QUANTIZER DESIGN FOR SOURCE LOCALIZATION IN SENSOR NETWORKS

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ABSTRACT

In this paper, we propose a quantizer design algorithm that is optimized for source localization in sensor networks. For these applications, the goal is to minimize the amount of information that the sensor nodes have to exchange in order to achieve a certain source localization accuracy. We show that to achieve this goal requires the use of "application-specific" quantizers. Our proposed quantizer design algorithm uses a cost function that takes into account the distance between the actual source position and the position estimated based on quantized data. We apply this algorithm to a system where an acoustic sensor model is employed for localization. For this case we introduce the Equally Distance-divided Quantizer (EDQ), designed so that quantizer partitions correspond to a uniform partitioning in terms of distance. Our simulations show the improved performance of our quantizer over traditional quantizer designs. They also show that an optimized bit allocation leads to significant improvements in localization performance with respect to a bit allocation that uses the same number of bits for each node.

1. INTRODUCTION

In sensor networks, multiple correlated observations are available from many sensors that can sense, compute and communicate. It should be noted that these sensors are battery-powered with strict limitations on wireless communication bandwidth, which motivates the use of data compression in the context of various tasks such as detection, classification, localization and tracking, which require data exchange between sensors. The basic strategy for reducing the overall energy usage in the sensor network would then be to decrease the communication cost at the expense of additional computation in the sensors [6].

One important sensor collaboration task with broad applications is source localization. In [1], localization based on acoustic energy measured at individual sensors is considered, where each sensor transmits unquantized acoustic energy readings to the central node, which then computes an estimate of the location of the source of these acoustic signals. Clearly, practical systems will require quantization of these energy readings before transmission and thus different quantizer designs should be compared in terms of localization error, defined as the average of the distance between the actual source location and its estimated value based on received quantized data.

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Since standard scalar quantizers are focused on minimizing the average distortion between the actual sensor reading and its quantized value, there is no guarantee that they will be optimal in the sense of minimizing localization error. Thus, the quantizer design should be "application-specific". That is, to design optimal quantizers, a new metric should be defined to maximize the accuracy of the application objective. For an example of application specific quantizer design for time-delay estimation see [4]. In the localization problem, the metric to minimize is the localization error. A challenging aspect of this problem is that, while quantization has to be performed independently at each node, the metric of interest is based on the localization error, which depends on the readings from *all* quantizers. Thus we have a problem where independent (scalar) quantizers for each node have to be optimized based on a global (vector) cost function.

To solve this problem, we will propose an iterative design algorithm, similar to that proposed in [3]. Furthermore, based on the approach from [1], we apply our algorithm to a system where an acoustic sensor model is considered. We also study the bit allocation problem (i.e., determining the number of bits to be used by each sensor) and we provide a solution with the introduction of the Equally Distance-divided Quantizer (EDQ), which is simple and provides good performance in the acoustic sensor model case. Overall, our experiments demonstrate the benefits of using application-specific designs. The bit allocation results show that bits should be distributed so as to lead to a partition of the sensor field that is as uniform as possible. Thus, for example, we will see that when several nodes are clustered together, the number of bits per node tends to be lower than when the same sensors are more spread out.

This paper is organized as follows. The problem formulation and target cost function are introduced in Section 2. The quantizer design algorithm is proposed in Section 3. In Section 4, we present an application to the case where an acoustic sensor model is employed. Section 5 discusses the bit allocation problem. Simulation results are given in Section 6 and the conclusions are found in Section 7.

2. PROBLEM FORMULATION

Suppose that there are M nodes in the sensor field S, which measure signals generated from a source assumed to be static during the localization process. We assume that the sensor reading at node i can be modeled by a function $f(x, x_i, \mathbf{P_i})$ where x is the source location, x_i is the position of node i, and $\mathbf{P_i}$ is the parameter vector for the sensor model. It is also assumed that the positions of all nodes $x_i, i = 1, ..., M$, are known and each node senses its observation $z_i(x, k)$ at time interval k, quantizes it with a given rate R_i, and sends it to a central node, where all sensor readings are used to obtain an estimate of the source location $\hat{x}$. As an example, $z_i(x, k)$ could be the energy of an acoustic signal during the k-th interval, where each interval has a predetermined duration. We assume that the central node will determine the location of a source based on $z_i(x, k)$'s obtained from all nodes. In some cases, one reading per node is used, while in other cases values of $z_i(x, k)$ for several k are needed for localization. Clearly, for $z_i(x, k)$ to be useful for localization, it must be a function of relative positions of the source and the node:

$$z_i(x, k) = f(x, x_i, \mathbf{P_i}) + w_i(k) \quad \forall i = 1, ..., M, \qquad (1)$$

and thus there exists some function $g(.)$ that can provide an estimate of the source location $\hat{x}$ based on quantized observations

$$\hat{x} = g(\alpha_1(z_1), ..., \alpha_M(z_M)), \qquad (2)$$

where $\alpha_i(.)$ is the encoder at node i. Obviously, the localization function $g(.)$ should be closely related to the sensor model $f(.)$.

To design the optimal quantizer at node i, i.e., the one that minimizes the localization error, we define a cost function $J_i(x)$ as follows

$$\begin{aligned} J_i(x) &= \| z_i - \hat{z}_i \|^2 + \lambda \| x - \hat{x} \|^2 \quad \forall x \in S && (3) \\ J_{avg} &= E(J_i(x)) = \int_S J_i(x)p(x)dx && (4) \end{aligned}$$

where J_{avg} is averaged based on prior knowledge of the probability density function of the source locations $p(x)$. If no prior information is available about the likelihood of possible source locations, $p(x)$ could be made uniform over the sensor field. For the purpose of training our quantizer we will generate a training set of observations $\{z_1(x, k), ..., z_M(x, k)\}$ based on the sensor model, $f(x, x_i, \mathbf{P_i})$ with a given choice of $p(x)$. Our algorithm is aimed at finding 2^{R_i} quantizer bins for i-th quantizer so that J_{avg} is minimized. The cost function $J_i(x)$ can be rewritten in terms of the M quantizers

$$\begin{aligned} &J_i(x, \alpha_i(z_i)) = \| z_i - \beta_i(\alpha_i(z_i)) \|^2 + \\ &\lambda \| x - g(\alpha_1(z_1(x, k)), \alpha_2(z_2(x, k)), ..., \alpha_M(z_M(x, k))) \|^2 \qquad (5) \end{aligned}$$

where $\beta_i(.)$ is the decoder corresponding to node i.

3. QUANTIZER DESIGN ALGORITHM

Our goal is to design a set of encoders, each operating independently on the observations of one node, so as to minimize the average of the cost function (5), when the entire vector $[\alpha_1(.), ...\alpha_M(.)]$ is used for localization. The algorithm should seek to design independent quantizers for each node, while taking into account their combined effect on localization. The generalized Lloyd algorithm (GLA) is used to design the encoder at each node. Note that the cost function in (5) is dependent on the encoders at other nodes, and thus we will propose an iterative procedure, where the quantizer at node i is optimized while the quantizers for the other nodes remain unchanged. This iterative method is based on that proposed in [3].

Note that the quantizer training phase makes use of information about all nodes, but when the resulting quantizers are actually used, each node quantizes the information available to it independently (equivalently, the cost function $J_i(x)$ is used for encoding after setting $\lambda = 0$.)

The proposed algorithm is summarized as follows. For simplicity $z_i(x, k)$ is written as $z_i(x)$.

Step1 : Initialize the encoders $\alpha_i(.), i = 1, ..., M$ with the dynamic range $[z_{min} \quad z_{max}]$. Set the thresholds ϵ_1 and ϵ_2, set $i = 1$, and set iteration indices $k = 0$ and $k_1 = 0$.
Step2 : Compute the cost function, (5).
Step3 : Compute the quantization regions V_i^j

$$V_i^j = \{x : J_i(x, \alpha_i = Q_i^j) < J_i(x, \alpha_i = Q_i^m), \forall m \neq j\} \quad (6)$$

where $j, m = 1, ..., 2^{R_i}$ and Q_i^j is the j-th quantizer bin of $\alpha_i(.)$.
Step4 : Compute the average cost $J_{avg}^k = E_x(J_i(x))$
Step5 : If $\frac{(J_{avg}^{k-1} - J_{avg}^k)}{J_{avg}^k} < \epsilon_1$ go to Step 7; otherwise continue
Step6 : $k = k + 1$. Update the quantization bin Q_i^j as follows.

$$\begin{aligned} \hat{z}_i^j &= E(z_i(x) | x \in V_i^j) \\ Q_i^j &= [b_i^{j-1} \quad b_i^j] \quad \forall j = 1, ..., 2^{R_i} && (7) \end{aligned}$$

where $b_i^j = \frac{1}{2}(\hat{z}_i^j + \hat{z}_i^{j+1})$, $b_i^0 = z_{min}$, $b_i^{2^{R_i}} = z_{max}$. Go to Step 2
Step7 : if $i < M \quad i = i + 1$ go to step 2;
else if $\frac{D^{k_1-1}(x,\hat{x}) - D^{k_1}(x,\hat{x})}{D^{k_1}(x,\hat{x})} < \epsilon_2$ Stop;
else $i = 1$; $k_1 = k_1 + 1$;Go to Step 2.
where $D^{k_1}(x, \hat{x}) = E(\| x - \hat{x} \|^2)$. As discussed above, the training set is generated based on known values of $\mathbf{P_i}$ and $p(x)$. A discussion of the robustness of our approach to model mismatches is left for Section 6.

4. APPLICATION TO ACOUSTIC SENSOR MODEL

As an example, we consider source localization based on acoustic signal energy as proposed in [1], where an energy decay model of sensor signal readings is used for localization based on unquantized sensor readings. The acoustic sensor model is given by

$$z_i(x, k) = g_i \frac{a}{(x - x_i)^\alpha} + w_i(k) \qquad (8)$$

where $z_i(x, k)$ is the signal energy measured by node i over a given time interval, and the parameter vector $\mathbf{P_i}$ consists of the gain factor of ith sensor g_i, an energy delay factor α, which is approximately equal to 2, and the combined noise term $w_i(k)$ for the measurement noise and the modelling error that might exist. In (8), it is assumed that the signal energy, a, is known and varied over $[a_{min} \quad a_{max}]$, and a source generates a constant energy during localization. Localization based on quantized observations is illustrated by Figure 1, where each ring-shaped area can be obtained from one quantized observation provided by a sensor. By computing the intersection of all the ring areas (one per sensor), it is possible to define the area where the source is expected to be located. Note that at least three observations are required to achieve a connected intersection. This can be written as follows

$$A = \bigcap_{i=1}^{M} A_i \quad A_i = \{x : g_i \frac{a}{(x - x_i)^\alpha} \in Q_i^j\} \qquad (9)$$

where A_i is the ring-shaped region obtained from the quantized observation $\hat{z}_i$ at node i and Q_i^j is the quantizer bin that $\hat{z}_i$ falls

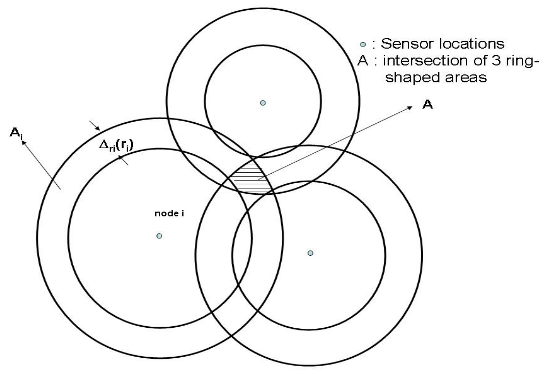

Fig. 1. Localization of the source based on quantized energy readings

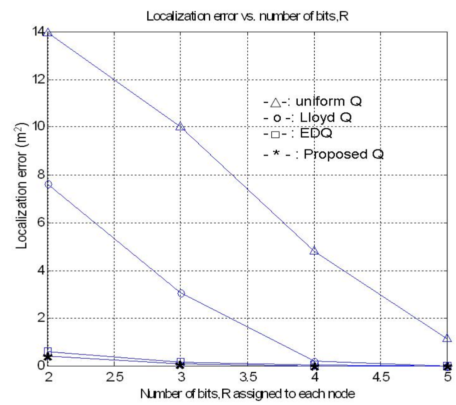

Fig. 2. Localization error vs. the number of bits, R_i assigned to each node. The localization error is given by $E(\parallel x - \hat{x} \parallel^2)$

into. If the source is uniformly distributed in the sensor field, the estimate, $\hat{x}$ would be the sample mean in the intersection A

$$\hat{x} = E(x|x \in A) \tag{10}$$

To avoid quantizer overload, the dynamic ranges of the M quantizers are initialized as $[z_{min} \quad z_{max}] = [\frac{a_{min}}{r^2_{max}} \quad \frac{a_{max}}{r^2_{min}}]$ where $[r_{min} \quad r_{max}]$ is the range within which each sensor is supposed to measure acoustic source energy. The values of r_{max} are set such that the probability that an arbitrary point inside the sensor field can be sensed simultaneously by at least 3 nodes should be close to 1 [5]. In this way, the likelihood of missing a source is minimized. To have finite dynamic ranges, the values of r_{min} are chosen as small non-zero values. Note that if more nodes are used, better quantization in each node is possible (the dynamic ranges will tend to be smaller). With this initialization step, the quantizer design as outlined in Section 3 can be used.

5. EQUALLY DISTANCE-DIVIDED QUANTIZER AND BIT ALLOCATION PROBLEM

Since each set of quantizers induces a partitioning of the sensor field, designing good quantizers for localization can be seen to be equivalent to making a good partition of the sensor field by adjusting the width, $\Delta_{r_i}(r_i)$ of the ring-shaped areas in Figure 1. If no prior information is available about the source location, $p(x)$ can be assumed to be uniform and thus choosing $\Delta_{r_i}(r_i)$ to achieve a uniform partitioning of the sensor field would seem to be a good choice. Intuitively, a uniform partitioning of the sensor field is more likely to be achieved when the ring-shaped areas have the same width, $\Delta_{r_i}(r_i) = const$ (this is certainly the case when the nodes are uniformly distributed). This consideration leads to the introduction of Equally Distance-divided Quantizers (EDQ), which can be viewed as uniform quantizers in distance, and such that $\Delta_{r_i}(r_i) = \frac{r_{max}-r_{min}}{2^{R_i}} \quad \forall i$. To justify the EDQ design, we performed a simulation (see Figure 2) which shows that EDQ provides good localization performance, which comes close to that achievable by our proposed optimal quantizer. EDQ has the added advantage of facilitating the solution of the bit allocation problem.

Given a total number of bits, $R_T = \sum R_i$, the goal is to minimize the localization error by allocating different number of bits to each node. Even though the GBFOS algorithm [2] provides the optimal bit allocation, it would also require extremely large computational load, since it relies on the calculation of rate-distortion points at each iteration step, and the quantizers should be redesigned using the algorithm of Section 3 for each candidate bit allocation. Instead, in our experiments we use the GBFOS algorithm along with EDQ, which does not require quantizer redesign for each candidate bit allocation.

6. SIMULATION

The proposed quantizer was designed by the algorithm in Section 3, using a training set with 1532 source locations generated with a uniform distribution in a sensor field of size $10 \times 10m^2$, where 5 nodes are randomly located (Figure 3). The model parameters are given by $a = 50, \alpha = 2, g_i = 1$ and $SNR = \infty$, and the localization error is computed by $E(\parallel x - \hat{x} \parallel^2)$. In Figure 2, the localization error is compared with traditional quantizers such as uniform quantizers and Lloyd quantizers ($\lambda = 0$). Since the proposed quantizer makes full use of the distributed property of the observations, it can be seen to provide improved performance over the traditional quantizers. This can be also explained in terms of the partitioning of the sensor field, which is plotted in Figure 3 and 4. It is easily seen that the our quantizer leads to a more uniform partitioning, which in turn reduces the average localization error. In this simulation, we assume that when the source is very close to one of the nodes, the node position becomes an estimate of the source position. The localization error due to this assumption can be reduced by lowering the value of r_{min} at the expense of large dynamic range.

The proposed quantizer was tested to see how it would work under various types of mismatches. In each test we modify one of the parameters with respect to what was assumed during training. The simulation results are tabulated in Table 1. In this experiment, 1481 and 1176 source locations in the sensor field 10×10, were generated with assumption of uniform distribution and normal distribution, respectively. Localization is performed using the true parameters, even when there is mismatch. The proposed quantizers showed good performance for the various parameter perturbations. That is, there is no need to redesign quantizers when there are tol-

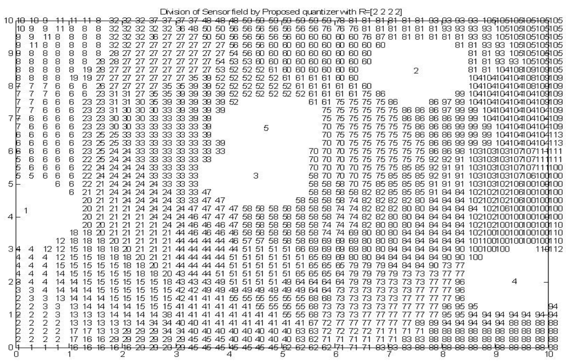

Fig. 3. Partitioning of Sensor field$(10\times 10m^2)$(grid$= 0.25\times 0.25$) by proposed quantizers. All partitioned regions are numbered, so that a region is filled with the same number ($R_i = 2$)

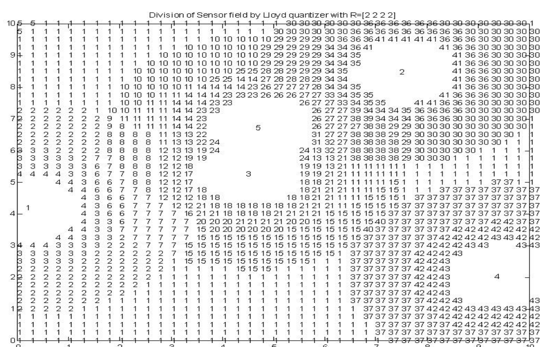

Fig. 4. Partitioning of Sensor field$(10\times 10m^2)$(grid$= 0.25\times 0.25$) by Lloyd quantizer. All partitioned regions are numbered ($R_i = 2$)

erable parameter mismatches. In a large sensor field, they also provided good results with respect to traditional quantizers.

In the same node configuration as in Figure 3, the bit allocation was conducted using EDQ to search for the optimal bit allocation R^*, that would give the minimum localization error. It can be seen that nodes 3 and 5 are so close to each other that they provide redundant information for localization and thus the optimal solution allocates few bits to both these nodes. In fact, in our example, at relatively low rates (an average of 2 bits per node) it is more efficient to send information from only three nodes (node 1,2 and 4), i.e., allocating zeros bits for the other two nodes (node 3 and 5). In Table 2, the localization errors are compared for several different bit allocations, showing that bit allocation is important to achieve good localization performance.

7. CONCLUSION

In this paper, we have presented a quantizer design algorithm for source localization in sensor networks. We have also discussed the bit allocation problem and introduced a simple quantizer that shows good performance when acoustic sensor models are employed. In our experiments, based on the acoustic sensor model, our approach outperforms traditional quantizers and provides good results under various types of mismatches. In the future, we will work on the case where the source signal energy is unknown. For this case, a new localization algorithm should be developed.

Source energy a	40	45	50	55	60
LE(normal)	0.0657	0.0706	0.0788	0.0982	0.1196
LE(uniform)	0.0710	0.0615	0.0727	0.0779	0.0946
Delay factor α	1.6	1.8	1	2.2	2.4
LE(normal)	0.1676	0.1247	0.0788	0.0467	0.0557
LE(uniform)	0.3070	0.1206	0.0727	0.0786	0.6941
Gain factor g_i	0.6	0.8	1	1.2	1.4
LE(normal)	0.0466	0.0657	0.0788	0.1196	0.1731
LE(uniform)	0.0713	0.0710	0.0727	0.0946	0.1326
SNR(dB)	20	40	60	80	100
LE(normal)	1.4700	0.1674	0.0822	0.0796	0.0788
LE(uniform)	2.3811	0.1235	0.0831	0.0738	0.0727

Table 1. Localization error(LE)(m^2) of the proposed quantizers under various mismatches. Localization error(LE) is given by $E(\| x-\hat{x} \|^2)$. LE(normal) is for normal distribution and LE(uniform) for uniform distribution. The proposed quantizers are designed with $R_i = 3, a = 50, \alpha = 2, g_i = 1$ and $SNR = \infty$ for uniform distribution.

Sets of bit allocations	EDQ	Proposed Quantizer
$R^* = [4 \quad 3 \quad 0 \quad 3 \quad 0]$	0.1533	0.1105
$R \quad = [3 \quad 4 \quad 0 \quad 3 \quad 0]$	0.1543	0.1227
$R \quad = [3 \quad 2 \quad 2 \quad 3 \quad 0]$	0.3005	0.2014
$R \quad = [2 \quad 2 \quad 2 \quad 2 \quad 2]$	0.6199	0.3975

Table 2. Localization error(m^2) for various sets of bit allocations where R^* was obtained by GBFOS using EDQ given $R_T = 10$.

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