SENSOR–FUSION CENTER COMMUNICATION OVER MULTIACCESS FADING CHANNELS

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ABSTRACT

We study the problem of communicating sensor readings over a multiaccess channel. Previous works focused on the approach that each sensor is allocated an orthogonal channel to transmit its data as in TDMA (Time-Division Multiple Access). In this paper, we propose an alternative method in which the sensors transmit *simultaneously* to deliver a noisy version of the *type* of sensor observations. We analyze the estimation/detection performance of this Type-Based Multiple Access (TBMA) approach in inferring a parameter θ. The data at different sensors are modeled as conditionally independent given θ. An asymptotic performance analysis is presented, and significant gains in estimation/detection performance and bandwidth usage are demonstrated.

1. INTRODUCTION

Main functions of wireless sensor networks include sensing of a physical phenomena, and the delivery of the sensed data. Since sensor data are correlated, the efficiency is improved by processing the data locally by a *fusion center* and then delivering a compressed version.

We consider the up-link communication between n sensors and a fusion center. The fusion center is interested in estimating or detecting a parameter θ which affects the distribution of sensor data. It is assumed that the data of each sensor, X_i, can take k possibilities (*i.e.,* $X_i \in \{1, \cdots, k\}$).[1] Moreover, $X_1, \cdots, X_n$ are assumed i.i.d. conditional on the parameter θ.

A conventional approach to the up-link problem is to allocate orthogonal dimensions to each sensor as in TDMA or FDMA. In this paper, we consider an alternative method in which transmissions from different sensors overlap with one another, but different *observations* are assigned orthogonal dimensions. That is, let $s_1, \cdots, s_k$ be orthonormal waveforms. In the proposed approach, sensor i transmits waveform $\sqrt{E} s_{X_i}$, where E is the energy available for transmission (assumed to be the same for all sensors). The received signal at the fusion center is modeled as

$$z = \sum_{i=1}^{n} h_i \sqrt{E} s_{X_i} + w, \tag{1}$$

where $h_i \in \mathbb{R}$ is the channel gain[2] from sensor i, and w is white Gaussian noise with power spectral density σ^2.

The basic idea of scheme (1) is easier to understand for the special case that all h_i are equal to 1. In this case,

$$z = \sum_{j=1}^{k} N_j \sqrt{E} s_j + w, \tag{2}$$

where N_j is the number of sensors that observe symbol j. After appropriate matched filtering and scaling by $1/n$, it is seen that the received signal z contains a noisy version of the *type* of sensor observations [2]. Since the type is a *sufficient statistic* for estimating θ, the communication could be considered lossless if the fusion center had noiseless access to the type. We later argue that the effect of noise w in (2) actually becomes negligible for n large enough.

In general, we shall call the scheme leading to (1) as the *Type-Based Multiple Access* (TBMA). This method is proposed by the authors in [1], [3], where its performance is analyzed for parameter estimation. Liu and Sayeed proposed the TBMA independently [4] in the context of detection. Estimation/detection over multiaccess channels problem has attracted considerable attention recently. However, previous works (*e.g.,* [5], [6]) focused on the analysis and design of systems with orthogonal allocation such as TDMA.

In this paper, we first present an asymptotic performance analysis of the TBMA in estimating a continuous parameter $\theta \in \mathbb{R}$. Our analysis assumes that the channel gains

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[1] In some networks such as the ones designed to detect existence of targets, the sensor data are indeed discrete. In others that are used for measurement, what we call $X_1, \cdots, X_n$ actually model *quantized* data, where identical quantizers are used at sensor nodes.

[2] The results of this paper can be easily extended to complex valued signals and channel gains (see [1]).

$h_1, \cdots, h_n$ are i.i.d., extending the analysis for $h_i = 1$ in [3]. In particular, we derive the asymptotic mean square error (MSE) with a variant of the maximum-likelihood (ML) estimator as $n \to \infty$.

We then derive the asymptotic *error exponents* for the ML detector in the binary hypothesis testing problem

$$\mathcal{H}_0 : \theta = \theta_0 \text{ vs. } \mathcal{H}_1 : \theta = \theta_1. \tag{3}$$

The ML detector over a fading channel is in general intractable. As a result, we propose a computationally feasible, yet asymptotically equivalent, version of it, and characterize its error exponents.

Our analysis indicates that both the detection and estimation performance of TBMA is superior to TDMA depending on the channel conditions. Surprisingly, if there is no channel fading (*i.e.,* all h_i are constant and identical), then the asymptotic estimation/detection performance of TBMA is as if the fusion center has noiseless access to $X_1, \cdots, X_n$. We provide a general comparison between the TDMA and TBMA approaches in the conclusions.

In Section 2, the estimation results are presented. Section 3 derives the error exponents for hypothesis testing. Numerical examples and comparison with orthogonal allocation methods such as TDMA are distributed within the sections. Section 4 concludes the paper.

2. PARAMETER ESTIMATION WITH TBMA

Let $p_\theta = [p_\theta(1) \cdots p_\theta(k)]^T$ be the probability mass function the data $X_1, \cdots, X_k$ comes from. Suppose that the channel gains $h_1, \cdots, h_n$ are i.i.d. with *non-zero* mean $h := \mathbb{E}(h_i)$ and $\sigma_h^2 := \text{Var}(h_i)$ (we will discuss zero-mean h_i's later). A sufficient statistic in estimation is the inner product between z and the waveforms $s_1, \cdots, s_k$. Let

$$y := \frac{1}{\sqrt{E}nh}[(z|s_1) \cdots (z|s_k)]^T \tag{4}$$

$$= \frac{1}{n}\sum_{i=1}^{n} \frac{h_i}{h} e_{X_i} + \tilde{w}, \tag{5}$$

where $e_1, \cdots, e_k$ are the standard basis vectors, and $\tilde{w} \sim \mathcal{N}(0, \frac{\sigma^2}{En^2}I)$.

In general, the distribution of y has a complicated form. However, the following lemma asserts that the distribution approaches the Gaussian as $n \to \infty$. Also, implicit in the lemma is that the effect of noise on the distribution of y disappears as $n \to \infty$ (notice that the power of $\tilde{w}$ is $O(1/n^2)$).

Lemma 1 *The asymptotic distribution of y is*

$$y \simeq \mathcal{N}(p_\theta, \frac{1}{n}\Sigma_\theta) \quad \textit{as } n \to \infty,^{3}$$

where $\Sigma_\theta = (1 + \frac{\sigma_h^2}{h^2})\textit{Diag}(p_\theta) - p_\theta p_\theta^T$.

Proof Follows almost directly from the law of large numbers and the multivariate central limit theorem. For the details see [1].

In general, the actual ML estimator based on y is computationally intractable. Nevertheless, the asymptotic distribution of y provided by Lemma 1 can be used to design an estimator for the TBMA. If the distribution of y were *exactly* equal to $\mathcal{N}(p_\theta, \frac{1}{n}\Sigma_\theta)$, then its pdf would be

$$f(y) = \frac{1}{(2\pi/n)^{\frac{k}{2}}} \exp\left(-\frac{n(y-p_\theta)^T\Sigma_\theta^{-1}(y-p_\theta) + \log|\Sigma_\theta|}{2}\right).$$

Given y, the ML estimator based on this distribution maximizes $f(y)$ with respect to $\theta \in \mathbb{R}$. Notice that this maximization is equivalent to minimizing the exponent. Moreover, the first term in the exponent has a factor n, and it dominates the minimization for large n. As a result, we propose the estimator $\hat{\theta}$ which minimizes

$$M(\theta) := (y-p_\theta)^T\Sigma_\theta^{-1}(y-p_\theta)$$

with respect to θ. We shall call this the *asymptotic ML* estimator. The following theorem provides the asymptotic MSE of $\hat{\theta}$.

Theorem 1 *Under certain regularity conditions*[4] *on the* $\{p_\theta : \theta \in \mathbb{R}\}$*, the estimator $\hat{\theta}$ satisfies*

$$\hat{\theta} \simeq \mathcal{N}(\theta, \frac{1+\frac{\sigma_h^2}{h^2}}{nI(\theta)}) \quad \textit{as } n \to \infty, \tag{6}$$

where $I(\theta) = \sum_{j=1}^{k} \frac{(dp_\theta(j)/d\theta)^2}{p_\theta(j)}$ *is the Fisher Information* [7] *in variable* X_i.

Proof See [1].

Equation (6) implies that the estimator $\hat{\theta}$ asymptotically achieves the *Cramer-Rao Bound* (CRB) $\frac{1}{nI(\theta)}$ [7] on the MSE if $\sigma_h^2 = 0$ (*i.e.,* all h_i's are constant). According to the CRB, no unbiased estimator $\hat{\theta}$, even the ones based on the exact data $X_1, \cdots, X_n$, can have lower MSE. In this respect, the TBMA scheme is *asymptotically optimal* in estimating the parameter θ. This conclusion was first obtained in [3]. On the other hand, the theorem also indicates that if σ_h^2/h^2 is large, then there is a significant gap between the CRB and the asymptotic MSE with TBMA.

We made simulations to check the validity of the asymptotic results for finite n. In general, we have observed that $(1 + \sigma_h^2/h^2)/nI(\theta)$ provides a reasonably accurate estimate for the MSE of TBMA. Fig. 1 considers the case that

[3]Notation "$\simeq$" means that y converges in probability to p_θ, and $\sqrt{n}(y - p_\theta)$ converges in distribution to $\mathcal{N}(0, \Sigma_\theta)$ as $n \to \infty$. The notation "$\simeq$" in the rest of the paper should be understood similarly.

[4]The theorem requires conditions such as three times differentiability of p_θ with respect to θ (see [1]).

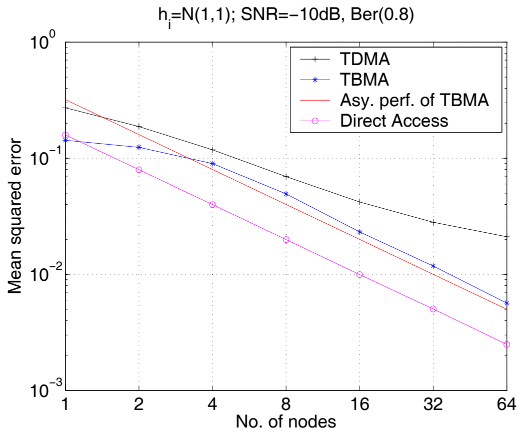

Fig. 1. Performance of TBMA in parameter estimation.

$X_1, \cdots, X_n$ are Bernoulli(θ) distributed, $h_i \sim \mathcal{N}(1,1)$, and SNR $= E/\sigma^2 = -10$dB. The antipodal constellation and the ML estimator are used with the TDMA (the ML estimator is based on the received signal at the fusion center, not based detected symbols).

So far we have assumed that $\mathbb{E}(h_i) \neq 0$. When $\mathbb{E}(h_i) = 0$, an interesting phenomena happens: TBMA fails to deliver the type. More precisely, what happens is that if we define $y = \frac{1}{\sqrt{En}}[(z|s_1) \ \cdots \ (z|s_k)]^T$ (observe the difference between this and (4)), then $y \to \mathcal{N}(0, \mathrm{Diag}(p_\theta))$ in distribution as $n \to \infty$. In other words, the pdf p_θ doesn't appear in the mean of the asymptotic distribution, but in the covariance. This indicates that when $h_i = 0$, even with the ML estimator the MSE does not go zero as $n \to \infty$. We have verified this fact via simulations, but can not include them here due to space limitations (see [1]).

In band-pass wireless communications, the h_i's typically contain a random phase $e^{j\rho}$, $\rho \sim$ uniform$[0, 2\pi]$. Consequently, the channels have a zero mean. In order to have $\mathbb{E}(h_i) \neq 0$, the received phases somehow need to be aligned. For this purpose, the use of *transmitter CSI* (Channel Side Information) is proposed in [1]. In case the transmitters can be made aware of their phases, the TBMA approach still works well [1].

3. HYPOTHESIS TESTING WITH TBMA

This section considers the binary hypothesis testing setup in (3). Lemma 1 indicates that the received signal y under hypothesis $\mathcal{H}_i$ becomes concentrated around p_{θ_i}. In hypothesis testing, errors mainly happen because the realization of y turns out to be close neither to p_{θ_0} nor to p_{θ_1}. To be able to assess the probability of such events, the theory of types (or, in general, the large deviations theory) can be used. As a notation, we let $D(Q||P)$ denote the *relative entropy* [2] between the pdfs Q and P. Similarly, for random variables X and Y, $D(Y||X)$ denotes the relative entropy between the pdfs of Y and X

Suppose that the channels $h_1, \cdots, h_n$ are i.i.d. and have non-zero mean h. Let $B_\epsilon(r)$ be the open ball in $\mathbb{R}^k$ centered at $r = [r_1 \ \cdots \ r_k]^T$ with radius $\epsilon > 0$. The next theorem characterizes the exponent of the probability that the realization of y turns out to be in $B_\epsilon(r)$.

Theorem 2 *Let $\epsilon > 0, r \in \mathbb{R}^k$ and $r \neq p_\theta$. Then,*

$$\lim_{n\to\infty} \frac{1}{n} \log \Pr\{y \in B_\epsilon(r)\} = -E_\theta(r) + O(\epsilon) \quad (7)$$

where $O(\epsilon)$ is a function which goes to zero as $\epsilon \to 0$,

$$E_\theta(r) = \min_{\tilde{p}} \{D(\tilde{p}||p_\theta) + \sum_{j=1}^{k} \tilde{p}(j) \Lambda(\frac{hr_j}{\tilde{p}(j)})\} \quad (8)$$

$$\Lambda(\frac{hr_j}{\tilde{p}(j)}) = \min_{\tilde{h}_i : \mathbb{E}(\tilde{h}_i) = \frac{hr_j}{\tilde{p}(j)}} D(\tilde{h}_i||h_i). \quad (9)$$

The theorem says that the probability of having y at location r is approximately $e^{-nE_\theta(r)}$ for large n. This interpretation enables us to understand the behavior of the ML detector for large n. When n is large, the ML detector partitions the space $\mathbb{R}^k$ into two sets such that the detection region for $\mathcal{H}_1$ is approximately equal to

$$\Gamma_1 = \{r \in \mathbb{R}^k : E_{\theta_1}(r) < E_{\theta_0}(r)\};$$

the reason for detecting $\mathcal{H}_1$ in Γ_1 is that the approximate likelihood $e^{-nE_{\theta_1}(r)}$ under $\mathcal{H}_1$ is bigger than the approximate likelihood $e^{-nE_{\theta_0}(r)}$ under $\mathcal{H}_0$. Similarly, $\Gamma_0 = \Gamma_1^c$ is the detection region for $\mathcal{H}_0$ for large n.

The computation of the likelihood function with TBMA is intractable in general. Consequently, the exact ML detector can not be directly implemented. The above interpretation of the ML detector, however, suggests an *asymptotic version* which detects $\mathcal{H}_i$ when y is in Γ_i. In general, the functions $E_\theta(r)$ are easier to compute than the exact likelihood.

Let $\alpha_n = \Pr\{\mathcal{H}_0 \to \mathcal{H}_1\}$, $\beta_n = \Pr\{\mathcal{H}_1 \to \mathcal{H}_0\}$ denote the type I and type II error probabilities.

Theorem 3 *The error exponent of the asymptotic ML detector is given by*

$$\eta := -\lim_{n\to\infty} \frac{1}{n} \log \alpha_n = \min_{r \in \partial\Gamma_1} E_{\theta_0}(r), \quad (10)$$

where $\partial\Gamma_1 = \{r : E_{\theta_0}(r) = E_{\theta_1}(r)\}$ is the boundary between Γ_1 and Γ_0. The exponent of β_n is same as that of α_n.

Notice that when all h_i's are constant and identical, $E_\theta(r) = D(r||p_\theta)$ if r is a probability vector, and is ∞ otherwise.

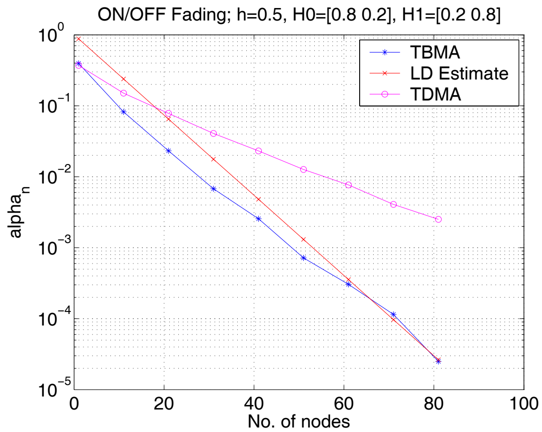

Fig. 2. Performance of TBMA in detection.

Therefore, the exponent provided by Theorem 3 is same as that of the true ML detector [2, p. 308] based on $X_1, \cdots, X_n$. *Hence, if all h_i are constant the TBMA scheme with the proposed ML-variant estimator gives the same asymptotic performance with the ML detector based on $X_1, \cdots, X_n$.*

Next, we will compute the error exponents for an example channel. Consider the ON/OFF channel, *i.e.,* h_i is Bernoulli $\{0, 1\}$ distributed with mean h. For this scenario, $\Lambda(\cdot)$ is the relative entropy function between two Bernoulli variables. Using the Lagrange multipliers method, it is easy to get

$$E_\theta(r) = RD(\tilde{r}||p_\theta) + R\log\frac{R}{h} + (1-R)\log\frac{1-R}{1-h},$$

where $R = h\sum_j r_j$ and $\tilde{r} = r/(\sum_j r_j)$. Theorem 3 gives the error exponent of α_n as

$$\eta = RC + R\log\frac{R}{h} + (1-R)\log\frac{1-R}{1-h},$$

where

$$C = -\min_{0\leq\lambda\leq 1}\log\left(\sum_j p_{\theta_0}^{\lambda}(j)p_{\theta_1}^{1-\lambda}(j)\right)$$

is the so-called *Chernoff information* [2], and $R = he^C/(1-h+he^C)$.

Simulation results for the hypothesis $\mathcal{H}_0 : X_i \sim$ Bernoulli(0.8), $\mathcal{H}_0 : X_i \sim$ Bernoulli(0.2) are given in Fig. 2. The LD (Large Deviations) estimate refers to $e^{-n\eta}$. SNR $= E/\sigma^2 =$ 3dB. For the TDMA, the antipodal constellation and the ML detector based the received signal are used. We see that the performance of TBMA with the asymptotic ML detector follows the LD estimate reasonably closely.

4. CONCLUSIONS

Signaling design for sensor networks is a new avenue for research. The problem is practically relevant, and is especially interesting, because it involves joint design of modulation and medium access.

In this paper we proposed the TBMA approach to communicate sensor data. One advantage of TBMA is that its bandwidth/time requirement is independent of the number of transmitting sensors (k orthogonal dimensions are needed). On the hand, the bandwidth requirement of the classical TDMA approach grows linearly with n. In a network with large n and small k, the TBMA is significantly more bandwidth efficient than TDMA.

The estimation/detection performance of TBMA depends on the type of channel fading. If the channel has non-zero mean, and small variance, the TBMA has very desirable MSE and error probability performance. In particular, if the sensor channels are constant and identical, then asymptotic performance of TBMA is as if the fusion center has *direct access* to $X_1, \cdots, X_n$. The performance of TDMA approach is sensitive to the amount of channel noise. On the hand, the performance of TBMA is not affected by the noise when n is large.

After the submission of this work, we have become aware of the paper by Liu and Sayeed [8] in which it is shown that the TBMA achieves the best error exponent in hypothesis testing in the non-fading scenario. Our analysis in this paper is also applicable in case of fading.

One drawback of TBMA is its poor performance in channels with zero-mean. Its our hope that this work will stimulate some interest to come up with improvements or better schemes for zero-mean channels.

5. REFERENCES

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