SPATIAL POWER SPECTRUM ESTIMATION BASED ON A MVDR-MMSE-MUSIC HYBRID BEAMFORMER

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ABSTRACT

In the computation of the Minimum Variance Distortionless Response (MVDR) beamformer, scaling is necessary in order to satisfy the unity gain constraint. When estimating the power in the close vicinity of a source the MVDR beamformer allocates a null at the direction of the source and the scaling factor becomes very large, and substantially increases the gain of the white noise. This situation can cause the non detection of a weaker source located in the vicinity of a higher power source. Comparing the equation of the MVDR beamformer to the Wiener-Hopf MMSE equation we observe that a initial estimate of the power can be used instead of scaling to satisfy the unity gain constraint. As an initial power estimate we propose to estimate the directions and powers of the dominant sources by applying the MUSIC algorithm; and to use this information to estimate the power spectrum using an artificially extended array. We also propose to use the dominant sources' powers and directions obtained with MUSIC to generate a spectrum composed of triangular windows.

1. INTRODUCTION

When computing the Minimum Variance Distortionless Response (MVDR) beamformer it is necessary to scale the beamformer $\mathbf{R}^{-1}\mathbf{s}(\theta)$ in order to satisfy the unity gain constraint. $\mathbf{R}$ is the signal correlation matrix and $\mathbf{s}(\theta)$ is the steering vector associated to the direction θ. Comparing the MVDR beamformer to the Wiener-Hopf Minimum Mean Square Error (MMSE) equation we observe that the scaling factor is equal to the power arriving from the direction θ. This motivates us to propose a new version of the IDMR beamformer [1], where an initial estimate of the power is used instead of scaling to satisfy the unity gain constraint.

Two methods are proposed to initially estimate the power. The first one is based on artificially expanding the number of array elements, and forming a parametric estimate of the correlation matrix based on the detected dominant signal sources. This increases the resolution of the initial power spectrum. The second method is to form triangles centered at the detected dominant power sources. In both methods the dominant power sources are detected by using MUSIC [2]; these methods are described in Section 4.

In Section 2 we compare the MVDR beamformer to the Wiener-Hopf equation. In Section 3 we briefly review the IDMR beamformer. In Section 4 we describe the two power windows methods used to initially estimate the power spectrum. In Section 5 we show the simulation results; and in Section 6 we present the conclusion.

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2. MVDR BEAMFORMER

The MVDR beamformer is the solution to

$$\min_{\mathbf{w}(\theta_1)} \quad S_{xx}(\theta_1) = \mathbf{w}^H(\theta_1)\mathbf{R}\mathbf{w}(\theta_1)$$
$$\text{subject to} \quad \mathbf{w}^H(\theta_1)\mathbf{s}(\theta_1) = 1, \tag{1}$$

where $\mathbf{R} = \mathrm{E}\{\mathbf{x}[n]\mathbf{x}^H[n]\}$ is the correlation matrix; $\mathbf{x}[n]$ is a $M \times 1$ vector representing the sampled signal at the array elements at time n; M is the number of array elements. Using the Method of Lagrange Multipliers and taking the gradient of the augmented objective function dictates that the solution to the constrained optimization problem in (1) may be computed as the solution to

$$\mathbf{R}\mathbf{w}(\theta_1) = \lambda(\theta_1)\mathbf{s}(\theta_1), \tag{2}$$

where the Lagrange multiplier $\lambda(\theta_1)$ serves to satisfy the unity gain constraint in (1). Observe that when signals arriving from different directions are uncorrelated the equation above is the Wiener-Hopf equation,

$$\begin{aligned} \mathrm{E}\{\mathbf{x}[n]\mathbf{x}^H[n]\}\mathbf{w}(\theta_1) &= \mathrm{E}\{d_{\theta_1}[n]\mathbf{x}[n]\} \Rightarrow \\ \mathbf{R}\mathbf{w}(\theta_1) &= \mathrm{E}\{|d_{\theta_1}[n]|^2\}\mathbf{s}(\theta_1), \end{aligned} \tag{3}$$

where $d_{\theta_1}[n]$ is the desired signal from the direction θ_1 at time n. The sampled signal $\mathbf{x}[n]$ is given by

$$\mathbf{x}[n] = \sum_{i=1}^{k} d_{\theta_i}[n]\mathbf{s}(\theta_i) + \mathbf{n}[n], \tag{4}$$

where $d_{\theta_i}[n]$ is the signal waveform of the ith signal source, $\mathbf{s}(\theta_i)$ is its corresponding steering vector, and k is the number of incident signals. The array noise at time n is represented by the $M \times 1$ vector $\mathbf{n}[n]$. Uncorrelated signals, i.e., for $i \neq j$ $\mathrm{E}\{d_{\theta_i}[n]d_{\theta_j}[n]\} = 0$ (assuming signals have a zero mean), imply that $\mathrm{E}\{d_{\theta_1}[n]\mathbf{x}[n]\} = \mathrm{E}\{|d_{\theta_1}[n]|^2\}\mathbf{s}(\theta_1)$. Comparing Eqn. (2) with Eqn. (3), we observe that $\lambda(\theta_1)$ corresponds to $\mathrm{E}\{|d_{\theta_1}[n]|^2\}$, which is the power of the signal arriving from the direction θ_1. This motivates us to replace the scaling factor $\lambda(\theta_1)$ in (2) with an initial power estimate. Taking this approach we expect to reduce the white noise gain when the beamformer is aimed at a look-direction close to a source direction.

3. INDIRECT DMR

The true correlation matrix $\mathbf{R}$ has the form

$$\mathbf{R} = \mathbf{A}\mathbf{P}\mathbf{A}^H + \sigma_n^2\mathbf{I}, \tag{5}$$

where $\mathbf{A}$ is called the signal direction matrix (SDM) and is given by $\mathbf{A} = [\mathbf{s}(\theta_1) \vdots \mathbf{s}(\theta_2) \vdots \dots \vdots \mathbf{s}(\theta_k)]$, where $\mathbf{s}(\theta_i)$ $i = 1, ..., k$ are the steering vectors associated with the directions θ_i of the k signal sources. The matrix $\mathbf{P}$ is the source-signal correlation matrix, where σ_i^2 is the ith element along the main diagonal and is equal to the power of the signal arriving from the direction θ_i. σ_n^2 is the power of the spatially and temporally white Gaussian noise. $\mathbf{I}$ is the $M \times M$ Identity matrix, and M is the number of sensors.

Eqn. (5) can be rewritten as:

$$\mathbf{A}\mathbf{P}\mathbf{A}^H = \mathbf{R} - \sigma_n^2\mathbf{I}. \tag{6}$$

Multiplying the left hand side of the equation above by the pseudo-inverse $\mathbf{A}^\dagger = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H$, and multiplying the right hand side by $\mathbf{A}^{\dagger H}$ yields:

$$\mathbf{P} = \mathbf{A}^\dagger(\mathbf{R} - \sigma_n^2\mathbf{I})\mathbf{A}^{\dagger H}. \tag{7}$$

The first step of the proposed algorithm is to apply MUSIC to the sample correlation matrix $\hat{\mathbf{R}}$ to estimate the dominant signal directions. MUSIC is used for this purpose because it can handle source-signal correlations. This was the original "selling point" of the MUSIC algorithm; it just cannot handle 100% correlated sources. Subsequently, the SDM, denoted $\hat{\mathbf{A}}$, is formed from the estimated signal directions, given the known form of the array manifold. $\hat{\mathbf{A}}$ along with the sample correlation matrix $\hat{\mathbf{R}}$ and an estimate of the noise power $\hat{\sigma}_n^2$, is used to estimate the source-signal correlation matrix $\hat{\mathbf{P}}$, according to Eqn. (7). The elements not along the main diagonal of $\hat{\mathbf{P}}$, arising from residual correlations between sources due to finite sample averaging (or even due to true correlation between sources) are discarded, giving rise to a diagonalized signal-source correlation matrix estimate denoted $\hat{\mathbf{P}}_d$. Substituting $\hat{\mathbf{A}}$, $\hat{\mathbf{P}}_d$, $\hat{\sigma}_n^2$ into Eqn. (5) yields a parametric estimate of the correlation matrix denoted $\mathbf{R}_{idmr}$, wherein the contributions due to residual correlations amongst sources are removed. The MVDR beamformer for the direction θ is then formed using $\mathbf{R}_{idmr}$ as

$$\mathbf{w}(\theta_1) = \lambda(\theta_1)\mathbf{R}_{idmr}^{-1}\mathbf{s}(\theta_1), \tag{8}$$

where $\lambda(\theta_1)$ is the scaling factor.

4. INITIAL POWER ESTIMATE

In section 2 we showed that $\lambda(\theta_1)$ in Eqn.(8) is the power arriving from the direction θ_1. Therefore this motivates us to replace $\lambda(\theta_1)$ with an initial power estimate. In this paper we propose two forms of generating an initial estimate of $\lambda(\theta_1)$. We refer to these techniques as extended array power windows and triangular power windows.

4.1. Extended Array Power Windows

We now artificially extend the size of the array with the objective of getting a better estimate of the power spectrum. Using the directions of the dominant sources found with MUSIC we form a SDM matrix $\hat{\mathbf{A}}_{ex}$ for an artificially extended array of M_{ex} elements. We then form a parametric estimate of the correlation matrix $\mathbf{R}_{ex}$ for the extended array as

$$\mathbf{R}_{ex} = \hat{\mathbf{A}}_{ex}\hat{\mathbf{P}}_d\hat{\mathbf{A}}_{ex} + \frac{M_{ex}}{M}\hat{\sigma}_n^2, \tag{9}$$

where the factor M_{ex}/M accounts for the array gain, so that the noise floor is kept at the same level as the obtained with the original array. The power of the noise due to each array element is reduced by a factor equal to the number of array elements. Therefore, if this factor is omitted the noise floor will be reduced by a factor of M/M_{ex} in relation to the noise floor obtained with the original array. The power from a direction θ is then estimated as,

$$\lambda_{ex}(\theta) = \frac{1}{\mathbf{s}_{ex}^H(\theta)\mathbf{R}_{ex}^{-1}\mathbf{s}_{ex}(\theta)}, \tag{10}$$

where $\mathbf{s}_{ex}(\theta)$ is the steering vector associated to the direction θ for the extended array of M_{ex} elements. $\lambda_{ex}(\theta)$ is then used in Eqn. (8) to compute the beamformer $\mathbf{w}_{ex}(\theta)$ associated to the look-direction θ.

4.2. Triangular Power Windows

With the triangular power windows we propose an initial spectrum estimate where the power spectrum in decibels is formed by triangles with vertices located at the estimated power values of the dominant sources. For a source direction, the ideal output power of the MVDR beamformer is simply the signal power. For noise directions, this ideal output power becomes σ_n^2/M, that is, the power of the white noise divided by the number of array elements. Observe that in Eqn. (7), the main diagonal entries of $\hat{\mathbf{P}}$ are good power estimates. So we artificially construct scaling factors for each look-direction according to the following rule,

- Given the base length of the triangle as 2Δ;
- From the sources' directions estimated with MUSIC; we select the one with azimuth θ_i closest to the looking direction θ;
- If $|\theta - \theta_i| = \delta < \Delta$, then the scaling factor is

$$\lambda(\theta) = \sigma_i^2 - \frac{\delta}{\Delta} \cdot (\sigma_i^2 - \sigma_n^2/M), \tag{11}$$

  where σ_i^2 is the i^{th} power estimate. Calculations are taken in decibels (dBs);
- Otherwise $\lambda(\theta) = \sigma_n^2/M$.

The above rule applies estimated signal powers as the scaling factor for source directions and gradually pulls them down to the noise level for noise directions. We refer to this as the triangular power windows.

5. SIMULATIONS

Simulations were conducted employing a linear array of $M = 24$ elements with half-wavelength spacing. The noise at the array elements was spatially and temporally white with a Gaussian distribution. There were 12 incident signals with the arrival angles' directions and respective SNRs given in Table 1. The sampled correlation matrix was computed with 24 snapshots; and the forward-backward average [3] was used.

Angle (deg)	SNR (dB)
-69.70	8.91
-50.90	-1.89
-46.60	5.79
-41.50	6.17
-34.70	15.66
0.00	-13.80
6.80	16.20
11.80	14.20
19.80	7.74
24.20	-2.59
47.90	15.20
86.90	-3.05

Table 1. Angles and SNRs of incident signals

In Fig. 1 the initial power estimate $\lambda_{ex}(\theta)$ is plotted using the extended array technique with $M_{ex} = 96$ array elements. In Fig. 1(a) we used the 10 dominant signals detected with MUSIC to form the extended correlation matrix; and in Fig. 1(b) we used the 11 dominant signals.

In Fig. 2 we used in Eqn. (8) the power estimate $\lambda_{ex}(\theta)$ showed in Fig. 1 to compute the IDMR-Ex beamformer $\mathbf{w}_{ex}(\theta)$. This beamformer was then used to compute the output power at a look-direction θ using,

$$S(\theta) = \mathbf{w}_{ex}^H(\theta)\hat{\mathbf{R}}\mathbf{w}_{ex}(\theta), \tag{12}$$

this is the estimated power plotted in Fig. 2. It is interesting to observe that when only 10 dominant signals are used to form $\lambda_{ex}(\theta)$ in Fig. 1(a) the signal at $86.9°$ is not detected. However, when using the IDMR beamformer to compute the power spectrum there is a significant increase of power near the source at $86.9°$ indicating the presence of a source. The signal is still detected because it is embedded in the sampled correlation matrix $\hat{\mathbf{R}}$; the peak is not sharp because the beamformers used for the neighboring angles have no information of the presence of this signal (it was not included in $\mathbf{R}_{idmr}$), and therefore does not allocate a null at its direction. Fig. 3(a) plots the triangular power windows with a base of $2\Delta = 3°$ using the 10 dominant signals detected with MUSIC; these values are then used as $\lambda(\theta)$ to estimate the power spectrum plotted in Fig. 3(b).

6. CONCLUSION

We have proposed an hybrid beamformer for spectrum estimation that combines the MVDR beamformer with the Wiener-Hopf MMSE equation. Instead of scaling the IDMR beamformer to satisfy the unity gain constraint, we propose scaling the IDMR beamformer by an initial power spectrum estimate based on the power of the dominant signals computed with MUSIC. Simulations reveal the effectiveness of this technique.

7. REFERENCES

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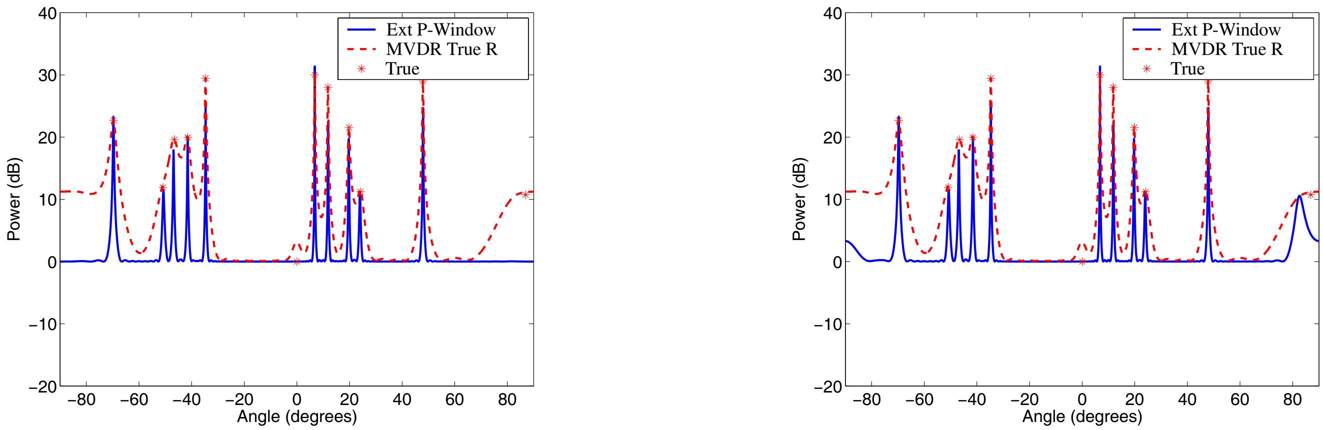
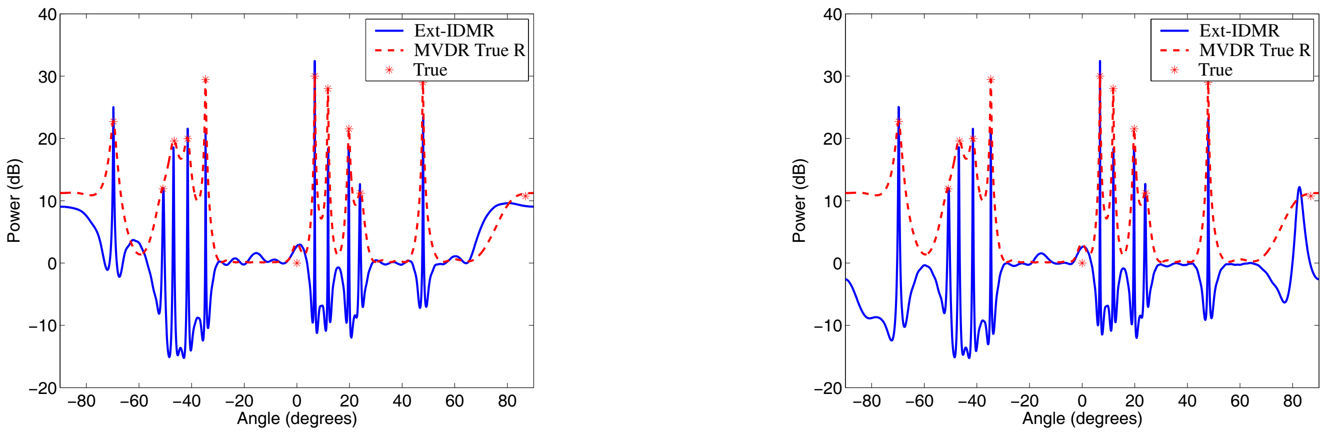


(a) 10 sources out of 12 detected by MUSIC

(b) 11 sources out of 12 detected by MUSIC

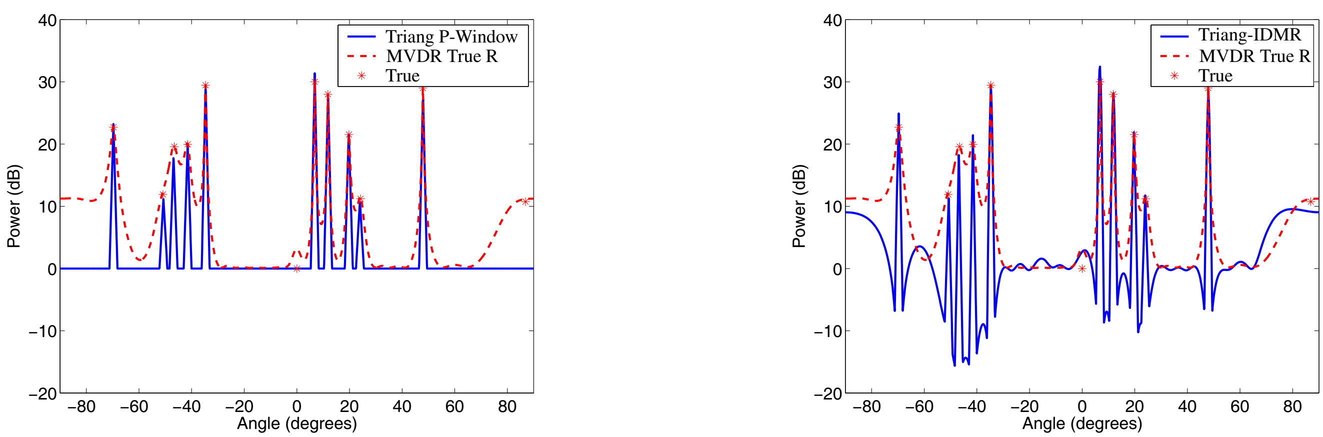
Fig. 1. Power windows computed with artificially extended array of 96 elements.



(a) 10 sources out of 12 detected by MUSIC

(b) 11 sources out of 12 detected by MUSIC

Fig. 2. Power Spectrum computed with Ext-IDMR.



(a) Triangular power window

(b) T-IDMR power spectrum

Fig. 3. Triangular power windows and T-IDMR power spectrum, 10 sources out of 12 detected by MUSIC.