DECENTRALIZED DETECTION IN DENSE SENSOR NETWORKS WITH CENSORED TRANSMISSIONS

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ABSTRACT

In this work we consider the problem of detection in a sensor network with censored transmission. The motivation for this is the improvement of the energy efficiency by means of allowing just positive detection transmission. Both, Bayes and NP tests are developed and the performance of NP test is studied using large deviation bounds on the error probability. We also show that these bounds using the KL divergence between the probability density functions of the observations under each hypothesis might be used as a criteria for determining the optimal exploration area or even the optimal strategy for energy efficiency. The "spanish hat" model for the probability of detection is used to example the performance of the proposed bound.

1. INTRODUCTION

In [1], the problem of binary distributed detection in the context of large-scale, dense sensor networks, was considered. The detection is based on an identical local binary detection rule, and the probability that a sensor detects a target is modeled by a function p_d that depends on the distance between the sensor and the target. In this paper we want to extend that work by considering censoring schemes to improve the energy efficiency of the detection process. Previously proposed censoring schemes (like [2]) are not applicable to our setting because we do not restrict local decisions to be based on a likelihood ratio test. Instead, we propose to use a simple censoring scheme in which only the sensors that detect the target transmit their position to the fusion center.

Following this approach, we analyze the simple problem of detecting a target knowing its position under both Bayes and Neyman-Pearson (NP) tests. For obtaining the log-likelihood ratio (LLR), we will first model the number of sensors that after detecting a target, successfully transmit their position to the fusion center. The performance of the NP test is analyzed using large deviation bounds on the error probability and a parametric approximation of p_d. We also provide, using these bounds, rules for designing the test for the exploration of a given spatial area.

The paper is organized as follows. In Section 2 we state the problem we are facing at and we introduce the basic notation of the paper. In Section 3, we present the modeling of the number of sensors included in the location task. Section 4 is devoted to the derivation of both NP and Bayes tests. Section 5 presents the bounds on error probability for the proposed test, Section 6 shows an example, and Section 7 concludes the paper with a discussion of the presented results.

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2. PROBLEM STATEMENT AND NOTATION

We consider that ℓ sensors are randomly deployed over a region $\mathcal{D} \in I\!R^2$, of area S, following a uniform distribution. Each sensor applies the same binary detection rule, not necessarily based on a LLR test. The probability of a positive detection ($Y = 1$) in a sensor located at coordinates $\boldsymbol{x}$ when a target is present at coordinates $\boldsymbol{x}^t$ is denoted as $p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha)$, where α is the probability of false alarm (PFA) of the sensor when no target is present. $p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha)$ is a non-increasing function of the euclidean distance between $\boldsymbol{x}$ and $\boldsymbol{x}^t$, it is determined by the underlying physics of the sensing process and the kind of the detection rule, and it is assumed to be known (more details can be found in [1]).

Given $\mathcal{D}$ and $\boldsymbol{x}^t$, we define two hypothesis, H_0 or null hypothesis for the case when no target is present, and H_1 or alternative hypothesis for the case when a target is present.

- Under hypothesis H_0, the joint pdf of $\boldsymbol{X}$ and Y (the result of the local detection process) is

$$f_{\boldsymbol{X},Y|H_0}(\boldsymbol{x}, y|H_0) = \frac{1}{S}\left(\alpha\,\delta[y-1] + (1-\alpha)\,\delta[y]\right)$$

  where δ is the Kronecker function.

- Under hypothesis H_1, the joint pdf of $\boldsymbol{X}$ and Y is

$$f_{\boldsymbol{X},Y|H_1}(\boldsymbol{x}, y|H_1) = \frac{1}{S}(p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha)\,\delta[y-1] + (1 - p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha))\,\delta[y])$$

At every sensing instant (automatic or beacon driven), each sensor independently decides to sense with probability p_s. We denote ℓ_s the number of sensors that sense. At the fusion center we receive only a set of $\ell_t \leq \ell_s$ coordinates $\{\boldsymbol{x}_i, i = 1, \cdots, \ell_t\}$ that corresponds to the positions of the sensors that have detected the target and that have transmitted successfully.

The information that is not available at the fusion center is: 1) the number of sensors that, after sensing, does not detect the target, ℓ_{nd}; 2) the positions of the above sensors, $\{\boldsymbol{x}_i^{nd}, i = 1, \cdots, \ell_{nd}\}$; 3) the number of sensors that have failed the transmission of a positive detection, ℓ_e, and; 4) the positions of the above sensors, $\{\boldsymbol{x}_i^{e}, i = 1, \cdots, \ell_e\}$.

Moreover, we will denote by $\ell_s = \ell_t + \ell_e + \ell_{nd}$.

3. MODELING THE NUMBER OF SENSORS

We start by modeling the number of active sensors, ℓ_s. Assuming a sensing probability p_s, the number of sensors ℓ_s that sense can be modeled as a random variable L_s with a binomial conditional probability density function

$$f_{L_s|L}(\ell_s|\ell) = \binom{\ell}{\ell_s} p_s^{\ell_s}(1-p_s)^{\ell-\ell_s},$$

for $0 \leq \ell_s \leq \ell,\ \ell_s \in \mathbb{Z}$, where L denotes the random variable modeling the number of sensor in the region $\mathcal{D}$.

Now lets proceed with the modeling of the number of sensors that are detecting a target. First we consider the influence of wireless transmission plus the Medium Access Control (MAC). To avoid an unnecessary notation complexity, we will group the influence of all the sources of errors involved into the transmission into a single probability of error of a message (or a packet), p_e. Although in some cases the consideration of a single probability of error does not lead to a realistic analysis of the energy efficiency of the network (mainly if retransmission are allowed in the MAC) in most other it will provide the exact solution.

Assuming that the expected probability of detection of a sensor in the region $\mathcal{D}$, denoted as $p_{\mathcal{D}}$, and the probability of error in the transmission, p_e, are independent, the probability of a successful transmission for a sensor is

$$p_t = (1-p_e)\cdot p_{\mathcal{D}}.$$

Given the number of sensors that effectively sense, ℓ_s, the number of sensors that detect and correctly transmit can also be modeled as a random variable, L_t, with a binomial distribution,

$$f_{L_t|L_s}(\ell_t|\ell_s) = \binom{\ell_s}{\ell_t} p_t^{\ell_t}(1-p_t)^{\ell_s-\ell_t},$$

for $0 \leq \ell_t \leq \ell_s,\ \ell_t \in \mathbb{Z}$.

We will provide the expressions for $p_{\mathcal{D}}$, which depend on the underlying hypothesis, at the end of Section 4. In the following, we will denote by $p_{t|i}$ and $p_{\mathcal{D}|i}$ the probability of a successful transmission and the average probability of detection of a sensor in area $\mathcal{D}$, respectively, under hypothesis H_i, $i \in \{0,1\}$.

Using this notation, the conditional distribution of the number of sensors ℓ_t that, after sensing, detect a target and transmit their position to the fusion center, given the total number of sensors, ℓ, and the hypothesis H_i, is

$$f_{L_t|L,H}(\ell_t|\ell,H_i) = \sum_{\ell_s=\ell_t}^{\ell} \binom{\ell_s}{\ell_t} p_{t|i}^{\ell_t}\left(1-p_{t|i}\right)^{\ell_s-\ell_t} \binom{\ell}{\ell_s} p_s^{\ell_s}\left(1-p_s\right)^{\ell-\ell_s}.$$

for $0 \leq \ell_t \leq \ell,\ \ell_t \in \mathbb{Z}$.

4. HYPOTHESIS DETECTION PROBLEM

At the fusion center, only positions of the sensors that, after sensing, detected a target and had a successful transmission, $\{\boldsymbol{x}_i\}_{i=1,\cdots,\ell_t}$, are known. To derive a LLR-based test, first of all, we have to determine the probability density function of the observation under each hypothesis (H_0 a target is not present, H_1 a target is present). Initially, we consider the positions to be the only significant observations. Therefore, we need the distribution of the random variable $\mathbf{X}$ modeling the positions

$$\mathbf{x} = [\boldsymbol{x}_1, \cdots, \boldsymbol{x}_\ell]^T.$$

Without lack of generality, and for the sake of simplifying the following notation, we consider that in a general case these positions are ordered as follows

$$\mathbf{x} = [\boldsymbol{x}_1, \cdots, \boldsymbol{x}_{\ell_t}, \boldsymbol{x}_1^e, \cdots, \boldsymbol{x}_{\ell_e}^e, \boldsymbol{x}_1^{nd}, \cdots, \boldsymbol{x}_{\ell_{nd}}^{nd}]^T.$$

When all sensor positions are known, and independence between sensors is assumed, the distribution of the random variable $\mathbf{X}$ modeling the positions, conditioned to each hypothesis is

$$f_{\mathbf{X}|H}(\mathbf{x}|H_i) = \prod_{i=1}^{\ell_t} f_{X|H,Y}(\boldsymbol{x}_i|H_i,1) \cdot \prod_{j=1}^{\ell_e} f_{X|H,Y}(\boldsymbol{x}_j^e|H_i,1) \prod_{k=1}^{\ell_{nd}} f_{X|H,Y}(\boldsymbol{x}_k^{nd}|H_i,0).$$

However, in the problem we are facing, only the positions $\{\boldsymbol{x}_k\}_{k=1,\cdots,\ell_t}$ are known at the fusion center. To circumvent this problem, we model the observations as the set of sensor positions along with the number of successful received lectures, i.e., the observations are modeled by a random variable $\boldsymbol{\Theta}$, modeling the following vector

$$\boldsymbol{\theta} = [\boldsymbol{x}_1, \cdots, \boldsymbol{x}_\ell, \ell_t]^T.$$

The joint probability density function of the observations is now given by

$$f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_i) = \prod_{i=1}^{\ell_t} f_{X|H,Y}(\boldsymbol{x}_i|H_i,1)\frac{f_{L_t|L,H}(\ell_t|\ell,H_i)}{S^{\ell-\ell_t}}.$$

By relying on the probability of detection function, $p_d(\boldsymbol{x}^t, \boldsymbol{x}_i, \alpha)$, it is straightforward to obtain

$$f_{X|H,Y}(\boldsymbol{x}_i|H_1,1) = \frac{p_d(\boldsymbol{x}^t,\boldsymbol{x}_i,\alpha)}{\int_{\mathcal{D}} p_d(\boldsymbol{x}^t,\boldsymbol{x},\alpha)\,d\boldsymbol{x}}, \tag{1}$$

and

$$f_{X|H,Y}(\boldsymbol{x}_i|H_0,1) = \frac{1}{S}. \tag{2}$$

Therefore, now we can obtain

$$f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_1) = \prod_{i=1}^{\ell_t} \frac{p_d(\boldsymbol{x}^t,\boldsymbol{x}_i,\alpha)}{\int_{\mathcal{D}} p_d(\boldsymbol{x}^t,\boldsymbol{x},\alpha)\,d\boldsymbol{x}} \frac{f_{L_t|L,H}(\ell_t|\ell,H_1)}{S^{\ell-\ell_t}},$$

and

$$f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_0) = \frac{1}{S^\ell}\cdot f_{L_t|L,H}(\ell_t|\ell,H_0).$$

Based on these expressions, the likelihood ratio can be written as

$$\Gamma = \frac{f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_1)}{f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_0)} = S^{\ell_t}\prod_{i=1}^{\ell_t} \frac{p_d(\boldsymbol{x}^t,\boldsymbol{x}_i,\alpha)}{\int_{\mathcal{D}} p_d(\boldsymbol{x}^t,\boldsymbol{x},\alpha)\,d\boldsymbol{x}} \frac{f_{L_t|L,H}(\ell_t|\ell,H_1)}{f_{L_t|L,H}(\ell_t|\ell,H_0)}.$$

The decision is usually defined in terms of such log-likelihood ratio

$$\gamma = \ln \Gamma \underset{H_1}{\overset{H_0}{\lessgtr}} \tau.$$

Under Bayes criteria, the threshold is easily set as [3]

$$\tau = \ln \frac{\pi_0(C_{10} - C_{00})}{\pi_1(C_{01} - C_{11})}$$

and under NP criteria, the threshold could be calculated by asymptotic gaussianity, like in [1].

To complete the expression for the test, we need the probabilities $p_{t|i}$ involved in $f_{L_t|L,H}(\ell_t|\ell, H_i)$. Given the distributions $f_{X|H,Y}(\boldsymbol{x}_i|H_i, 1)$ for the positions of sensors detecting a target under each hypothesis, (1) and (2), it is straightforward to obtain

$$p_{t|1} = (1 - p_e) \cdot p_{\mathcal{D}|1},$$

and

$$p_{t|0} = (1 - p_e) \cdot p_{\mathcal{D}|0},$$

where

$$p_{\mathcal{D}|1} = \frac{\int_{\mathcal{D}} \left[p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha)\right]^2 \, d\boldsymbol{x}}{\int_{\mathcal{D}} p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha) \, d\boldsymbol{x}},$$

and

$$p_{\mathcal{D}|0} = \alpha.$$

5. LARGE DEVIATION BOUNDS ON THE PROBABILITY OF ERROR

In this section, we will bound the probability of error in the hypothesis test by using large deviation bounds in the form of error exponents. If ϵ_n is the probability of error (of some kind) obtained with n observations, the error exponent is defined as

$$\lim_{n\to\infty} -\frac{1}{n} \ln \epsilon_n$$

In NP test, the best error exponent is given by the Stein's lemma, that applied to our problem says that for any $\alpha_n \in (0, 1)$

$$\lim_{n\to\infty} -\frac{1}{n} \ln \beta_n = D(f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_0) \| f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_1)),$$

where $D(f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_0) \| f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_1))$ denotes the Kullback-Leibler (KL) divergence [4] between the probability density functions of the observations under each hypothesis. We will use the notation $D(H_0\|H_1)$ for short.

In Bayes tests (assuming that $C_{10} - C_{00} = C_{01} - C_{11}$), the best achievable error exponent is the Chernoff information. Due to space limitation, in this paper we will obtain the large deviation bound only for NP tests, although a development parallel to the one followed in [1] could be performed.

The KL divergence, in our problem, takes the form

$$D(H_0||H_1) = \int_{\boldsymbol{\theta}} \frac{1}{S^\ell} \cdot f_{L_t|L,H}(\ell_t|\ell, H_0) \cdot \ln \frac{f_{L_t|L,H}(\ell_t|\ell, H_0) \cdot S^{\ell-\ell_t}}{S^\ell \cdot \prod_{i=1}^{\ell_t} f_{X|H_1,Y}(\boldsymbol{x}_i|H_1, 1) \cdot f_{L_t|L,H}(\ell_t|\ell, H_1)} d\boldsymbol{\theta},$$

which becomes

$$D(H_0||H_1) = \sum_{\ell_t=0}^{\ell} f_{L_t|L,H}(\ell_t|\ell, H_0) \cdot \left\{ \ln \frac{f_{L_t|L,H}(\ell_t|\ell, H_0)}{f_{L_t|L,H}(\ell_t|\ell, H_1)} - \ell_t \cdot \ln S - \ell_t \frac{1}{S} \int_{\mathcal{D}} \ln f_{X|H_1,Y}(\boldsymbol{x}|H_1, 1) \, d\boldsymbol{x} \right\}.$$

6. ONE EXAMPLE

We provide an example of the performance of the test using the "spanish hat" model for the probability of detection. This model is defined as

$$p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha) = \begin{cases} (1-\beta) & \text{if } ||\boldsymbol{x}^t - \boldsymbol{x}||_2 < r_o \\ \alpha & \text{otherwise} \end{cases},$$

where r_o is the range of the sensor, and β is the probability of misdetection. For simplicity, we use circular exploration areas, $\mathcal{D}$, of radius R, centered at the target position $\boldsymbol{x}^t$. For this choice, we have the following values

$$p_{\mathcal{D}|1} = \begin{cases} (1-\beta) & \text{if } R \le r_o \\ \dfrac{\pi r_o^2(1-\beta)^2 + \pi(R^2 - r_o^2)\alpha^2}{\pi r_o^2(1-\beta) + \pi(R^2 - r_o^2)\alpha} & \text{if } R > r_o \end{cases},$$

$$\int_{\mathcal{D}} \ln f_{X|H_1,Y}(\boldsymbol{x}|H_1, 1) \, d\boldsymbol{x} = \begin{cases} \pi R^2 \ln \frac{1-\beta}{C} & \text{if } R \le r_o \\ \pi r_o^2 \ln \frac{1-\beta}{C} + \pi(R^2 - r_o^2) \ln \frac{\alpha}{C} & \text{if } R > r_o \end{cases},$$

where constant $C = \int_{\mathcal{D}} p(\boldsymbol{x}^t, \boldsymbol{x}, \alpha) \, d\boldsymbol{x}$, is in this case

$$C = \begin{cases} \pi R^2(1-\beta) & \text{if } R \le r_o \\ \pi r_o^2(1-\beta) + \pi(R^2 - r_o^2)\alpha & \text{if } R > r_o \end{cases}.$$

Figure 1 shows the KL divergence obtained for a sensor range $r_o = 1$, as a function of the radius, R, of the circular area $\mathcal{D}$. The following values have been assumed: $\alpha = 0.1$, $\beta = 0.1$, $p_e = 0.1$, $p_s = 0.5$ and $\ell = 50$ sensors.

It can be seen how, as expected, the divergence keeps constant as the size of the exploration area is smaller than the sensor range. In this case, all sensors in the area exhibit the same detection capability. However, as R becomes greater than the sensor range r_o, some sensors in the area have the target out of their range, and therefore are non informative in the test. This has the effect of decreasing the divergence.

Although this is a simple example, because the "spanish hat" model is only a first order approximation of a probability of detection function, it can be seen that this measure can be used to define the optimal size for the exploration area.

The evolution of the divergence as a function of sensing probability, p_s, and the probability of transmission error, p_e, is plotted in Figure 2.

Again, the obtained results are the expected ones. As the probability of transmission error increases, the divergence decreases, and the opposite happens with the sensing probability.

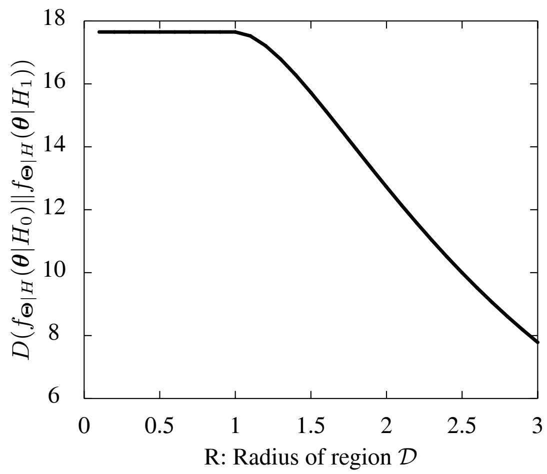

Fig. 1. Divergence $D(f_{\Theta|H}(\boldsymbol{\theta}|H_0)\|f_{\Theta|H}(\boldsymbol{\theta}|H_1))$ for the "spanish hat" model and a circular area $\mathcal{D}$ centered at the target position, $\boldsymbol{x}^t$, as a function of the radius R of the area.

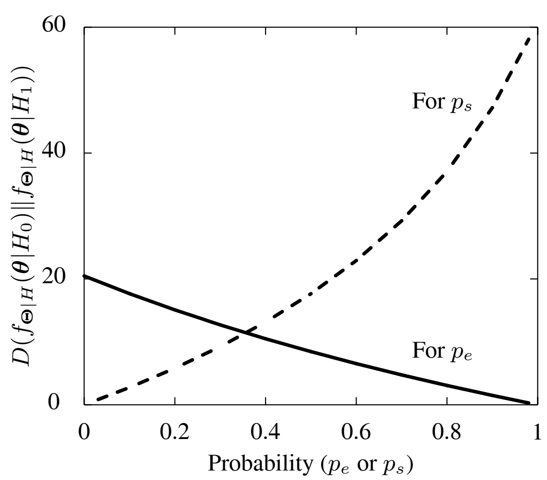

Fig. 2. $D(f_{\Theta|H}(\boldsymbol{\theta}|H_0)\|f_{\Theta|H}(\boldsymbol{\theta}|H_1))$ for the "spanish hat" model and a circular area $\mathcal{D}$ centered at the target position, $\boldsymbol{x}^t$, as a function of the sensing probability, p_s, and the probability of transmission error, p_e.

This simple example shows that this measure can be used for the goal of designing parameters of the network such as the sensing probability (p_s), or the design of the medium access code or the error protecting codes.

7. DISCUSSION

In this paper, we proposed a censoring scheme to improve the energy efficiency of the detection process. As the wireless communication with the fusion center is one of the most energy consuming operation, we propose to transmit only the positions of the sensors that have detected a target. Moreover, for the sake of enlarging the life of the sensor network, each sensor only senses with a previously specified probability, p_s.

Based on a probability of detection model, which depends on the underlying physics of the sensing process and is assumed to be known, we have derived the NP and Bayes test by defining a set of observations including the number of successful lectures, ℓ_t, received at the fusion center.

The probability of error of the NP test is analyzed using the Stein's lemma, and therefore, the Kullback-Leibler divergence between the joint probability density functions conditioned to both hypothesis has been obtained as a measure of performance.

This measure has been shown to be useful in several tasks concerning the network design and operation. For instance, to obtain the optimal size of the exploration area or to select the p_s parameter or the probability of error p_e (which is defined by the MAC and by the error protection code). The divergence can explain the capability of discrimination between both hypothesis and how the different parameters affect this divergence. This information can be used to obtain a trade-off between different requirements.

For instance, when the goal is to define the optimal strategy in terms of a trade-off between energy cost and probability of detection, this measure is also valuable. In this case, representing the average energy versus the divergence would be helpful. In this case, further work is necessary to model the cost per transmission and to obtain a measure of the probability of detection/energy cost ratio obtained with the proposed test.

8. REFERENCES

[1] Antonio Artés-Rodríguez, "Decentralized detection in sensor networks using range information," in *Proceedings of the ICASSP 2004*, Montreal, Canada, May 2004.

[2] C. Rago, P. Willett, and Y. Bar-Shalom, "Censoring sensors: a low communication-rate scheme for distributed detection," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 32, no. 2, pp. 554–568, Apr. 1996.

[3] Steven M. Kay, *Fundamentals of statistical signal processing: detection theory*, Prentice-Hall, Inc., Upper Saddle River, New Jersey, 1998.

[4] T.M. Cover and J.A. Thomas, *Elements of information theory*, John Wiley & Sons, New York, NY, 1991.