AN LMMSE DETECTOR FOR A SPREAD-SPECTRUM SYSTEM BASED ON RANDOM PERMUTATIONS OVER FREQUENCY SELECTIVE FADING CHANNELS

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ABSTRACT

This article deals with the joint detection of bits for a spread-spectrum system based on random permutations, in the case of a transmission over frequency-selective channels. A linear minimum mean-squares error approach is studied, whose objective consists of mitigating both the multi-access interference and the inter-symbole interference. The proposed system is compared with the DS-CDMA system. The theoretical study is confirmed by simulation results.

1. INTRODUCTION

Spread spectrum systems consitute one of the most important multiple-access techniques considered for the third generation of mobile systems. Direct-sequence CDMA (DS-CDMA) and Frequency-Hopping CDMA are the most two common. However, other techniques can be investigated, which present equivalent spreading capabilities, and allow multiple access. In particular, Periodic Clock Changes (PCC), which are a particular case of linear periodic time varying filters, are useful for multiple access because of their spreading properties [5], [7]. In this article, a sub-class of discrete PCC, called permutations, is studied. The joint detection in the case of Random Permutation-based Multiple Access (RPMA) has been studied in previous works [1], [2], [5], [7]. Moreover, many detectors can be found in the literature for CDMA systems (see for instance [6], [8] and references therein). In this paper we study a Linear Minimun Mean Squares Error (LMMSE) detector, in the case of a transmission over frequency selective fading channels. Section 2 presents the random permutation technique, and the RPMA signal model in its continuous and discrete forms. The LMMSE detector is studied in Section 3 for the RPMA system, as well as for the DS-CDMA system; theoretical bit error rates (BER) are then provided. Simulation results are given in section 4, which show a comparison between the RPMA system and a DS-CDMA system based on Gold codes.

2. PROBLEM FORMULATION

2.1. The permutation process

Consider a sequence of equiprobable bits $(b_n)_{n\in\mathbb{Z}}$ ($b_n \in \{-1;+1\}$). This sequence is modulated using an antipodal baseband code (for instance NRZ or biphase), with duration T and waveform pattern $m(t)$. The modulated process $Z(t)$ is then defined by $Z(t) \triangleq \sum_{n\in\mathbb{Z}} b_n m(t-nT)$. This process is sampled with a sampling period T_s, such that $N_s \triangleq T/T_s$ is an integer number. N_s is then the number of samples per bit. Let $(Z_n)_{n\in\mathbb{Z}}$ denote this sampled sequence. This sequence is transformed into a new sequence U_n as follows: considering blocks of N_b consecutive bits, the N_sN_b samples Z_n corresponding to a given block are permuted using an uniformly distributed permutation of the set $\{1,\ldots,N_sN_b\}$ (this permutation is the same for all blocks). The new sequence U_n is the resulting sequence, where samples are permuted block by block, from those of Z_n. The autocorrelation function and the power spectral density of the sequence $(U_n)_{n\in\mathbb{Z}}$ can be derived from those of the sequence $(Z_n)_{n\in\mathbb{Z}}$; in particular, the power spectral density of $(U_n)_{n\in\mathbb{Z}}$ is spreaded by a factor N_s with respect to the one of $(Z_n)_{n\in\mathbb{Z}}$ [5]. Consequently, this permutation procedure, which can be regarded as a particular case of periodic clock changes [4], is a spread-spectrum technique. We propose in this paper to use this technique as a multiple access system, and to study the associated multi-user detection problem.

Denote $\mathbf{b}_r = \left[b_{(r-1)N_b+1},\ldots,b_{rN_b}\right]^T$ as the rth block of bits, and $\mathbf{m} \triangleq [m_1,\ldots,m_{N_s}]^T$ as the result of the sampling of the waveform pattern $m(t)$. Then, the rth block of sequence $(Z_n)_{n\in\mathbb{Z}}$, i.e. the vector $\mathbf{Z}_r = \left[Z_{(r-1)N_sN_b+1},\ldots,Z_{rN_sN_b}\right]^T$ can be expressed as: $\mathbf{Z}_r = M^T\mathbf{b}_r$, where $M = \mathbf{m}^T \otimes \mathbf{I}_{N_b}$ is the $N_b \times (N_sN_b)$ matrix defined by

$$M = \begin{bmatrix} \mathbf{m}^T & 0 & \cdots & \\ 0 & \ddots & \ddots & \\ & \cdots & & \mathbf{m}^T \end{bmatrix}$$

($\otimes$ denotes the Kronecker product, and $\mathbf{I}_n$ is the identity matrix of order n). Now, if P denotes the $(N_sN_b)\times(N_sN_b)$ permutation matrix, the rth block of sequence $(U_n)_{n\in\mathbb{Z}}$, i.e. the vector $\mathbf{U}_r = \left[U_{(r-1)N_sN_b+1},\ldots,U_{rN_sN_b}\right]^T$ can be written as: $\mathbf{U}_r = P\mathbf{Z}_r = PM^T\mathbf{b}_r$. The sequence $(U_n)_{n\in\mathbb{Z}}$ is then transformed into a continuous-time process using a rectangular waveform signaling, yielding the process

$$x(t) \triangleq \sum_{r\in\mathbb{Z}} \sum_{j=1}^{N_sN_b} \left(PM^T\mathbf{b}_r\right)_j \rho\left(t - jT_s - rN_bT\right)$$

where $(v)_j$ denotes the jth component of any vector v, and $\rho(t)$ is the indicator function over $[0;T_s]$.

2.2. Modelling of the multi-user signal

Consider the asynchronous transmission of K users using the spread-spectrum technique given in the above section. Let $\mathbf{b}_{r,k}$, P_k and $x_k(t)$ denote respectively the rth block of bits, the permutation matrix, and the continuous-time process associated to user k. The case of a flat fading channel has been studied in [2]. Here, we consider transmissions over frequency-selective fading channels. More precisely, the radio channel of the k-th user is given by [1]:

$$c_k(t) = \sum_{l=1}^{L} c_{k,l}\delta\left(t - \tau_{k,l}\right)$$

where $c_{k,l}$ is the (complex) gain of the lth path of the kth user, $\tau_{k,l}$ is the propagation delay, and δ is the Dirac function. It is assumed in this paper that the coherence time of the channel is greater than the signal duration ; consequently, the coefficients $c_{k,l}$ can be considered as constant for all users' bit. The continuous-time received signal is a finite length signal formed by B blocks. It can then be expressed as:

$$\begin{aligned} y(t) = &\sum_{r=1}^{B}\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=1}^{N_sN_b} c_{k,l}\left(P_kM^T\mathbf{b}_{r,k}\right)_j \times \\ &\rho\left(t - jT_s - rN_bT - \tau_{k,l}\right) + n(t) \end{aligned}$$

where $n(t)$ is an additive white Gaussian noise (AWGN) with variance σ^2, independent of the transmitted signals.

2.3. The discrete asynchronous signal model

The first step of the detection consists of passing the continuous received signal $y(t)$ through a filter bank. These filters are matched to the signaling function $\rho(t)$ delayed by the delays $\tau_{k,l}$, i.e., there is a matched filter for all users and all delays. More precisely, the output of these matched filters at time u is given by:

$$\int y(t)\rho\left(t - u - \tau_{k,l}\right)dt\ ,\quad k = 1, \ldots, K;\ l = 1, \ldots, L$$

Define variables $y_{k,l,r}(j)$ as the outputs of the above matched filters at times $u = (j + rN_bN_s)\,T_s$, for $j = 1, \ldots, N_sN_b$ and $r = 1, ..., B$, i.e.

$$y_{k,l,r}(j) \triangleq \int y(t)\rho\left(t - jT_s - rN_bT - \tau_{k,l}\right)dt\ ,$$

for $k = 1, \ldots, K$, $l = 1, \ldots, L$, $j = 1, \ldots, N_sN_b$, and $r = 1, ..., B$. Define then vectors $\mathbf{y}_{k,l,r} \triangleq [y_{k,l,r}(1), y_{k,l,r}(2), \ldots, y_{k,l,r}(N_sN_b)]^T$, $\mathbf{y}_{l,r} \triangleq [\mathbf{y}_{1,l,r}^T, \mathbf{y}_{2,l,r}^T, \ldots, \mathbf{y}_{K,l,r}^T]^T$, $\mathbf{y}_r \triangleq [\mathbf{y}_{1,r}^T, \mathbf{y}_{2,r}^T, \ldots, \mathbf{y}_{L,r}^T]^T$, and $\mathbf{y} \triangleq [\mathbf{y}_1^T, \mathbf{y}_2, \ldots, \mathbf{y}_B^T]$. Define in the same manner $\mathbf{b}_{k,r}$, $\mathbf{b}_r$, and $\mathbf{b}$.Vectors $\mathbf{y}$ and $\mathbf{b}$ are thus the concatenation of all outputs of the matched filters, and of all bits, respectively. $\mathbf{y}$ is a sufficient statistic for $\mathbf{b}$. Indeed, given the fact that the data are independent and equiprobable, and that the noise is AWGN, the optimal detector consists of minimizing the error $\int(y(t) - \widetilde{y}(t))^2dt$ with respect to $\mathbf{b}$. Now, it is not difficult to show that this error only depends on the received signal $y(t)$ through the variables $y_{k,l,r}(j)$. Consequently, the proposed detector is based on the statistic $\mathbf{y}$, which is referred to as the data. Then, $\mathbf{y}$ can be expressed as:

$$\mathbf{y} = \mathbf{\Lambda\Pi C b} + \mathbf{n} \tag{1}$$

where $\mathbf{n}$ is a zero-mean Gaussian vector with covariance matrix $\sigma^2\mathbf{\Lambda}$ and matrices $\mathbf{\Lambda}$, $\mathbf{\Pi}$, and $\mathbf{C}$ are detailed in Appendix 6.1. The objective of the detector derived below is to mitigate simultaneously the multiple-access and inter-symbol interferences, along with the aditive noise.

[1] In fact, the number of paths is generally not equal for all users. However, one can consider L as the maximum of the path lengths, annulling gains $c_{k,l}$ if necessary.

Note that matrices $\mathbf{\Lambda}$, $\mathbf{\Pi}$, and $\mathbf{C}$ are huge, since they have dimensions $(N_sKN_bLB) \times (N_sKN_bLB)$, $(N_sKN_bLB) \times (KN_b LB)$, and $(KN_bLB) \times (KN_bB)$, respectively. However, the memory and computationnal costs can be drammatically reduced given the fact that they are sparse matrices. For instance, using parameters used for fig. 1 (see section 4), matrix $\mathbf{\Lambda}$ contains only 0.11% of non-zero elements. Also, matrices $\mathbf{\Pi}$ and $\mathbf{C}$ have KN_sN_bLB and KN_bLB non-zero elements, respectively.

3. THE LMMSE DETECTOR

3.1. The RPMA case

It is assumed in this section that the channel coefficients are known by the receiver. The LMMSE estimator consists of estimating bits $\widehat{b}_{k,r}(j)$ by a linear transformation of the received data $\mathbf{y}$, i.e.

$$\widehat{b}_{k,r}(j) \triangleq sign\left(\mathbf{h}_{k,r,j}{}^T\mathbf{y}\right)$$

where $\mathbf{h}_{k,r,j}$ is the vector which minimizes the mean-square error $E\left[\left(b_{k,r}(j) - \mathbf{h}^T\mathbf{y}\right)^2\right]$ with respect to $\mathbf{h} \in \mathbb{R}^{N_sKN_bLB}$. This minimization can be performed for all bits by considering the problem in the following matrix form:

$$\min_{\mathbf{H}} E\left[\|\mathbf{b} - \mathbf{Hy}\|^2\right]$$

(the matrix norm is defined by $\|A\| \triangleq \left(trace\left(AA^T\right)\right)^{1/2}$). This problem can be solved using the Bayesian Gauss-Markov theorem ([3], p. 391): given eq. (1), and the fact that i) $\mathbf{b}$ is a zero-mean vector with covariance matrix equal to $\mathbf{I}_{KN_bB}$, and ii) $\mathbf{n}$ is a zero-mean Gaussian vector with covariance matrix $\sigma^2\mathbf{\Lambda}$, the optimal matrix $\mathbf{H}$ is given by

$$\mathbf{H} = \left(\sigma^2\mathbf{I}_{KN_bB} + \mathbf{C}^H\mathbf{RC}\right)^{-1}\mathbf{C}^H\mathbf{\Pi}^T$$

where $\mathbf{R} = \mathbf{\Pi}^T\mathbf{\Lambda\Pi}$ is a symmetric matrix (since $\mathbf{\Lambda}$ is symmetric). The estimated bit vector $\widehat{\mathbf{b}}$ can then be written as $\widehat{\mathbf{b}} = sign\left(\mathbf{Hy}\right)$.

3.2. The DS-CDMA case

We propose to compare the LMMSE approach for RPMA and DS-CDMA systems. With DS-CDMA, the bits $b_k(j)$ of the k-th user are modulated by a signature waveform $s_k(t)$ which is assumed to be zero outside the interval $[0, T]$. The signal transmitted by user k is then

$$\widetilde{x}_k(t) \triangleq \sum_{j\in\mathbb{Z}} b_k(j)s_k(t - jT)\ ,$$

and the received signal, when N_b bits are transmitted per user, can be expressed as:

$$\widetilde{y}(t) = \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{j=1}^{N_b} c_{k,l}b_k(j)s_k(t - jT - \tau_{k,l}) + n(t)$$

In that case, the sufficient statistic vector $\widetilde{\mathbf{y}}$ is formed by the outputs of the filters matched to signatures $s_k(t - jT - \tau_{k,l})$, $k = 1, \ldots, K$, $l = 1, \ldots, L$, $j = 1, \ldots, N_b$. $\widetilde{\mathbf{y}}$ can then be written as:

$$\widetilde{\mathbf{y}} = \widetilde{\mathbf{\Lambda}}\widetilde{\mathbf{C}}\mathbf{b} + \widetilde{\mathbf{n}}$$

where $\widetilde{\mathbf{\Lambda}}$ is a matrix depending on the correlations between (shifted) signatures, $\widetilde{\mathbf{C}}$ is a matrix depending on the channel coefficients, and $\widetilde{\mathbf{n}}$ is a zero-mean Gaussian vector with covariance matrix $\sigma^2\widetilde{\mathbf{\Lambda}}$. Using the Bayesian Gauss-Markov theorem, the LMMSE decision is given by $\widehat{\mathbf{b}}=sign(\widetilde{\mathbf{H}}\widetilde{\mathbf{y}})$, where

$$\widetilde{\mathbf{H}}=\left(\sigma^2\mathbf{I}_{KN_b}+\widetilde{\mathbf{C}}^H\widetilde{\mathbf{\Lambda}}\widetilde{\mathbf{C}}\right)^{-1}\widetilde{\mathbf{C}}^H .$$

3.3. Performance

Define $\mathbf{Q}\triangleq\mathbf{C}^H\mathbf{R}\mathbf{C}$ with $\mathbf{R}=\mathbf{\Pi}^T\mathbf{\Lambda}\mathbf{\Pi}$ for the RPMA (resp. $\mathbf{Q}\triangleq\widetilde{\mathbf{C}}^H\mathbf{R}\widetilde{\mathbf{C}}$ with $\mathbf{R}=\mathbf{\Lambda}$ for the DS-CDMA). The decision for $\mathbf{b}$ can be expressed as: $\widehat{\mathbf{b}}=sign\left(\mathbf{W}\mathbf{y}+\mathbf{N}\right)$, where $\mathbf{W}\triangleq\left(\sigma^2\mathbf{I}+\mathbf{Q}\right)^{-1}$ $\mathbf{Q}$ and $\mathbf{N}$ is a zero-mean complex Gaussian vector with covariance matrix $\mathbf{\Sigma}\triangleq\sigma^2\left(\sigma^2\mathbf{I}+\mathbf{Q}\right)^{-1}\mathbf{Q}\left(\sigma^2\mathbf{I}+\mathbf{Q}\right)^{-1}$.

It can then be shown using Bayes' formula that the BER for the m-th bit of vector $\mathbf{b}$ is given by:

$$P_m=2^{1-|\mathbf{b}|}\sum_{\mathbf{d}\in\{-1;+1\}^{|\mathbf{b}|},\mathbf{d}_m=-1}Q\left(\frac{\mathbf{W}_{m,m}+\sum_{j\neq m}\mathbf{W}_{m,j}\mathbf{d}_j}{\mathbf{\Sigma}_{m,m}}\right)$$

where $Q(x)\triangleq\int_x^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-u^2/2}du$, and $|\mathbf{b}|$ is the length of vector $\mathbf{b}$ ($|\mathbf{b}|=KN_bB$ for RPMA, $|\mathbf{b}|=KN_b$ for DS-CDMA). Now, the computational cost of this formula grows exponentially with K and/or N_b and/or B: it is therefore useless in practice. However, using the Central Limit theorem, it is possible to use a Gaussian approximation, which yields to:

$$P_m\simeq Q\left(\left(|\mathbf{W}_{m,m}|^2/\left(\mathbf{\Sigma}_{m,m}+\sum_{j\neq m}|\mathbf{W}_{m,j}|^2\right)\right)^{1/2}\right)$$

Now, this BER has been obtained with fixed channel coefficients. It should then be averaged with respect to the distribution of the coefficient vector (e.g., a complex Gaussian distribution, with covariance matrix derived for instance from the so-called WSSUS assumption). However, this cannot be achieved analytically. The multiple integration can be done numerically for a very small number of coefficients, i.e. for a small number of users and a small number of paths (typically, $K=2$ and $L=2$). Otherwise, the integration can also be approximated by forming a Markov chain using the Metropolis algorithm, which allows to obtain, after a sufficient burn-in, an estimated value which converges to the actual one. However, the convergence can be very slow, when the number of coefficients is moderately high. In such case, one resorts to Monte-Carlo simulations to approximate the mean BER. Finally, this mean BER would be obtained for fixed users delays. Now, these offsets can also be considered as random variables, and the previous mean BER should also be averaged with respect to the distribution of these offsets. Here again, this can only be done by using Monte-Carlo simulations.

4. SIMULATION RESULTS

Many Monte-Carlo simulations have been performed to validate the theoretical results. In order to obtain the theoretical BERs as a function of the Signal-to-Noise Ratio (SNR) for any set of model parameters, it is necessary to have an expression of this SNR with respect to the parameters. This study is achieved in Appendix 6.2. Fig. 1 shows a comparison between theoretical and simulated (using 200 Monte-Carlo runs) BERs for the RPMA case. The parameters used for these simulations are: $K=4$ users, $N_b=4$ bits per block, $B=10$ blocks, the spreading factor is equal to $N_s=8$, and the number of paths is $L=4$. Delays and channel coefficients have been randomly chosen. This figure presents the BERs obtained for the 3^{rd} bit of the 4^{th} block for the 4 users. Clearly, simulations confirm the theoretical study. The differences between the results are due to the difference between channels of each user, which implies that the components of each user in the received signal have different powers. Now, the LMMSE detector mitigates in general the so-called near-far effect, for CDMA [8], as well as for RPMA [2]. However, in the case of frequency selective channels, this mitigation is much more difficult, whereas it could be seen that it is much more efficient than with a simple matched filter at the receiver. Moreover, it could also be seen (which is not shown on this figure, for clarity reasons), that, even for a same user, there are differences between results associated to different bits of a given block. Indeed, different correlations between bits of a given user are assigned by the random permutations, thus the bits do not have the same power.

The LMMSE detector for the RPMA system is next compared with the corresponding detector for DS-CDMA systems. The CD MA transmission has been performed using Gold codes, with 7 chips per signature, which is similar to the number of "chips" $N_s=8$ for the RPMA system. Here, a set of Gold codes is defined by a particular choice of 4 codes among 9 possible codes, along with a random delay from 0 to 6 chips. Given the fact that the chip duration is identical for both systems, the delays for CDMA case have been normalized with respect to those of RPMA, in order to conserve the same inter-symbol interference. Moreover, the performance of the detector depends on the choice of the set of permutations (or codes) of the users. Consequently, the comparison between RPMA and CDMA cannot be done using only one set of permutations and one set of codes, but must be achieved for a large number of sets. Thus, Fig. 2 and 3 show the theoretical results obtained for the RPMA and the DS-CDMA for 100 random permutation sets and 100 code sets, respectively. It can be concluded that both multiple access techniques perform similarly for this context of transmission.

5. CONCLUSION

This paper studied a spread-spectrum system based on random permutations (RPMA), in the case of a multi-user transmission over frequency selective fading channels. A linear MMSE detector has been proposed, which attempts to reduce simultaneously the multiple-access interference, the inter-symbol interference, and the additive noise. Theoretical performance has been derived, which is validated by Monte-Carlo simulations. A discussion on the choice of the permutations can be found in [1]: an optimal set of permutations (in the sense that it minimizes the error probability) can be obtained by using particular optimization algorithms, such as simulated-annealing algorithm or genetic algorithm. The LMMSE detector for RPMA was next compared with the LMMSE detector derived for DS-CDMA: theoretical curves prove that both methods yields quite similar results in terms of BER. The next step of this study will consist of considering the case of time-selective (along with frequency-selective) fading channels. In such a case, one can expect that the RPMA system perform better than the CDMA system: indeed, since the bits are spreaded in time with the RPMA, this system should be less sensitive with respect to fades, and there should then be less bursts of errors. For the same reason, the channel coefficient estimation should give better results in the RPMA case. This estimation is necessary in practice, where the assumption of known coefficients, which is required by the LMMSE detector, seems quite unrealistic. Another technique to overcome the problem of the knowledge of these coefficients would be to consider an adaptive version of the LMMSE detector. These issues are currently under investigation.

6. APPENDIX

6.1. Expression of vector $\mathbf{y}$

Denote $\tau_{k,l}^n \triangleq \tau_{k,l} + nN_bT$. $\tau_{k,l}^n$ can be expressed in an unique way as $\tau_{k,l}^n = \nu_{k,l}^n T_s + \varepsilon_{k,l}^n$, where $\nu_{k,l}^n$ is an integer, and $\varepsilon_{k,l}^n \in [0, T_s[$. Define now $\underline{\beta}_{n,n'}^{l,l'}(k,k') \triangleq \frac{1}{T_s}|\varepsilon_{k,l}^n - \varepsilon_{k',l'}^{n'}| 1_{\varepsilon_{k,l}^n > \varepsilon_{k',l'}^{n'}}$, $\overline{\beta}_{n,n'}^{l,l'}(k,k') \triangleq \frac{1}{T_s}|\varepsilon_{k,l}^n - \varepsilon_{k',l'}^{n'}| 1_{\varepsilon_{k',l'}^{n'} > \varepsilon_{k,l}^n}$ (where $1_{a>b} = 1$ if $a > b$ and $1_{a>b} = 0$ otherwise), and $\alpha_{n,n'}^{l,l'}(k,k') \triangleq 1 - \frac{1}{T_s}|\varepsilon_{k,l}^n - \varepsilon_{k',l'}^{n'}|$. Then, define the $(N_sN_b) \times (N_sN_b)$ matrix $\Lambda_{n,n'}^{l,l'}(k,k')$ such that its j-th row is $[0, \ldots, 0, \underline{\beta}_{n,n'}^{l,l'}(k,k'), \alpha_{n,n'}^{l,l'}(k,k'), \overline{\beta}_{n,n'}^{l,l'}(k,k'), 0, \ldots, 0]$, where $\alpha_{n,n'}^{l,l'}(k,k')$ occurs at the $(j + \nu_{k,l}^n - \nu_{k',l'}^{n'})$-th column. Let $\Lambda_{n,n'}^{l,l'}$ be the block matrix whose block (k,k') is $\Lambda_{n,n'}^{l,l'}(k,k')$, and $\mathbf{\Lambda}_{n,n'}$ be the block-matrix whose block (l,l') is $\Lambda_{n,n'}^{l,l'}$. Finally, denote $\mathbf{\Lambda}$ as the block-matrix whose block (n,n') is $\mathbf{\Lambda}_{n,n'}$. It can be shown that $\mathbf{\Lambda}$ is a symmetric matrix.

Define $\mathbf{\Pi} \triangleq \mathbf{I}_B \otimes (\mathbf{I}_L \otimes \Pi)$ where Π is a $(N_sKN_b) \times (KN_b)$ block-diagonal matrix, whose k-th diagonal block is the $(N_sL) \times L$ matrix P_kM^T; define also $\mathbf{C} \triangleq \mathbf{I}_B \otimes C$, where C is a $(KN_bL) \times (KN_b)$ block matrix such that $C \triangleq \left[C_1^T, C_2^T, \ldots, C_L^T\right]^T$, with $C_l \triangleq diag\left([c_{1,l}, c_{2,l}, \ldots, c_{K,l}]\right) \otimes \mathbf{I}_{N_b}$. Then, it can be shown that $\mathbf{y}$ can be written as:

$$\mathbf{y} = \mathbf{\Lambda\Pi C b} + \mathbf{n}$$

6.2. Autocorrelation and power of the received signal

Denote $\widetilde{y}(t) \triangleq y(t) - n(t)$ as the uncorrupted received signal, and $y_k(t) \triangleq c_k(t) * x_k(t)$. Assuming an antipodal modulation and equiprobable bits, the random processes $x_k(t)$ and $y_k(t)$ are zero-mean. Thus, since all users and all channels are statistically independent, the total power $P_{\widetilde{y}}$ is obtained by $P_{\widetilde{y}} = \sum_{k=1}^K P_{y_k}$ where P_{y_k} is the power of the process $y_k(t)$. For fixed (i.e. non-random) channel coefficients,

$$P_{y_k} = \sum_{l,l'=1}^{L} c_{k,l} c_{k,l'}^* K_{x_k}(\tau_{k,l} - \tau_{k,l'})$$

where $K_{x_k}(.)$ is the autocorrelation function of the process $x_k(t)$. It can be shown that

$$K_{x_k}(\tau) = \left(1 - \underline{\tau'}\right) K_{U_k}\left(\overline{\tau'}\right) + \underline{\tau'} K_{U_k}\left(\overline{\tau'} + 1\right)$$

where $\tau' = \frac{\tau}{T_s}$, $\overline{\tau'}$ is the integer part of τ', $\underline{\tau'} = \tau' - \overline{\tau'}$ is the decimal part, and $K_{U_k}(.)$ is the autocorrelation function of the sequence $(U_n)_{n\in\mathbb{Z}}$ associated to user k (this autocorrelation can be obtained from the expression given in [5]).

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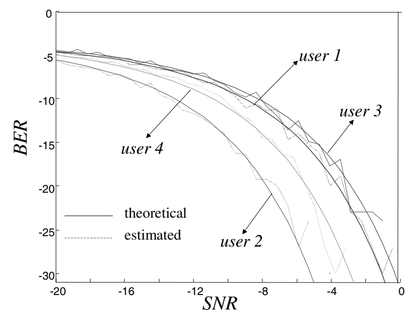

Fig. 1: Theoretical and simulated BERs for the RPMA system.

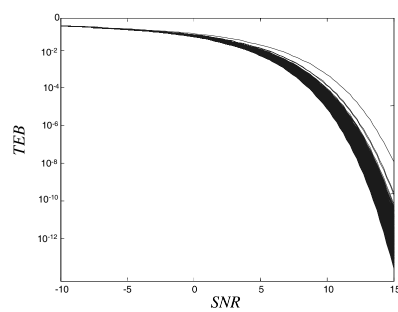

Fig. 2: BERs for 100 different random permutation sets.

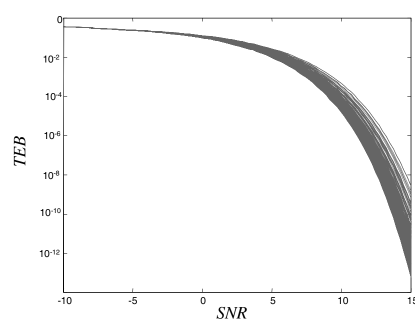

Fig. 3: BERs for 100 different Gold code sets.