very small fraction of the ROI, which is the area surrounding the target, the received signal power is significantly larger than 0. By ignoring the border effect of the ROI, we assume the target is located at the center of the ROI, without any loss of generality.

2.2. Decision Fusion Rule

Since it is difficult to estimate p_{d_i}s at local sensors, the fusion center is forced to treat every sensor equally. As proposed in [5], the system level decision is made by first counting the number of detections made by local sensors and then comparing it with a threshold T:

$$\Lambda = \sum_{i=1}^{N} I_i \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{7}$$

where $I_i = \{0,\ 1\}$ is the local decision made by sensor i.

3. PERFORMANCE ANALYSIS

3.1. System Level False Alarm Rate

At the fusion center level, the probability of false alarm P_{fa} is

$$P_{fa} = \sum_{N=T}^{\infty} p(N) Pr\{\Lambda \geq T | N, H_0\} \tag{8}$$

Obviously, for a given N, under hypothesis H_0, Λ follows a Binomial (N, p_{fa}) distribution, and

$$P_{fa} = \sum_{N=T}^{\infty} p(N) \sum_{i=T}^{N} \binom{N}{i} p_{fa}^i (1 - p_{fa})^{N-i} \tag{9}$$

It is well known that the Kurtosis of a Poisson distribution is $3 + \frac{1}{\lambda}$. As λ increases, the Kurtosis of this Poisson distribution approaches that of a Gaussian distribution, and its distribution has a light tail. As a result, when λ is large, the probability mass of N will concentrate around the average value (λ), and

$$\sum_{N_1}^{N_3} \frac{e^{-\lambda}\lambda^N}{N!} \approx 1 \tag{10}$$

where $N_1 = \left\lfloor \lambda - 6\sqrt{\lambda} \right\rfloor$ and $N_3 = \left\lceil \lambda + 6\sqrt{\lambda} \right\rceil$.

Hence, for a large λ, a "typical" N is also a large number. The probability that N takes a small value is negligible. For example, when $\lambda = 1000$, $Pr\{N < 810\} = 2.4 \times 10^{-10}$. Therefore, when λ is large enough, (9) can be calculated by using Laplace-DeMoivre approximation [6]:

$$P_{fa} \simeq \sum_{N=N_2}^{N_3} \frac{\lambda^N e^{-\lambda}}{N!} Q\left(\frac{T - N p_{fa}}{\sqrt{N p_{fa}(1 - p_{fa})}} \right) \tag{11}$$

where $N_2 = max(T,\ N_1)$.

3.2. System Level Probability of Detection

In [5], through approximation by using Central Limit Theorem, we derived the system level P_d when the number of sensors N is large:

$$Pr\{\Lambda \geq T | N, H_1\} \simeq Q\left(\frac{T - N\bar{p_d}}{\sqrt{N\bar{\sigma^2}}} \right) \tag{12}$$

where

$$\bar{p_d} = \frac{2\pi}{b^2} \int_0^{\frac{b}{2}} C(r) r dr + \left(1 - \frac{\pi}{4}\right) p_{fa} \tag{13}$$

$$\begin{aligned} \bar{\sigma^2} = \frac{2\pi}{b^2} \int_0^{\frac{b}{2}} &\left[1 - C(r)\right] C(r) r dr \\ &+ \left(1 - \frac{\pi}{4}\right) p_{fa}(1 - p_{fa}) \end{aligned} \tag{14}$$

and

$$C(r) = Q\left(\tau - \sqrt{P_0/(1 + \alpha r^n)}\right) \tag{15}$$

Similar to the derivation of (11), we have

$$P_d \simeq \sum_{N=N_2}^{N_3} \frac{\lambda^N e^{-\lambda}}{N!} Q\left(\frac{T - N\bar{p_d}}{\sqrt{N\bar{\sigma^2}}} \right) \tag{16}$$

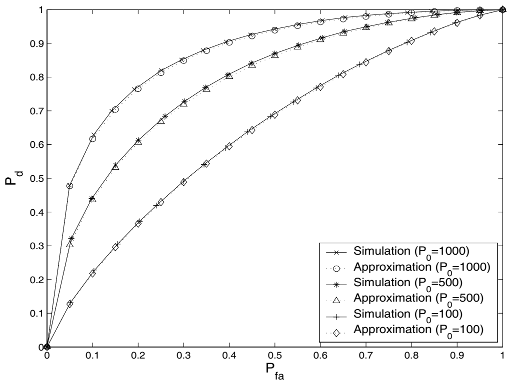

Fig. 2. ROC curves obtained by calculation and simulations. $N = 1000$, $n = 2$, $b = 100$, $\alpha = 200$, and $\tau = 0.77,\ 0.73,\ 0.67$ for $P_0 = 1000,\ 500,\ 100$, respectively.

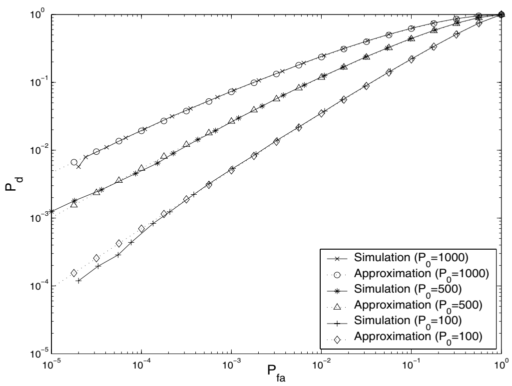

Fig. 3. ROC curves obtained by calculation and simulations. System parameters are the same as those listed in Fig. 2.

3.3. Simulation Results

In Figs. 2 and 3, the receiver operative characteristic (ROC) curve obtained by using approximations in Section 3.1 and 3.2 and that by simulations are plotted. The simulation results in Figs. 2 and 3 are based on 10^5 and 10^6 Monte Carlo runs, respectively. From Figs. 2 and 3, it is clear that the results by using approximations are very close to those obtained by simulations, even when the system level P_{fa} is very low (Fig. 3).

3.4. Asymptotic Analysis

In (11), we know that

$$max\left(T,\ \left\lfloor \lambda - 6\sqrt{\lambda} \right\rfloor\right) \leq N \leq \left\lceil \lambda + 6\sqrt{\lambda} \right\rceil \qquad (17)$$

Hence, as $\lambda \to \infty$, we have $N \to \lambda$, if $T \leq \left\lceil \lambda + 6\sqrt{\lambda} \right\rceil$. Assume that the system level threshold is in the form of

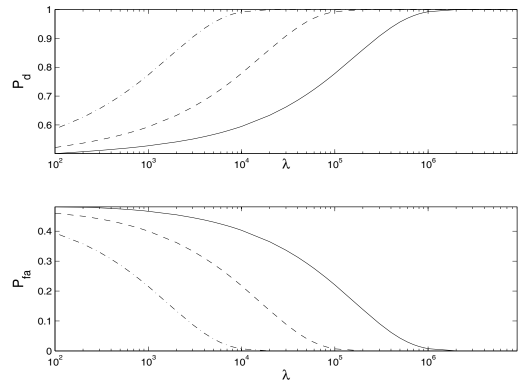

Fig. 4. System level performances as functions of λ. $n = 2$, $b = 100$, $\alpha = 200$, and $\tau = 0.5$. Dashdot line: $P_0 = 1000$, dashed line: $P_0 = 500$, solid line: $P_0 = 100$.

$T = \beta\lambda$, we have

$$P_{fa} \simeq \sum_{N=N_2}^{N_3} \frac{\lambda^N e^{-\lambda}}{N!} Q\left(\frac{(\beta - p_{fa})\sqrt{\lambda}}{\sqrt{p_{fa}(1-p_{fa})}}\right) \qquad (18)$$

Similarly, from (16) we have

$$P_d \simeq \sum_{N=N_2}^{N_3} \frac{\lambda^N e^{-\lambda}}{N!} Q\left(\frac{(\beta - \bar{p_d})\sqrt{\lambda}}{\sqrt{\bar{\sigma^2}}}\right) \qquad (19)$$

Therefore, when $\lambda \to \infty$, if $\beta < p_{fa}$, $P_{fa} = P_d = 1$; if $p_{fa} < \beta < \bar{p_d}$, $P_{fa} = 0$ and $P_d = 1$; if $\beta > \bar{p_d}$, $P_{fa} = P_d = 0$. As a result, as long as β takes a value between p_{fa} and $\bar{p_d}$, as $\lambda \to \infty$, the WSN's detection performance will be perfect with $P_d = 1$ and $P_{fa} = 0$. In Fig. 4, P_d and P_{fa} as functions of λ are plotted. It is clear that the P_d converges to 1 as λ increases and P_{fa} converges to 0. In this example, we set β such that $\beta = \frac{p_{fa}+\bar{p_d}}{2}$. Another conclusion is that when λ is large enough, even for a small SNR_0, the system can achieve a very good detection performance.

4. THRESHOLD FOR LOCAL SENSORS

In this paper, we will find the optimum local sensor level threshold τ by maximizing the so-called deflection coefficient, which is defined as

$$D(\Lambda) = \frac{[E(\Lambda|H_1) - E(\Lambda|H_0)]^2}{Var(\Lambda|H_0)} \qquad (20)$$

In the case of $Var(\Lambda|H_1) = Var(\Lambda|H_0)$, this is in essence the SNR of the detection statistic.

Under hypothesis H_0, we have

$$E(\Lambda|N, H_0) = Np_{fa} \qquad (21)$$

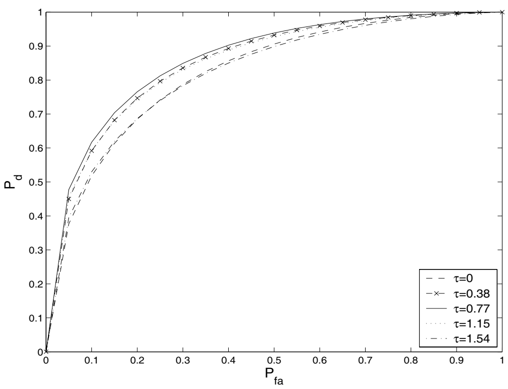

Fig. 5. ROC curves for system with different τ. $\lambda = 1000$, $n = 2, b = 100, \alpha = 200, SNR_0 = 30dB$.

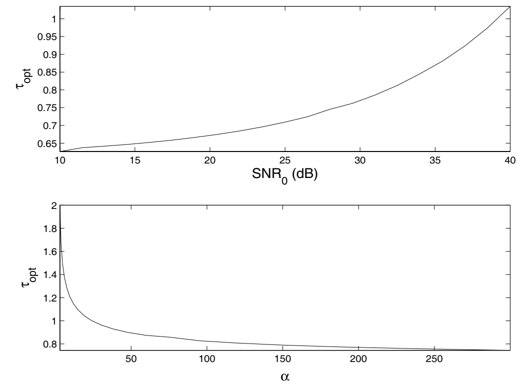

Fig. 6. Optimal τ_{opt} as functions of SNR_0 and α. $\lambda = 1000$, $n = 2$, $b = 100$. In top figure: $\alpha = 200$; in bottom figure: $SNR_0 = 30dB$.

and

$$Var(\Lambda|N, H_0) = Np_{fa}(1 - p_{fa}) \tag{22}$$

With (21) and (22), it is easy to show that

$$E(\Lambda|H_0) = \lambda p_{fa} \tag{23}$$

and

$$Var(\Lambda|H_0) = \lambda p_{fa} \tag{24}$$

Similar to the derivation of (23), we have

$$E(\Lambda|H_1) = \lambda \bar{p_d} \tag{25}$$

Therefore, the deflection coefficient is

$$D(\tau) = \frac{\lambda[\bar{p_d}(\tau) - p_{fa}(\tau)]^2}{p_{fa}(\tau)} \tag{26}$$

The optimum τ can be found by maximizing $D(\tau)$ with respect to τ.

As we can see in Fig. 5, the ROC curve corresponding to the optimal threshold τ_{opt} (0.77) is above those for other thresholds, meaning that τ_{opt} provides the best system level performance. In Fig. 6, τ_{opt} as functions of SNR_0 and α are shown. It is clear that τ_{opt} is a monotone increasing function of SNR_0 and a monotone decreasing function of α. This is because with a strong target signal (high SNR_0 and low α), by adopting a higher threshold, local sensors lower their false alarm rate, while at the same time they can still maintain a relatively high probability of detection.

5. CONCLUSIONS

We have analyzed the performance of a decision fusion rule that is based on the total number of detections made by local sensors, for a WSN with a random number of sensors. The number of sensors in a ROI has been modeled as a Poisson random variable. We have shown that even at very low SNR, this fusion rule can achieve a very good system level detection performance given that there are, on an average, a sufficiently large number of sensors deployed in the ROI.

To achieve a better system performance, we also design an optimum threshold at the local sensors by maximizing the deflection coefficient. Guidelines on how to choose this optimal threshold are provided for various system parameters.

6. REFERENCES

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