## **EXPLOITING TEMPERATURE DEPENDENCY IN THE DETECTION OF NQR SIGNALS**

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# ABSTRACT

Nuclear Quadrupole Resonance (NQR) offers an unequivocal method of detecting and identifying land mines. Unfortunately, the practical use of NQR is restricted by the low signal to noise ratio (SNR), and means to improve the SNR are vital to enable a rapid, reliable and convenient system. In this paper, we develop a non-linear least squares detector exploiting the temperature dependency of the NQR frequencies as a way to enhance the SNR. Numerical simulations on both synthetic and real measured data indicate an excellent performance of the method.

## 1. INTRODUCTION

The use of land mines to impede the movement of enemy troops has a long history, dating back to 1277 when the Chinese used explosives to repel the invading Mongols. The problem has escalated since, and it is estimated that there are about 110 million active land mines in the world today, killing or injuring an average of 70 people every day. Removing them at the present rate will take more than 500 years and US\$33 billion. For obvious reasons, the detection of land mines has attracted a lot of attention in the literature, and a variety of different approaches have been suggested (see, e.g., [1]). The idea of using Nuclear Quadrupole Resonance (NQR) to detect explosives, and in particular land mines, goes back to the early 1950s, when the British Army suggested the possibility to researchers working with Nuclear Magnetic Resonance [2]. The topic has attracted significant interest since; one of the primary reason for this interest is that the NQR signal offers a unique signature, differentiating it from most other mine detection techniques that suffer from trying to detect non-unique features. Further, current methods of detecting land mines have serious disadvantages; metal detectors, for example, have difficulties in magnetics soils and with mines of low metal content, and ground penetrating radar in clay or wet and conducting soils, and with mines very close to the ground surface. Although mine detection using NQR faces problems

with interference and with the very low signal-to-noise ratio (SNR), recent reports indicate that the unique NQR signature offers exceptionally high probability of detection [2–5]. NQR is a radiofrequency (RF) technique in which the observed frequencies depend on the interaction between the electric quadrupole moment of the nucleus and the electric field gradient generated at the nuclear site by external charges. All common high explosives contain <sup>14</sup>N, a quadrupolar nucleus generating three sets of resonance frequencies, providing an unequivocal method of detecting and identifying an explosive, as well as estimating its quantity and depth. Because of its high specificity, there is little or no interference from other nitrogen-containing materials that may be present - such as the mine casing or fertilizer in the soil. As with a metal detector, a specifically designed planar RF antenna is placed close to ground level and fed with a sequence of RF pulses at or close to the NQR frequency of the explosive to be detected. The same antenna is then used to detect the weak signals emitted by the explosive following the excitation. These signals are of two types: free induction decays immediately following the pulse, and echoes observed midway between a string of pulses, the latter having the advantage that a large number of signals can be averaged in a short time to improve sensitivity [2,6-8]. The important difference from metal detection is that it is the explosive that is detected, not any feature of the mine, so the false alarm rate is low. However, the NQR signals can be very weak, particularly from the common explosive TNT. Worth noting is that NQR is also of particular interest due the possibility of using the technique to detect explosives at airports and other public places, as well as for detecting narcotics<sup>1</sup>. The few current publicly available approaches to detection of the NQR signal are mainly based on linear filtering via the Fast Fourier Transform or on matched filtering assuming a reliable estimate of the temperature of the target [3,9]. These methods are limited due to phase and intensity uncertainties in the NQR signal as well as the difficulty in accurately measuring the temperature under ground. Given an accurate temperature estimate, one may combine the dominant frequency responses to a single response with higher

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<sup>&</sup>lt;sup>1</sup>Many narcotics contain nitrogen, enabling the use of the same NQR techniques being developed for the detection of land mines.

signal-to-noise ratio (SNR) [10]. Unfortunately, an imprecise temperature estimate prevents the shifted peaks from adding up accurately. Herein, we present a natural extension to the work in [10], forming a nonlinear least squares (NLS) approach, exploiting the fact that the shifts of the spectral lines depend in a known way on temperature; by matching the measured data to the data model formed over a range of possible temperatures, the (unknown) temperature yielding the best match is found. The combined response for this temperature is then used as a detection variable. The proposed method is evaluated using both simulated data, and real NQR data obtained from measurements on a TNT sample. Both these evaluations indicates a strong gain for the proposed method as compared to current state of the art Fourier-based techniques.

#### 2. DATA MODEL

The NQR signal can be well modeled as a sum of d damped sinusoids [9–11]

$$y(t) = \sum_{k=1}^{d} \alpha_k e^{-\beta_k t + i\omega_k(\tau)t} + w(t),$$
 (1)

for  $t = 1, \dots, N$ , where  $\alpha_k$  and  $\beta_k$  denote the (complex) amplitude and the damping constant of the kth sinusoid, respectively. Typically, all the spectral lines will have approximately the same damping constants, say  $\beta_0$ , which may not vary significantly with temperature, but may vary between samples. We will initially not exploit this fact in an effort to allow for cases when the damping constants are measurably different; the resulting general algorithm then simplifies in a natural way if we let  $\beta_k \approx \beta_0$ . Further,  $\omega_k(\tau)$  is the frequency shifting function of the kth sinusoidal component due to the (unknown) temperature of the explosive sample, and w(t) is an additive *colored* noise. An important point to note is that the number of damped sinusoids, as well as the frequency shifting function for each spectral line  $\omega_k(\tau)$ , may be assumed to be *known*, whereas  $\alpha_k$ ,  $\beta_k$ , as well as the temperature of the explosive sample,  $\tau$ , are *unknown*. For NQR signals of many explosive samples, particularly TNT, the frequency shifting function at likely land mine temperatures can be well modeled as [8]

$$\omega_k(\tau) = 2\pi(a_k - b_k\tau) \tag{2}$$

where  $a_k$  and  $b_k$ , for k = 1, ..., d, are given constants. Often, the relative ratio between the modulus of the signal amplitudes,  $|\alpha_k|$ , are approximately known for a given explosive sample. To exploit this knowledge, we will let  $\alpha_k = \rho_k \delta_k$ , where  $\delta_k$  denotes the *a priori* known scaling.

## 3. NON-LINEAR LEAST SQUARES DETECTOR

For derivation purposes, we will herein model w(t) as a complex *white* Gaussian noise. This is a quite crude approximation, but we note that the NLS estimator will for sinusoidal estimation *asymptotically* achieve the same performance as the maximum likelihood estimator even in the colored noise case [12]. Rewrite (1) as

$$\mathbf{y}_N = \mathbf{D}_{\tau,\beta} \boldsymbol{\rho} + \mathbf{w}_N, \tag{3}$$

where

$$\mathbf{y}_N = \begin{bmatrix} y(1) & \cdots & y(N) \end{bmatrix}^T \tag{4}$$

$$\mathbf{D}_{\tau,\beta} = \mathbf{E}_{\tau} \odot \mathbf{F}_{\beta} \tag{5}$$

$$\mathbf{E}_{\tau} = \begin{bmatrix} \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots \end{bmatrix}$$
(6)

$$\mathbf{F}_{\boldsymbol{\beta}} = \begin{bmatrix} \delta_1 e^{-\beta_1} & \cdots & \delta_d e^{-\beta_d} \\ \vdots & \ddots & \vdots \end{bmatrix}$$
(7)

$$\left[\begin{array}{ccc}\delta_1 e^{-\beta_1 N} & \cdots & \delta_d e^{-\beta_d N}\end{array}\right]$$

$$\boldsymbol{\rho} = \begin{bmatrix} \rho_1 & \cdots & \rho_d \end{bmatrix}$$
(8)

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_1 & \cdots & \beta_d \end{bmatrix}^T \tag{9}$$

with  $(\cdot)^T$  and  $\odot$  denoting the transpose and the Hadamard (elementwise) product, respectively. Further,  $\mathbf{w}_N$  is defined similar to  $\mathbf{y}_N$ . Using (3), the NLS estimate can be obtained as (see, e.g., [13])

$$\left\{\hat{\boldsymbol{\beta}}, \hat{\boldsymbol{\rho}}, \hat{\boldsymbol{\tau}}\right\} = \arg\min_{\boldsymbol{\beta}, \boldsymbol{\rho}, \boldsymbol{\tau}} \| \mathbf{y}_N - \mathbf{D}_{\boldsymbol{\tau}, \boldsymbol{\beta}} \boldsymbol{\rho} \|_2^2, \qquad (10)$$

where  $\|\cdot\|_2$  denotes the 2-norm, yielding the least-squares estimate of  $\rho$  as

$$\hat{\boldsymbol{\rho}} = \left( \mathbf{D}_{\tau,\beta}^* \mathbf{D}_{\tau,\beta} \right)^{-1} \mathbf{D}_{\tau,\beta}^* \mathbf{y}_N, \tag{11}$$

where  $(\cdot)^*$  denotes the conjugate transpose, which inserted into (10) yields the maximization

$$\left\{\hat{\boldsymbol{\beta}}, \hat{\boldsymbol{\tau}}\right\} = \arg\max_{\boldsymbol{\tau}, \boldsymbol{\beta}} \| \boldsymbol{\Pi}_{\mathbf{D}_{\boldsymbol{\tau}, \boldsymbol{\beta}}} \mathbf{y}_N \|_2^2, \qquad (12)$$

where

$$\mathbf{\Pi}_{\mathbf{D}_{\tau,\beta}} = \mathbf{D}_{\tau,\beta} \left( \mathbf{D}_{\tau,\beta}^* \mathbf{D}_{\tau,\beta} \right)^{-1} \mathbf{D}_{\tau,\beta}^*.$$
(13)

Minimizing (12) for the general case of d unknown damping constants results in a (d+1)-dimensional search, over  $\tau$  and  $\beta$ , each requiring about  $\mathcal{O}(Nd^2)$  operations to compute. However, exploiting the fact that one often can approximate  $\beta_k \approx \beta_0$ , the maximization in (12) can be obtained by a 2-D search, over temperature and the common damping constant  $\beta_0$ ; initial estimates for both these parameters exist, and only a quite limited search region is required. The detection variable is thus selected as

$$\xi = \max_{\tau,\beta_0} \| \mathbf{\Pi}_{\mathbf{D}_{\tau,\beta_0}} \mathbf{y}_N \|_2^2, \tag{14}$$

where  $\Pi_{\mathbf{D}_{\tau,\beta_0}}$  is defined as in (13), but with  $\beta_k = \beta_0$ . We note that the matrix inversion in (13) may be poorly conditioned for specific  $\tau$  due to the resulting closely spaced frequency components. To alleviate this problem, we employ a low rank approximation technique, noting that a least-squares solution can be found using the singular value decomposition. Let

$$\mathbf{Q} \stackrel{\triangle}{=} \mathbf{D}_{\tau,\beta}^* \mathbf{D}_{\tau,\beta} = \mathbf{U} \boldsymbol{\Sigma} \mathbf{V}^*, \qquad (15)$$

where  $\Sigma$  is a diagonal matrix containing the *d* singular values of  $\mathbf{Q}$  on the diagonal, and where  $\mathbf{U}$  and  $\mathbf{V}$  are unitary matrices. Further, let  $\sigma_l$  denote the *l*th singular value of  $\mathbf{Q}$ , and note that the solution minimizing  $\|\mathbf{Q}\mathbf{x} - \mathbf{D}_{\tau,\beta}^*\mathbf{y}_N\|_2$  can be found as [14]

$$\hat{\mathbf{x}} = \sum_{l=1}^{\bar{d}} \sigma_l^{-1} \mathbf{U}_l^* \mathbf{D}_{\tau,\boldsymbol{\beta}}^* \mathbf{y}_N \mathbf{V}_l,$$
(16)

where  $\mathbf{U}_l$  and  $\mathbf{V}_l$  denote the *l*th column of  $\mathbf{U}$  and  $\mathbf{V}$ , respectively, and where  $\tilde{d}$  is the rank of  $\mathbf{Q}$ , or alternatively the selected low-rank approximation of  $\mathbf{Q}$ . Using (16), (14) can be expressed as

$$\xi = \max_{\tau, \beta_0} \mathbf{y}_N^* \mathbf{D}_{\tau, \beta}^* \hat{\mathbf{x}} .$$
(17)

If the temperature region of interest includes very closely spaced, or even overlapping, spectral lines, one should select  $\tilde{d} = d - 1$ , otherwise  $\tilde{d} = d$ . In the numerical simulations below, we have used the former.

## 4. NUMERICAL EXAMPLES

In this section, we examine the performance of the proposed detector using both simulated and real NQR data measured by the NQR group at King's College London. A typical NQR measurement allows for the estimation of a number of consecutive decaying echoes for every transmitted RF pulse, with each echo consisting of 256 data samples. The damping of the NQR signal depends on the explosive examined; typically RDX decays much more rapidly than TNT, and it is therefore easier to detect. Here, the examined real NQR data is obtained as the NQR response from a TNT sample at  $\tau = 298$  K; the simulated data has been generated to mimic such a signal, and is from (1), with a damping factor of  $\beta_0 = 0.0015$  and with (uniform) random initial phases, and an autoregressive noise model derived from



Figure 1: Performance gain as a function of the measured echo number for a real NQR signal.

real NQR data. Typically, current techniques only measure the response of a single *a priori known* resonance frequency [15]; to ensure the most beneficial performance for this approach, we will herein allow it to have perfect knowledge of the sample temperature, so that the most dominating resonance frequency is exactly known. We denote this the demodulation approach with perfect temperature knowledge (DMA-p). Typically, it is difficult to estimate the sample temperature with more than 5 degrees (K) accuracy; as a comparison, we therefore also include the estimate for a sample with 5 degrees offset, terming this the demodulation approach with realistic temperature knowledge (DMA-r). Figure 1 illustrates the performance gain of the detectors as the gain factor between the detection thresholds for a sample containing TNT and for one without TNT, as a function of the measured echo number. The methods have been evaluated on a single echo of real NQR data; the figure illustrates how the gain decreases for the higher order (more damped) echoes. A gain larger than one indicates a probable detection of the TNT sample. Here, exact temperature and damping knowledge has been assumed for both the DMA-p and the NLS methods; as the temperature of the reflecting sample will typically not be exactly known, the gain of the DMA-r is given as a reference. We again stress that the NLS method will not suffer from such temperature uncertainties, and the gain for the NLS method evaluated with a temperature uncertainty will yield the same gain as if the temperature was exactly known. Figure 2 shows the gain factor for simulated data, as a function of the signal to noise ratio (SNR), defined as  $SNR = \sigma_w^{-2} \sum_{k=1}^d |\alpha_k|^2$ , where  $\sigma_w$  denotes the standard deviation of w(t). Both these figures indicates a strong gain of the NLS method as compared to the DMA method, even when the latter is allowed to have perfect knowledge of the sample temperature. Figure 3 illustrates the receiver-operator curve, showing the probability of a correct detection, as a function of the probability of false alarm, for the 8th echo of a real NQR signal.



Figure 2: Performance gain as a function of the SNR for a simulated NQR signal.



Figure 3: The receiver-operator curve for real NQR data.

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