

ENERGY-EFFICIENT DIGITAL FILTERING USING ML-BASED ERROR CORRECTION (ML-EC) TECHNIQUE

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ABSTRACT

In this paper, we present a maximum likelihood-based error correction (ML-EC) technique which achieves significant power savings in digital filtering. Although voltage overscaling (VOS) can achieve high energy efficiency, it can introduce “soft errors” which severely degrade the performance of the filter. The proposed scheme detects, estimates and corrects these soft errors via an ML-based algorithm that achieves up to 47% power savings without any SNR loss and up to 60% power savings with a 1.5 dB SNR loss for an example case study of a frequency-selective low-pass filter.

1. INTRODUCTION

Due to the growing power demands on portable and wireless applications, techniques for reducing power in DSP applications have gained importance. Numerous techniques have been proposed to achieve low-power in DSP systems [1]-[4]. Among them, supply voltage reduction has been perhaps most widely used due to the quadratic dependency of supply voltage on power dissipation [1].

Previously, an approach referred to as voltage overscaling (VOS) was proposed to achieve significant power savings [3, 4]. VOS implies reduction of supply voltage below the point where the critical path delay begins to exceed the sampling period, *i.e.*

$$V_{dd} = k_{vos} V_{dd-crit}, \quad 0 < k_{vos} < 1 \quad (1)$$

where k_{vos} is voltage overscaling factor (VOF) and $V_{dd-crit}$ is the voltage at which the critical path delay equals the sampling period. Hence, whenever an input pattern excites a path which causes larger delay than the sampling period, a transient error, called a “soft error”, occurs, thereby degrading the SNR performance in the main DSP (MDSP) block. Hence, effective soft error mitigation becomes necessary for successful power savings via VOS.

So far, a number of techniques to mitigate soft error have been proposed [3, 4]. A prediction-based error control

(PEC) scheme [3] estimates the MDSP output which in this case is a digital filter by using a forward linear predictor. A reduced-precision redundancy (RPR) scheme [4] replaces the potentially corrupted MDSP output with the output of MDSP replica, when an error is detected.

In this paper, we propose a soft error correction technique based on a maximum likelihood (ML) principle for low-power digital filtering. The ML-EC technique is distinct from previous PEC or RPR schemes, which use the estimator output to replace the noisy MDSP output whenever soft errors occur. The ML-based method estimates the errors, and removes them from the noisy MDSP output. Consequently, SNR degradation due to VOS can be restored completely up to certain point of k_{vos} . Even for a severely scaled VOS conditions, we show that the performance degradation of the ML-based method can remain small. Simulations suggest that the ML-EC technique outperforms the conventional PEC [3] and RPR [4] approaches.

The remainder of this paper is organized as follows. Section 2 describes soft error characteristics and presents the ML-EC technique. Simulation results are presented in Section 3.

2. ML-BASED ERROR CORRECTION (ML-EC) TECHNIQUE

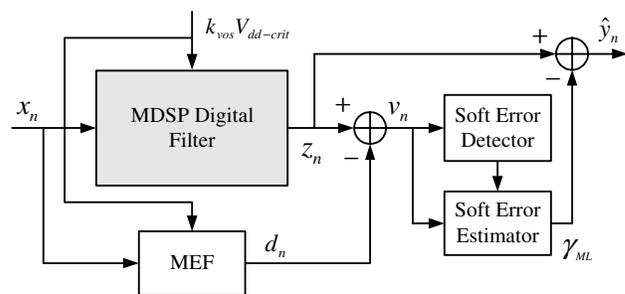


Fig. 1. Block diagram of the proposed ML-EC technique.

The ML-EC technique is depicted in Fig. 1. When a soft error is detected, we estimate the amplitude and subtract the estimate from the noisy MDSP output. In order to perform this task, we employ an MDSP estimation filter (MEF) which generates an estimate of the MDSP output derived from a minimum mean square error (MMSE) criterion. An ML-based error correction can then be applied after subtracting the MEF output from the noisy MDSP output.

2.1. Soft Error Characteristics

Soft errors occur when the supply voltage is scaled down beyond the point where the critical path delay exceeds the sampling period. Since most arithmetic units are based on LSB-first computation, soft errors tend to appear in the MSBs. The bits in a multiplier output are divided into two portions : error-prone bits (EB), or MSBs which are affected by VOS, and safe bits (SB), or unaffected LSBs. For example, in Fig. 2, the number of SB and EB are 5 and 3, respectively. Since the error occurs only in EB, we can deduce that possible amplitudes of the soft error are spaced by 2^M where M is the number of SB. Note that as k_{vos} decreases, the number of SB decreases and hence the error spacing becomes smaller.

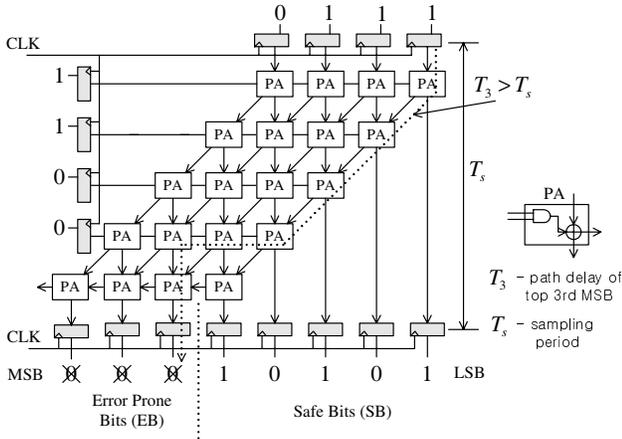


Fig. 2. Illustration of soft error characteristics in a simple 4-by-4 carry ripple multiplier.

2.2. Main DSP Estimation Filter (MEF)

In the presence of soft errors, the output z_n of an N -tap MDSP filter can be expressed as

$$z_n = y_n + \gamma = \sum_{k=0}^{N-1} h_k x_{n-k} + \gamma, \quad (2)$$

where x_n , γ and h_k are the system input, the soft error and the filter coefficients respectively, and y_n is the error free output of the MDSP filter.

The MEF estimates the desired output, y_n by using a reduced order $L (< N)$ and precision $P (< B)$ replica filter. Due to the reduced order and precision, the MEF introduces quantization noise q_n and estimation noise r_n i.e.

$$q_n = d_{on} - d_n \quad (3)$$

$$r_n = y_n - d_{on} \quad (4)$$

where d_{on} is the full precision estimate of y_n , and d_n is its quantized version. For effective error correction, we generate the residual signal v_n by subtracting d_n from z_n , i.e.

$$v_n = z_n - d_n = (y_n + \gamma) - (d_{on} - q_n) \quad (5)$$

$$= \gamma + (r_n + q_n). \quad (6)$$

Hence, in (6), the soft error γ , which we should detect, is embedded in the pure estimation noise r_n and the quantization noise q_n .

The MMSE MEF estimate, minimizing the mean squared error between d_{on} and y_n , is obtained by the Wiener-Hopf equation given by

$$\begin{aligned} \mathbf{w}_0 &= \mathbf{R}_x^{-1} \mathbf{p} \\ &= \mathbf{R}_x^{-1} \cdot \sum_{k=0}^{L-1} h_k \begin{bmatrix} r_{xx}(k) \\ \vdots \\ r_{xx}(k - L' + 1) \end{bmatrix} \end{aligned} \quad (7)$$

where \mathbf{R}_x is the L -by- L autocorrelation matrix of x_n and $r_{xx}(\cdot)$ is the autocorrelation function of x_n . The power of r_n can be easily shown as

$$\sigma_r^2 = \sigma_y^2 - \mathbf{p}^T \mathbf{R}_x^{-1} \mathbf{p} \quad (8)$$

where σ_y^2 is the variance of y_n .

Next, let us derive the quantization noise power σ_q^2 . We assume that all the quantization noise is due to truncation and all signals are represented in two's complement form in $[-1, 1)$. The input x_n consists of the truncated term x_{qn} and the quantization error term Δx_n . It can be shown that the power of Δx_n is given by

$$E[\Delta x_n^2] = \frac{1}{6} \left(\frac{2}{2^{2P}} - \frac{3}{2^{B+P}} + \frac{1}{2^{2B}} \right) \quad (9)$$

where Δx_n is assumed to be uniformly distributed. In addition, the coefficient w_n consists of the truncated term w_{qn} and remaining error term Δw_n . The quantization noise power, σ_q^2 can be shown to be

$$\begin{aligned} \sigma_q^2 &= \sum_{j=0}^{L'-1} \sum_{k=0}^{L'-1} \Delta w_j r_{xx}(j-k) \Delta w_k \\ &\quad + \sum_{k=0}^{L'-1} w_k^2 E[\Delta x_n^2] \end{aligned} \quad (10)$$

where we assume that x_n has zero mean, and x_n and Δx_n are uncorrelated, and that w_{qk} and Δw_k are known and non-random. From (8) and (10) and assuming that r_n and q_n are uncorrelated, the variance of the signal $g_n (= r_n + q_n)$ can be written

$$\sigma_g^2 = \sigma_r^2 + \sigma_q^2. \quad (11)$$

Fig. 3 illustrates the distributions of z_n and $v_n (= z_n - d_n)$ after applying the MEF. The variance of the residual output v_n is smaller than that of the MDSP output z_n . By exploiting this spacing property of the soft errors, we can detect and estimate the soft error with improved accuracy.

2.3. ML-based Soft Error Detection and Estimation

Based on observation of v_n so far, soft errors can be detected and estimated via an ML-based algorithm. If we suppose that x_n has a Gaussian distribution with zero mean, then r_n is also Gaussian. If the precision of MEF, P is sufficiently large, the effect of q_n can be approximated such that the distribution of $r_n + q_n$ is Gaussian with $N(0, \sigma_r^2 + \sigma_q^2)$.

2.3.1. Soft Error Detection

The hypothesis test for the soft error detection can be formulated as

$$\begin{aligned} H_0 &: v_n = r_n + q_n \\ H_1 &: v_n = r_n + q_n + \gamma \end{aligned} \quad (12)$$

where γ has a value among the set $\Omega = \{k2^{-B+M} | k \in \mathcal{Z}, k \in [-2^{B-M}, 2^{B-M}]\}$ due to the spacing property. Since γ is an unknown parameter, this can be interpreted as a composite hypothesis test with generalized likelihood ratio test (GLRT) given by [5]

$$\Lambda(v_n) = \max_{\gamma \in \Gamma} \frac{P(v_n | \gamma, H_1)}{P(v_n | H_0)} \quad (13)$$

$$= \max_{\gamma \in \Gamma} \frac{\frac{1}{\sqrt{2\pi}\sigma_g} \exp\left(-\frac{(v_n - \gamma)^2}{2\sigma_g^2}\right)}{\frac{1}{\sqrt{2\pi}\sigma_g} \exp\left(-\frac{v_n^2}{2\sigma_g^2}\right)} \underset{H_0}{\overset{H_1}{\geq}} \tau \quad (14)$$

where τ is an appropriately selected threshold. If we denote γ_{\max} as the argument γ which maximizes the left term of (14), the GLRT rule is rewritten as

$$\frac{v_n^2 - (v_n - \gamma_{\max})^2}{2\sigma_g^2} \approx \frac{v_n^2}{2\sigma_g^2} \underset{H_0}{\overset{H_1}{\geq}} \ln \tau \quad (15)$$

where the approximation is based on the fact that γ_{\max} becomes the ML estimator of γ . Specifically, the approximation holds since $v_n^2 \gg (v_n - \gamma_{\max})^2$ under H_1 , and $v_n - \gamma_{\max} \approx 0$ under H_0 . Rewriting (15), we obtain an approximate rule given by

$$\left| \frac{v_n}{\sigma_g} \right| \underset{H_0}{\overset{H_1}{\geq}} \tau' = \sqrt{2 \ln \tau}. \quad (16)$$

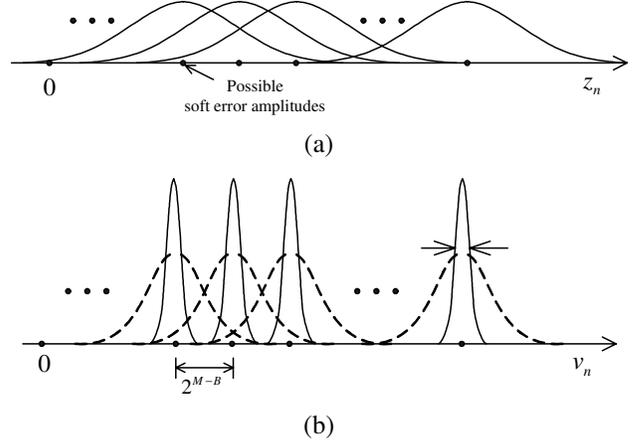


Fig. 3. Distributions of (a) the noisy MDSP output $z_n (= y_n + \gamma)$ and (b) the residual output $v_n (= g_n + \gamma)$

The corresponding false alarm and miss probability of the GLRT detector are, respectively, given by

$$P_e^F = 2Q(\tau') \quad (17)$$

$$P_e^M = Q\left(-\tau' - \frac{\gamma_0}{\sigma_g}\right) - Q\left(\tau' - \frac{\gamma_0}{\sigma_g}\right) \quad (18)$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp(-t^2/2) dt$ and γ_0 is the actual error amplitude. Since P_e^M is a monotonically increasing function of σ_g , we deduce that the reduced variance due to the MEF improves the performance of the GLRT detector.

2.3.2. Soft Error Estimation

The ML estimate of the detected soft error is obtained by finding γ_{\max} which maximizes $P(v_n | \gamma, H_1)$. The ML estimate, γ satisfying this condition is given by

$$\begin{aligned} \gamma_{ML} &= 2^{-B+M}i, \quad \text{where} \\ |v_n - 2^{-B+M}i| &\leq |v_n - 2^{-B+M}j| \quad \text{for } \forall j \end{aligned} \quad (19)$$

where i and j are integers between $[-2^{B-M}, 2^{B-M}]$. Essentially, the ML-based estimator selects the closest candidate from v_n among all possible. The variance of γ_{ML} , which measures the performance of the ML estimate, is given by

$$\begin{aligned} E[(\gamma_{ML} - \gamma)^2] &= 2c^2 \sum_{i=0}^{\infty} i^2 \left[Q\left(\frac{c}{\sigma_g} \left(i - \frac{1}{2}\right)\right) \right. \\ &\quad \left. - Q\left(\frac{c}{\sigma_g} \left(i + \frac{1}{2}\right)\right) \right], \end{aligned} \quad (20)$$

where $c = 2^{-B+M}$. Since small σ_g leads to small variance of γ_{ML} , we see that MEF also improves the accuracy of ML-based estimator.

3. SIMULATIONS AND RESULTS

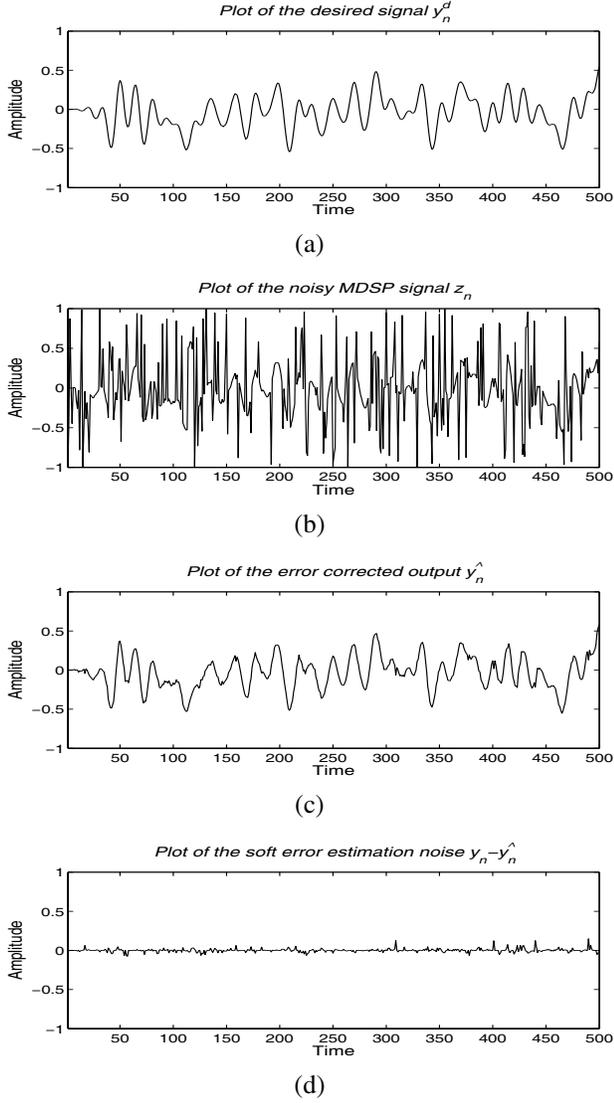


Fig. 4. Several signal plots when k_{vos} is set to 0.6. (a) y_n , (b) $z_n (= y_n + \gamma)$, (c) \hat{y}_n and (d) $y_n - \hat{y}_n$.

In this section, we present simulation results to evaluate the performance of ML-based method. The simulation is performed based on $0.25\mu\text{m}$, 2.5V CMOS process technology. We designed a 26-tap FIR low-pass filter with pass-band of $[0, \pi/5]$. The filter input and coefficients are quantized into 16 bit precision, and a bandlimited signal plus the white Gaussian noise is used as the input signal. Without VOS, the SNR measured at the MDSP output is 26.47 dB. Hence, if we set the tolerable noise margin to 1.47 dB, we should meet a 25 dB SNR requirement.

Fig. 4 (a) and Fig. 4 (b) show the clean desired signal y_n

and noisy output z_n due to VOS at $k_{vos} = 0.6$. Fig. 4 (c) and Fig. 4 (d) show plots of the restored signal and correction error when the MEF has 4 taps and 12 bits of precision. Fig. 5

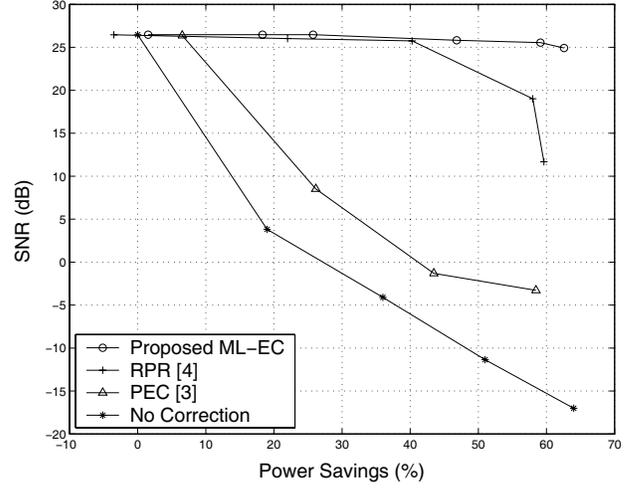


Fig. 5. SNR performance vs. power saving (%).

shows the SNR performance vs. power savings in case of an uncorrected, the ML-EC scheme, and the algorithms of [3, 4]. It is shown that the ML-EC technique achieves up to 47% power savings without any SNR loss and up to 60% power savings subject to 1.47 dB SNR loss which satisfies the desired SNR requirement. From these results, we observe that the ML-based algorithm shows promising performance and outperforms the conventional PEC [3] and RPR [4].

4. REFERENCES

- [1] B. Davari, R. H. dennard and G. G. Shahidi, "CMOS scaling for high-performance and low power - the next ten years," in *Proceedings of IEEE*, vol. 83, pp. 595-606, April 1995.
- [2] A. P. Chandrakasan and R. W. Brodersen, "Minimizing power consumption in digital CMOS circuits," in *Proc. of IEEE*, vol. 83, pp. 498-523, April 1995.
- [3] R. Hedge and N. Shanbhag, "Soft digital signal processing," *IEEE Trans. on VLSI*, vol. 9, no. 6, pp. 813-823, Dec. 2001.
- [4] B. Shim, S. Sridhara, N. Shanbhag, "Reliable low-power digital signal processing via reduced precision redundancy," *IEEE Trans. on VLSI*, vol. 12, pp. 497-510, May 2004.
- [5] H. V. Poor, *An Introduction to Signal Detection and Estimation, 2nd Edition*, Springer, 1994.