A COMPLETE FAMILY OF QUASI-ORTHOGONAL SPACE-TIME CODES

*Yu Chang, Yingbo Hua**

Department of Electrical Engineering
University of California
Riverside, California, USA, 92521
Email: {ychang, yhua}@ee.ucr.edu

ABSTRACT

For two-transmitter systems, there are (full-rate) orthogonal space-time block codes (O-STBC). For four-transmitter systems, there are (full-rate) quasi-orthogonal space-time block codes (QO-STBC). The orthogonality of such codes makes a decoder attractively simple with only little compromise of optimality of coding performance. A complete family of O-STBC is well understood. But for QO-STBC, only ad hoc examples have been reported in the literature. In this paper, we provide a systematic construction of a complete family of 4×4 QO-STBC. We show that there are only three independent QO-STBC and all other QO-STBC can be constructed by trivial variations of any three independent codes. Indeed, all 4×4 QO-STBC in the literature can be constructed in such a way. Furthermore, we show a connection between three independent QO-STBC and the Hurwitz-Radon families of matrices. A complete set of the HR families of size four is also discovered.

1. INTRODUCTION

Design and analysis of space-time block codes (STBC) for multiple-transmitter systems have been an active field of research since the work published in [1] and [18]. STBC is aimed to exploit the channel diversity between multiple transmitters and multiple receivers to improve the rate of reliable data transmission and/or the performance of bit error rate. STBC is also useful for cooperative relays in wireless mobile networks [2], [10], [8], [7].

A detailed review of STBC is available in [4] and [11]. For two-transmitter systems, the most prominent STBC are the Alamouti -type codes that are orthogonal and hence allow the maximum likelihood (optimal) detection to be performed independently on each of individual symbols. Unfortunately, such an orthogonal code does not exist for more than two transmitters unless a reduction of data rate is tolerated [17], [20]. However, for four-transmitter systems, there are (quasi-orthogonal) QO-STBC that allow the maximum likelihood detection to be performed independently on pairs of symbols [9]. With a simple modification of symbol constellations, an QO-STBC can be made to achieve a full diversity and a high coding gain [15], [14], [5], [16].

Despite the attractive features of QO-STBC, the existing QO-STBC reported in the literature are ad hoc. They are ad hoc because a question like "how many more QO-STBC are there?" is not answered. In this paper, we provide a systematic construction of a complete family of (4×4) QO-STBC. We show that there are only three independent QO-STBC codes and all other QO-STBC can be constructed by trivial variations of any three independent codes. Specifically, given a QO-STBC matrix $S(s_1, s_2, s_3, s_4)$ of four symbols (s_1, s_2, s_3, s_4), numerous QO-STBC matrices can be constructed as follows:

$$C(s_1, s_2, s_3, s_4) = P_r S(\pm s_{k_1}^{(*)}, \pm s_{k_2}^{(*)}, \pm s_{k_3}^{(*)}, \pm s_{k_4}^{(*)}) P_c \quad (1)$$

where (k_1, k_2, k_3, k_4) is a permutation of $(1, 2, 3, 4)$, $\pm$ is a plus or minus sign, the superscript $^{(*)}$ denotes the presence or absence of complex conjugation, P_r permutes the rows and/or reverses the sign of none or some rows, and P_c permutes the columns and/or reverses the sign of none or some columns. (Two codes are said to be mutually dependent if they are related to each other via (1).) Indeed, all 4×4 QO-STBC reported in the literature can be constructed in such a way. We will also show that all these three independent codes can be constructed from the Hurwitz-Radon (HR) families of matrices.

In Section 2, we review the HR families of (integer) matrices and present a new theorem on the complete set of the HR families of size four. In Section 3, we first present two more theorems. One is that for any two 4×1 (linear-and-integer coded) vectors to be orthogonal to each other, they must be orthogonal through pairs of 2×1 subvectors. The other is that only three independent codes are necessary to produce a complete family of QO-STBC via (1). In Section 4, we show the connections between three independent codes and the QO-STBC matrices available in the literature. The proofs of our theorems are outlined in the Appendix.

2. THE HR FAMILIES OF MATRICES

As shown in [3], within the space of $L \times L$ integer matrices, there is a family of m matrices $\{A_0, A_1, ..., A_{m-1}\}$ satisfying: $A_k A_k^T = I_L$ (the $L \times L$ identity matrix), $A_k = -A_k^T$ unless $A_k = I_L$ and $A_k^T A_l = -A_l^T A_k$ $(k \neq l)$, where the maximum value m_{max} of m is governed by L as follows. Let $L = 2^a b$ where b is odd, $a = 4c + d$ and $0 \leq d \leq 3$, then $m_{max} = 8c + 2^d$. Within such a family, one member is I_L.

A family of matrices defined above is called a Hurwitz-Radon (HR) family of matrices. All HR matrices can be constructed from the following elementary matrices P, Q and R [3]:

$$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; Q = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; R = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

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For $L = 2$, an HR family consists of I_2 and R. The Alamouti code matrix can be constructed by using this HR family, i.e.,

$$C(s_1, s_2) = [R\underline{x}_1 + j\underline{x}_2, \underline{x}_1 + jR\underline{x}_2] = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix}$$

where $\underline{x}_1$ and $\underline{x}_2$ are two 2×1 real vectors, $s_1 = \underline{x}_1(2) + j\underline{x}_2(1)$ and $s_2 = \underline{x}_1(1) + j\underline{x}_2(2)$.

For $L = 4$, a known HR family consists of the following four matrices: $Q_0 = I_4$; $Q_1 = R \otimes I_2$; $Q_2 = P \otimes R$; $Q_3 = Q \otimes R$ where $\otimes$ is the Kronecker product. But more generally, we have discovered the following:

Theorem 1: All HR families of matrices of size four have the following two possible forms: $\{Q_0; \pm Q_1; \pm Q_2; \pm Q_3\}$ and $\{Q_0; \pm G_1; \pm G_2; \pm G_3\}$ where $G_1 = Q_1[-I_2 \otimes Q]$, $G_2 = Q_2[Q \otimes (RP)]$, and $G_3 = Q_3(Q \otimes I_2)$.

3. INDEPENDENT CODES OF QO-STBC

In this section, we show that only three independent codes are needed for (1) to produce all QO-STBC matrices of the symbol set (s_1, s_2, s_3, s_4) where each element in a code matrix is $\pm s_k^{(*)}$. But first, we have the following theorem:

Theorem 2: Let $\underline{s} = \underline{r} + j\underline{i}$ be a 4×1 complex vector where the first term is the real part and the second term is the imaginary part. Define $\underline{p} = T_r\underline{r} + jT_i\underline{i}$ where T_r and T_i are unitary integer matrices. Then, this orthogonality $\underline{s}^H\underline{p} = 0$ holds if and only if a pair of elements in $\underline{s}$ is orthogonal to the corresponding pair in $\underline{p}$ and the other pair of elements in $\underline{s}$ is orthogonal to the other corresponding pair in $\underline{p}$.

Given the above theorem, the next theorem follows:

Theorem 3: If used in (1), the following three independent codes produce all possible QO-STBC matrices for the symbol set (s_1, s_2, s_3, s_4) where each element in an QO-STBC matrix has the form $\pm s_k^{(*)}$:

$$S_1(s_1, s_2, s_3, s_4) = \begin{pmatrix} s_1 & -s_4 & s_2^* & -s_3^* \\ s_2 & s_3 & -s_1^* & -s_4^* \\ s_3 & -s_2 & -s_4^* & s_1^* \\ s_4 & s_1 & s_3^* & s_2^* \end{pmatrix}$$

$$S_2(s_1, s_2, s_3, s_4) = \begin{pmatrix} s_1 & s_4 & s_2^* & -s_3^* \\ s_2 & s_3 & -s_1^* & s_4^* \\ s_3 & s_2 & -s_4^* & s_1^* \\ s_4 & s_1 & s_3^* & -s_2^* \end{pmatrix}$$

$$S_3(s_1, s_2, s_3, s_4) = \begin{pmatrix} s_1 & -s_4 & -s_2^* & -s_3^* \\ s_2 & s_3 & s_1^* & -s_4^* \\ s_3 & s_2 & -s_4^* & s_1^* \\ s_4 & -s_1 & s_3^* & s_2^* \end{pmatrix}$$

All of the above three independent codes can be expressed in terms of the HR matrices. Let the real and imaginary parts of each symbol s_k be expressed as $s_k = r_k + ji_k$. It is not difficult to verify the following results. For the first independent code,

$$\begin{aligned} S_1 = {} & [Q_0\underline{r}_1, -Q_1\underline{r}_1, Q_3\underline{r}_1, -Q_2\underline{r}_1] + \\ & j[Q_2\underline{i}_1, -Q_3\underline{i}_1, -Q_1\underline{i}_1, Q_0\underline{i}_1] \end{aligned} \quad (2)$$

where $\underline{r}_1 = [r_1, ..., r_4]^T$ and $\underline{i}_1 = [i_3, i_4, -i_1, -i_2]^T$. For the second independent code,

$$\begin{aligned} S_2 = {} & [-Q_3K\underline{r}_2, Q_2\underline{r}_2, Q_0K\underline{r}_2, Q_1^T\underline{r}_2] + \\ & j[-Q_0K\underline{i}_2, Q_1^T\underline{i}_2, Q_3K\underline{i}_2, Q_2\underline{i}_2] \end{aligned} \quad (3)$$

where $\underline{r}_2 = [-r_2, -r_1, r_4, r_3]^T$, $\underline{i}_2 = [i_1, -i_2, i_3, -i_4]^T$, and $K = -I_2 \otimes Q$. For the third independent code,

$$\begin{aligned} S_3 = {} & [G_0\underline{r}_3, -G_1K\underline{r}_3, -G_3\underline{r}_3, -G_2K\underline{r}_3] + \\ & j[-G_3\underline{i}_3, -G_2K\underline{i}_3, G_0\underline{i}_3, -G_1K\underline{i}_3] \end{aligned} \quad (4)$$

where $\underline{r}_3 = \underline{r}_1$ and $\underline{i}_3 = [i_2, -i_1, i_4, -i_3]^T$.

4. THE PREVIOUSLY PUBLISHED QO-STBC

We now provide the specific expressions of the previously published QO-STBC in terms of the above three independent codes.

4.1. From the first independent code

The code by Papadias and Foschini in [12] can be described as:

$$\begin{aligned} C_{PF}(s_1, s_2, s_3, s_4) &= \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & -s_1 & s_2 \\ s_4^* & s_3^* & -s_2^* & -s_1^* \end{pmatrix} \\ &= P_1S_1(s_2^*, s_3, s_1, s_4^*)P_2 \end{aligned}$$

where

$$\begin{aligned} P_1 &= P_{+(1,3),+(2,1),+(3,2),+(4,4)} \\ P_2 &= P_{+(1,1),-(2,4),-(3,2),-(4,3)}. \end{aligned}$$

Here, $P_{\pm(i_1,j_1),\pm(i_2,j_2),\pm(i_3,j_3),\pm(i_4,j_4)}$ denotes a matrix where the entries at (i_1, j_1), (i_2, j_2), (i_3, j_3), (i_4, j_4) are ± 1 and all other entries are zero.

One code given by Hou, Lee and Park, i.e., (20) in [6], can be described by:

$$\begin{aligned} C_{HLP,1}(s_1, s_2, s_3, s_4) &= \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2 & s_1 & -s_4 & s_3 \\ -s_3^* & s_4^* & s_1^* & -s_2^* \\ -s_4^* & -s_3^* & s_2^* & s_1^* \end{pmatrix} \\ &= P_3S_1(s_1, -s_4^*, -s_3^*, -s_2)P_4 \end{aligned}$$

where

$$\begin{aligned} P_3 &= P_{+(1,1),+(2,4),+(3,3),+(4,2)} \\ P_4 &= P_{+(1,1),+(2,2),-(3,4),+(4,3)}. \end{aligned}$$

4.2. From the second independent code

The code by Ran, Hou and Lee, i.e., (10) in [13], actually is:

$$\begin{aligned} C_{RHL}(s_1, s_2, s_3, s_4) &= \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & -s_4^* & s_3^* \\ s_3^* & -s_4^* & -s_1^* & s_2^* \\ s_4 & s_3 & s_2 & s_1 \end{pmatrix} \\ &= S_2(s_1, s_2^*, s_3^*, s_4)P_5 \end{aligned}$$

where $P_5 = P_{+(1,1),+(2,4),+(3,2),-(4,3)}$.

The second code by Hou, Lee and Park, i.e., (16) in [6], actually is:

$$\begin{aligned} C_{HLP,2}(s_1, s_2, s_3, s_4) &= \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2 & s_1 & s_4 & s_3 \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ -s_4^* & -s_3^* & s_2^* & s_1^* \end{pmatrix} \\ &= P_6S_2(s_1, -s_4^*, -s_3^*, s_2)P_7 \end{aligned}$$

where $P_6 = P_3$ and $P_7 = P_{+(1,1),+(2,2),-(3,3),+(4,4)}$.

The code by Tirkkonen, Boariu and Hottinen in [19] actually is

$$C_{TBH}(s_1, s_2, s_3, s_4) = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_3 & s_4 & s_1 & s_2 \\ -s_4^* & s_3^* & -s_2^* & s_1^* \end{pmatrix}$$
$$= P_8 S_2(s_1, -s_2^*, -s_4^*, s_3) P_9$$

where

$$P_8 = P_{+(1,1),+(2,2),+(3,4),+(4,3)}$$
$$P_9 = P_{+(1,1),-(2,3),-(3,2),+(4,4)}.$$

4.3. From the third independent code

The code by Jafarkhani in [9] simply is

$$C_J(s_1, s_2, s_3, s_4) = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{pmatrix}$$
$$= S_3(s_1, -s_2^*, -s_3^*, s_4) P_{10}$$

where $P_{10} = P_{+(1,1),-(2,4),+(3,2),+(4,3)}$.

5. CONCLUSION

We have shown that using (1) together with only three independent codes, one can produce *all* QO-STBC. This result unifies all previously published QO-STBC as well as numerous "hidden" QO-STBC.

6. APPENDIX

6.1. Proof of Theorem 1

Here we describe all possible (non-identity) members in the HR families of size 4. An HR matrix F of size four can be written as $F = \begin{pmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{pmatrix}$ where $B_{i,l}$ is a 2×2 block matrix. Since $F^T = -F$ we have $B_{i,i}^T = -B_{i,i}$ for $i = 1, 2$ and $B_{1,2}^T + B_{2,1} = 0$. Therefore $B_{1,1}$ and $B_{2,2}$ are either the zero matrices or the HR matrices of 2×2. Since F is an integer matrix and unitary, each column of F has one and only one entry of ± 1 and all other entries of the column are zero. Then, we have either

$$\left\{ \begin{array}{l} B_{1,1} = B_{2,2} = 0 \\ B_{1,2}^T = -B_{2,1} \neq 0 \end{array} \right. \quad \text{or} \quad \left\{ \begin{array}{l} B_{1,1} \text{ is a } 2 \times 2 \text{ HR matrix} \\ B_{2,2} \text{ is a } 2 \times 2 \text{ HR matrix} \\ B_{1,2} = B_{2,1} = 0 \end{array} \right. \qquad (5)$$

Through an exhaustive search, we have found that any possible HR matrix has to come from those defined in Theorem 1.

To show that Q_i and G_i defined in Theorem 1 can not co-exist in one family, we first observe that $G_i = Q_i K_i$ where $K_i \neq 0$. If Q_i and G_i co-exist in one family, then $Q_i^T G_i = -G_i^T Q_i$. Combining the above two equations, we have $Q_i^T Q_i K_i = -K_i^T Q_i^T Q_i$, i.e., $K_i = 0$, which is a contradiction. By a further exhaustive search, we have found that Q_i and G_l can not co-exist within a family even if $i \neq l$.

Therefore, the two possible structures shown in Theorem 1 are applicable to all HR families of size four.

6.2. Proof of Theorem 2

Taking the real and imaginary parts of $\underline{s}^H \underline{p} = 0$ separately, we have $t_1 + t_2 = 0$ and $t_3 = 0$ where $t_1 = \underline{s}_r^T T_r \underline{s}_r$, $t_2 = \underline{s}_i^T T_i \underline{s}_i$, and $t_3 = \underline{s}_i^T T_r \underline{s}_r - \underline{s}_i^T T_i^T \underline{s}_r$.

Because of the independence of t_1 and t_2, $t_1 + t_2 = 0$ implies $t_1 = 0$ and $t_2 = 0$. Because both of the equations hold for any vectors, we have $T_r = -T_r^T$ and $T_i = -T_i^T$.

From $t_3 = 0$, we have $T_r = T_i^T$.

From $t_1 = 0$, we have

$$\sum_{l=1}^{4} [\sum_{i=1}^{4} \underline{s}_r(i) T_{r,i,l}] \underline{s}_r(l) = 0 \Rightarrow \sum_{l=1}^{4} \pm \underline{s}_r(i_l) \underline{s}_r(l) = 0 \qquad (6)$$

where $T_{r,i,l}$ is the (i, l) entry of T_r, and among $[T_{r,1,l}, \ldots, T_{r,4,l}]^T$ there is only one non-zero entry, which is ± 1 . We also know $i_l \neq l$, or otherwise (6) could not hold. For (6) to hold for any $\underline{s}_r$, the four terms in (6) must be divided into two groups, and the two terms in each group cancel each other. In other words, the indices of the two terms in each group must be (i_l, l) and (l, i_l). Without loss of generality (up to a permutation of the symbols), these terms can grouped as $\pm\{\underline{s}_r(1)\underline{s}_r(2) - \underline{s}_r(2)\underline{s}_r(1)\}$ and $\pm\{\underline{s}_r(4)\underline{s}_r(3) - \underline{s}_r(3)\underline{s}_r(4)\}$. Then, we can write $T_r = diag(\pm Q, \pm Q)$ (up to a permutation of the symbols). Furthermore, up to a sign change of all symbols, there are two possible forms for T_r, i.e., $diag(Q, Q)$ or $diag(Q, -Q)$.

Since $T_r = T_i^T$, the corresponding T_i must be $diag(-Q, -Q)$ or $diag(-Q, Q)$, respectively.

When $T_r = diag(Q, Q)$ and $T_i = diag(-Q, -Q)$, we have $\underline{p} = [s_2^*, -s_1^*, s_4^*, -s_3^*]$. When $T_r = diag(Q, -Q)$ and $T_i = diag(-Q, -Q)$, we have $\underline{p} = [s_2^*, -s_1^*, -s_4^*, s_3^*]$. In each of the above two cases, we have the familiar form of 2×2 orthogonality.

6.3. Proof of Theorem 3

Our proof is constructive. Without loss of generality, we will fix the first column of our code matrix T to be $[s_1, s_2, s_3, s_4]^T$. Furthermore, we will construct T such that its first two columns are orthogonal to the last two columns. By using Theorem 2, we have no more than the following two possibilities for T (up to the variations defined by (1)):

$$T_1 = \begin{pmatrix} s_1 & * & s_2^* & * \\ s_2 & * & -s_1^* & * \\ s_3 & * & -s_4^* & * \\ s_4 & * & s_3^* & * \end{pmatrix} \quad \text{or}$$

$$T_2 = \begin{pmatrix} s_1 & * & -s_2^* & * \\ s_2 & * & s_1^* & * \\ s_3 & * & -s_4^* & * \\ s_4 & * & s_3^* & * \end{pmatrix}$$

where $*$ (not in superscript) denotes a unspecified entry. From T_1, we have no more than the following two possibilities:

$$T_{1,1} = \begin{pmatrix} s_1 & * & s_2^* & -s_3^* \\ s_2 & * & -s_1^* & s_4^* \\ s_3 & * & -s_4^* & s_1^* \\ s_4 & * & s_3^* & -s_2^* \end{pmatrix} \quad \text{or}$$

$$T_{1,2} = \begin{pmatrix} s_1 & * & s_2^* & -s_3^* \\ s_2 & * & -s_1^* & -s_4^* \\ s_3 & * & -s_4^* & s_1^* \\ s_4 & * & s_3^* & s_2^* \end{pmatrix}$$

Similarly, from T_2, we have no more than another pair of possibilities:

$$T_{2,1} = \begin{pmatrix} s_1 & * & -s_2^* & -s_3^* \\ s_2 & * & s_1^* & s_4^* \\ s_3 & * & -s_4^* & s_1^* \\ s_4 & * & s_3^* & -s_2^* \end{pmatrix} \text{ or}$$

$$T_{2,2} = \begin{pmatrix} s_1 & * & -s_2^* & -s_3^* \\ s_2 & * & s_1^* & -s_4^* \\ s_3 & * & -s_4^* & s_1^* \\ s_4 & * & s_3^* & s_2^* \end{pmatrix}$$

Adding all together, we have no more than the following four possibilities:

$$T_{1,1} = \begin{pmatrix} s_1 & s_4 & s_2^* & -s_3^* \\ s_2 & s_3 & -s_1^* & s_4^* \\ s_3 & s_2 & -s_4^* & s_1^* \\ s_4 & s_1 & s_3^* & -s_2^* \end{pmatrix},$$

$$T_{1,2} = \begin{pmatrix} s_1 & -s_4 & s_2^* & -s_3^* \\ s_2 & s_3 & -s_1^* & -s_4^* \\ s_3 & -s_2 & -s_4^* & s_1^* \\ s_4 & s_1 & s_3^* & s_2^* \end{pmatrix},$$

$$T_{2,1} = \begin{pmatrix} s_1 & s_4 & -s_2^* & -s_3^* \\ s_2 & s_3 & s_1^* & s_4^* \\ s_3 & -s_2 & -s_4^* & s_1^* \\ s_4 & -s_1 & s_3^* & -s_2^* \end{pmatrix},$$

$$T_{2,2} = \begin{pmatrix} s_1 & -s_4 & -s_2^* & -s_3^* \\ s_2 & s_3 & s_1^* & -s_4^* \\ s_3 & s_2 & -s_4^* & s_1^* \\ s_4 & -s_1 & s_3^* & s_2^* \end{pmatrix}.$$

Furthermore, it is easy to verify that

$$T_{1,2} = P_{11} T_{2,1}(s_4, s_2, s_3, s_1) P_{12}$$

where

$$P_{11} = P_{+(1,4),+(2,2),+(3,3),+(4,1)}$$
$$P_{12} = P_{+(1,1),+(2,2),-(3,4),-(4,3)}.$$

Therefore, we have only three independent codes: $S_1 = T_{1,2}$, $S_2 = T_{1,1}$, and $S_3 = T_{2,2}$.

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