AN IDEA FOR QUANTIZATION WITH DATA ASSOCIATION

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ABSTRACT

Quantization for estimation is explored for the case that it must be performed jointly with data association; that is, the case in which measurements are of uncertain origin. Data association requires some sort of gating of distributed observations, and a censoring strategy is proposed. Several quantization philosophies are explored, specifically uniform quantization, uniform quantization with measurement exchangeability incorporated (the "type" method), and uniform quantization of sorted measurements. It is shown, perhaps surprisingly, that the third scheme preserves more information that may be useful for estimation; and a simple procedure for optimal fused estimation based on this third scheme is given. Interestingly, when compared in terms of rate-distortion curve, the schemes two and three perform similarly; their censored versions offer further improvement in performances due to the uncertain-origin property of the measurements.

1. INTRODUCTION

Data association [1, 2] refers to the practical concern that estimation must be done despite measurements being unlabeled: a target-originated measurement, if one is available, is among a group of false "clutter" measurements, and it is up to the estimation routine to decide which is which. In this paper we are interested in fused estimation of distributed observations that have been quantized and shared over a bandlimited channel: what is the impact of data association on quantization?

The paper is organized as follows. In section 2 we describe the model we shall explore: the goal is distributed estimation of a common one-dimensional quantity whose prior uncertainty is Gaussian and which is observed with additive noise among a background of unlabeled clutter random observations whose cardinality is itself a random variable (we take its distribution as Poisson). In section 3 we explore several uniform quantization strategies, and of special Peter Willett

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interest we have the "type" method, the "sorted" method, and the censored versions of these; the sorted approach appears complicated due to its dependency, but we are able to provide a neat and efficient fusion rule. We stress this point: in order to appropriately compare the "type" and "sorted" methods, it is mandatory to employ for both of them an optimal fusion rule. Usually adopted approximations of the fusion rule, valid for high resolution quantizers (see for instance [7]) are no longer applicable; accordingly, a fair amount of mathematics provides us a simple and plugin formula for this.

The type method is more frugal with bandwidth, while the sorted method provides better estimation: in section 4 we explore the tradeoff under the assumption that the quantizer output is processed by means of optimal source coding. We then discover via a rate-distortion function that both offer similar performance. In this section it is also found that the simple censoring rule that no more than the average number of observations per sensor ought to be quantized offers robust performance. In section 5 we summarize, and the basic message will be that a relatively simple uniform approach to quantization appears to be a strong competitor, at least as long as source coding is used. The "sorted" method is appropriate for systems that avoid complicated source coding and operate at a fixed and very low transmission rate. Finally, data censoring may be a valid alternative, due to the uncertain-origin property of the measurements.

This paper is a condensed version of [6], and some of the notation and mathematical development is more clearly given in that reference.

2. BACKGROUND

2.1. Model

The joint pdf of the aggregate X and of the number of measurements M, for a given target location θ (a-prior itself Gaussian), is

$$F(\boldsymbol{X}, \boldsymbol{M} | \boldsymbol{\theta}) = \prod_{j=1}^{J} f(\boldsymbol{x}_j, m_j | \boldsymbol{\theta}) .$$
 (1)

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The indicated conditional pdf pertaining to the *j*-th sensor may be found in the following form [1, 2]. For $m_j > 0$,

$$f(\boldsymbol{x}_{j}, m_{j} | \boldsymbol{\theta}) = \frac{\mathcal{P}(m_{j} - 1; \lambda) P_{d}}{m_{j} V^{m_{j} - 1}} \sum_{k=1}^{m_{j}} \mathcal{N}(x_{jk}; \boldsymbol{\theta}, \sigma) + \frac{\mathcal{P}(m_{j}; \lambda) (1 - P_{d})}{V^{m_{j}}}, \qquad (2)$$

while for $m_j = 0$ (i.e., no measurements at all at sensor j), we have

$$f(\boldsymbol{x}_j, m_j = 0 | \boldsymbol{\theta}) = \mathcal{P}(0; \lambda)(1 - P_d) .$$
(3)

in which \mathcal{P} denotes a Poisson pmf. This paper confines itself to the case that each sensor performs a scalar **uniform quantization** of the observed data. Assuming good source coding there is little to be gained by using quantizers whose characteristics are specifically tailored to the input data statistics [5]; as a matter of fact, in this case, the uniform quantizer is known to be nearly optimum [3], [4].

3. SYSTEM DESIGNS

We assume that the remote sensors operate as uniform quantizers with some modest signal processing capability. Such capabilities are used for data ordering, censoring, and source coding — details will be given shortly. As to the FC, we look for some simple, occasionally sub-optimal, fusion rule. With these problem constraints in mind we now introduce practical quantization and estimation strategies in the following four subsections (the first of these is trivial, and is for comparison). Performance analysis is then provided in a following section.

3.1. MS_{∞} : Unquantized MS

This ideal case (number of quantization bits $n \to \infty$) has little to do with a practical system, but functions here as a bound on the achievable MSE of the estimate.

3.2. MS_q: Quantized MS

Let us assume now that the channels between remote sensors and FC are capacity constrained. Accordingly, we assume that each remote sensor computes a *quantized* version of the measured data vectors, and transfers it through the channel toward the FC. In this section we first describe how the FC evaluates its fused MS estimate, say $\hat{\theta}_{MS_q}$. It turns out, not surprisingly, that only the empirical pmf (we shall call this the *type*) of the quantized data vector from each sensor is used by the FC to obtain $\hat{\theta}_{MS_q}$. Accordingly, we then recommend that each sensor send over the channel only its *type* (again, our use of the "type" means that the communication strategy delivers only the number of observations in each quantization bin, and does not label these), since the labelling of the observations by sensor j is arbitrary and irrelevant to estimation of θ .

3.3. MS_{ord-q}: Ordered-quantized MS

Let $\boldsymbol{x}^{ord} = [x_{\pi(1)}, x_{\pi(2)}, \dots, x_{\pi(m_j)}]^T$ denote the ordered version of the data vector \boldsymbol{x} (subscripts j dropped for clarity). Then we know $i < t \Rightarrow |x_{\pi(i)}| \leq |x_{\pi(t)}|$. We propose to apply the quantization process to the vector \boldsymbol{x}_j^{ord} instead of \boldsymbol{x}_j . When this procedure is used, we shall use the notation $\boldsymbol{q}_j^{ord} = \mathcal{Q}_n(\boldsymbol{x}_j^{ord})$ for the quantizer output¹, and further define the corresponding aggregate as $\boldsymbol{Q}^{ord} = \{\boldsymbol{q}_1^{ord}, \boldsymbol{q}_2^{ord}, \dots, \boldsymbol{q}_J^{ord}\}$. We have the following:

$$\begin{cases} \operatorname{MSE}\{\boldsymbol{Q}^{ord}\} \leq \operatorname{MSE}\{\mathcal{T}(\boldsymbol{Q})\}, \\ H(\boldsymbol{q}_{j}^{ord}) \geq H(type(\boldsymbol{q}_{j})), \end{cases}$$
(4)

in which the first (along with a neat explicit means to compute the optimal estimate from the ordered data) is shown in [6]; and the second follows from the (obvious) fact that we cannot send just the "type".

3.4. C-MS_q & C-MS_{ord-q}: Censored versions of MS_q & MS_{ord-q}

We can apply data censoring to either of the above estimation strategies: MS_q and MS_{ord-q} . In a censored scheme each sensor will retain only a given number m_j^* of observations, out of m_j . As an extreme case of censoring we take the "nearest-neighbor" data association idea [1] that a "hard" assignment is made that whatever measurement is nearest to the target's predicted location is target-generated, and all others clutter. Here we are interested in $m_j^* \ge 1$.

4. PERFORMANCE ANALYSIS

We refer to the following:

- V = 7 10 (screen size).
- $P_d = 0.8$ (detection probability).
- $\lambda = 2 5$ (average number of false alarms).
- $\sigma_{\theta} = 1$ (a-priori standard deviation of the r.v. θ).
- $\sigma = 0.1$ (measurement noise standard deviation).
- $\theta_0 = 0$ (mean value of the r.v. θ).

Most results here are for j = 2 sensors; larger-network results are given in [6]; results tend to be similar.

¹We wish to avoid confusion here: please, note that q_j^{ord} is **not** necessarily the (modulus) ordered version of q_j , because the ordering and the quantization operations are not commutative. They would be commutative if the ordering were not a **modulus** ordering.



Fig. 1. MSE of the θ estimates versus the quantization bit number n.

A first comparison among the four estimation strategies is made on the basis of the MSE achieved with a prescribed number n of quantization bits. The comparison, based on 10^4 Monte Carlo runs, is provided in Fig. 1 for J = 2 and $\lambda = 3$. We see that the MS_q and MS_{ord-q} estimates converge, for large n, toward the MSE lower bound of MS_{∞} (unquantized data). On the other side (lower n) the curves move toward the upper bound, represented by the a-priori uncertainty $\sigma_{\theta}^2 = 1$. We also see that the MSE pertaining to the estimate based on ordered-and-quantized data Q^{ord} is always lower than that based on the MS_q strategy, with the relative difference increasing as the quantization coarsens. As mentioned earlier, Q^{ord} contains more information about θ than Q.

As to the truncated strategies (C-MS_q, C-MS_{ord-q}), the curves in Fig. 1 refer to the particular choice $m^* = 3$. In this case, we see that at intermediate values of n the MSE is in the same order of magnitude as that of the MS_q and MS_{ord-q} estimators. However, there is a *saturation effect* at large n where the lower bound of MS_{∞} cannot be reached. The reason why we choose $m^* = \lambda$ is that deeper censoring implies greater MSEs; more details later. It is interesting that in Fig. 1 we have C-MS_q preferable to MS_q for single-bit quantization: this is because the censoring procedure is actually preceded by an *ordering* step that removes outlying (likely to be clutter) measurements.

As discussed earlier, we assume that each sensor employs an optimal source coding technique such that the average data rate is just the entropy of the vector to be sent. Fig. 2 refers to J = 2 and $\lambda = 3$. The curves labelled with "MS_q (no-type)" are those of $H(q_j)$: apparently, the entropy grows linearly with n. Specifically, look at the case of n = 1, where $H(type(q_j))$ is substantially lower than $H(q_i^{ord})$. However, the two curves approach each other for



Fig. 2. Entropies of the data vectors to be sent over the channel, as a function of the number of quantization bits n. We take this as a proxy for the communication load implied by the strategies.

larger n.

Referring for instance to the uncensored curves in Fig. 1, we see that MS_{ord-q} has a lower MSE than MS_q ; but we also discover that it requires a higher transmission rate from Fig. 2. From a theoretical viewpoint, the lesson learned is summarized in eqs. (4), that sorting the measurements prior to quantization provides greater accuracy, but the expense of higher required bandwidth. How do these effects trade off? To answer, we assume that the required bit-rate R_b equals the entropy of the vector to be sent (i.e., that an efficient source code is in use). For a given number of bits of quantization n we blend Fig. 1 (MSE vs. n) and Fig. 2 (entropy vs. n). Parametrized from these we can build Fig. 3 which shows (MSE, R_b) pairs.

The censored curves (with $m^* = \lambda$) give the best performance up to a certain R_b . Above this, due to the censoring, the MSE cannot be reduced further and the best option becomes the uncensored case (or a censored approach with a larger m^* , see below). The crossing point is clearly visible in Fig. 3, at $R_b \approx 17$ bits/scan.

Let us now return on the choice of m^* , the censoring level. An instance is offered in Fig. 4: the curve MS_{ord-q} serves as reference, and the others give the rate-distortion behavior by varying the number of samples sent. It is seen that $m^* = 2 = \lambda - 1$ rapidly saturates to a fixed value of MSE. The censoring depth $m^* = 3 = \lambda$ is a good choice for most (MSE, R_b) pairs: only at high R_b is $m^* = 4 = \lambda + 1$ preferable. (The same result is obtained for other values of λ .)



Fig. 3. MSE of the θ estimates versus the bit rate R_b (bits/scan).

5. CONCLUSIONS

With the advent of dense sets of *cheap* sensors, as an attractive alternative to relatively few expensive ones operating under the trivial "reporting responsibility" fusion scheme, the issue of how best to represent information for transmission over bandlimited channels - and of course how to fuse such information when it arrives — has once again risen to importance. There was a period of prior interest, and there were many interesting and clever results such as nonuniform quantization strategies, and theory relating to convergence, asymptotics, and to rate-distortion functions. Rather surprisingly, what has been missing is discussion of a highly practical concern: data association. In many surveillance applications there are false alarms and missed detections: there is "measurement origin uncertainty" and a variable number of measurements per sensor per scan to be communicated.

Our findings are as follows:

- Scheme (2) is always preferable to scheme (1), since its bandwidth is lower.
- Scheme (3) is preferable to schemes (1) and (2) in terms of MSE when the number of bits of quantization is held fixed. This surprising fact comes about from the information preserved via the sorting operation.
- While Scheme (3) may at first appear onerous in terms of fusion, we have provided an explicit way to compute its fused estimate.
- When both MSE and bandwidth are taken into account, the empirical rate-distortion functions show that schemes (2) and (3) are similar in performance.
- The censoring schemes (4) and (5) are preferable to (2) and (3) in rate-distortion terms.



Fig. 4. MSE of the θ estimates versus the bit rate R_b (bits/scan), with several choices of m^* .

• An appropriate censoring level appears to be the number of false alarms per screen.

We would like to stress that the numbers turn out, in fact, to be rather heartening. For example, even with an average of **five false measurements per gate**, it is possible to achieve almost optimal performance using approximately only 20 bits per sensor (per dimension, per scan)!

6. REFERENCES

- Y. Bar-Shalom, and X. Li, *Multitarget-Multisensor Tracking: Principles and Techniques*, Storrs, CT: YBS Publishers, 1995.
- [2] S. Blackman and R. Popoli, *Design and Analysis of Modern Tracking Systems*, Artech House (Boston), 1999.
- [3] A. Gersho and R. M. Gray, *Vector Quantization and Signal Compression*. Kluwer Academic Publishers, 1992.
- [4] H. Gish and J. N. Pierce, "Asymptotically Efficient Quantizing", *IEEE Trans. on Information Theory*, Vol. IT-14, No. 5, pp. 676-683, September 1968.
- [5] R. M. Gray and D. L. Neuhoff, "Quantization", *IEEE Trans. on Information Theory*, Vol. IT-44, No. 6, pp. 2325-2383, October 1998.
- [6] V. Matta, S. Marano and P. Willett, "Quantization for Distributed Estimation with Data Association", *to appear in IEEE Transactions on Signal Processing*, 2004.
- [7] F. Palmieri, S. Marano, P. Willett, "Measurement Fusion for Target Tracking Under Bandwidth Constraint", *Proceedings of the 2001 IEEE Aerospace Conference*, Big Sky MT, March 2001.