NON-MAPPING BACK SB-ESPRIT FOR COHERENT SIGNALS

Yanbo Xue, Jinkuan Wang

School of Information Science & Engineering
Northeastern University, Shenyang 110004, China
E-mail: {yxue, wjk}@mail.neuq.edu.cn

Yimin Zhang

Center for Advanced Communications
Villanova University, Villanova, PA 19085, USA
E-mail: yimin.zhang@villanova.edu

ABSTRACT

Subband-based eigenstructure method for bearing estimation is suggested in [1] and [2], while both methods require a mapping back manipulation after the subband frequency is estimated. In this paper, we propose a non-mapping back method to estimate the fullband bearings in the subband domain. The proposed method enhances the signal energy and reduces the root mean square error (RMSE). Simulations results show the decorrelation capability of coherent sources with appropriate subband division.

1. INTRODUCTION

Recently, most multiple source detection techniques in array processing are based on the eigenstructure decomposition of the covariance matrix. Among them, ESPRIT [3] has been widely used for Direction-of-Arrival (DOA) estimation, harmonic analysis, frequency estimation, delay estimation, and the combinations thereof. ESPRIT has advantages over another widely used multiple signal classification (MUSIC) [4] method because, compared to MUSIC, ESPRIT requires lower computational burden by exploiting the rotational invariance between two subarrays. However, ESPRIT experiences significant performance degradation when the signal-to-noise ratio (SNR) is low. The performance becomes even poorer when coherent signals imping.

On the other hand, subband-based method for array processing has been studied by many researchers. In particular, [1], [2] examined the subband partitioning of the spatial frequencies in array processing for enhanced DOA estimation. A subband-based ESPRIT (SB-ESPRIT) method is proposed in [2] which makes use of the fascinating features of subband signal processing, such as the lower minimum prediction error, closer entropy rate to the source, and improved whiteness of signals [5]. When coherent signals imping, SB-ESPRIT can decorrelate them by decomposing them into different subbands. The SB-ESPRIT, in essence, is a beamspace approach [6, 7].

However, SB-ESPRIT has two major problems related to its implementation and performance. First, the method requires a procedure to mapping the subband spatial frequency back to the fullband. This manipulation is necessary because of the widening of the spatial frequency spacing [8]. Second, the reduction of computational load in the singular value decomposition (SVD) is achieved at the expense of compromising the output SNR.

In this paper, we propose a modified SB-ESPRIT algorithm for DOA estimation. The proposed method is based on the pre-expansion of the sensor array such that the mapping back manipulation after the estimation of bearings is no longer required. The modified SB-ESPRIT first expands the array, and then uses SB-ESPRIT to decompose the fullband signal into several subbands. The combination of ESPRIT estimations at each subbands yields the fullband bearings, which is guaranteed by the pre-expansion of the array. The rotational invariance of our proposed method is proven.

2. DATA MODEL

Consider a uniform linear array (ULA) with K isotropic sensors spaced by a distance d. D ($D < K$) narrowband plane waves imping from the far-field with the same center frequency ω_0. The bearings of the D signals are denoted as $\theta_1, \theta_2, \cdots, \theta_D$. The received data sampled at the k-th sensor can be expressed as

$$x_k(n) = \sum_{i=1}^{D} s_i(n) \mathrm{e}^{-j\omega_0 (k-1) \sin\theta_i d/c} + w_k(n). \qquad (1)$$

In matrix form, we have

$$\mathbf{x}(n) = \mathbf{A}\mathbf{s}(n) + \mathbf{w}(n), \qquad (2)$$

where $\mathbf{x}(n)$, $\mathbf{s}(n)$, and $\mathbf{w}(n)$ denote respectively the $K \times 1$ received data vector, the $D \times 1$ wavefront vector, and the $K \times 1$ additive noise vector. Define $\omega_i = \omega_0 \sin\theta_i d/c$ as the equivalent spatial frequency of the i-th wavefront. Then,

This work is supported by Key Program of Science and Technology from the Ministry of Education of China, under Grant no. 02085.

the mixing matrix $\mathbf{A} \in \mathcal{C}^{K\times D}$ can be expressed as

$$\mathbf{A}(\omega) = [\mathbf{a}(\omega_1), \mathbf{a}(\omega_2), \cdots, \mathbf{a}(\omega_D)] \tag{3}$$

where $\mathbf{a}(\omega_i) = [1, \mathrm{e}^{-j\omega_i}, \cdots, \mathrm{e}^{-j(K-1)\omega_i}]^{\mathrm{T}}$ denotes the steering vector corresponding to the spatial frequency ω_i, and superscript T denotes transpose. (1) can be rewritten as

$$x_k(n) = \sum_{i=1}^{D} s_i(n)\mathrm{e}^{-j(k-1)\omega_i} + w_k(n), \tag{4}$$

where $s_i(n) = |s_i(n)|\mathrm{e}^{-j\phi_i}$ is the complex source waveform, where the phase ϕ_i is uniformly distributed over $[0, 2\pi)$. Assume that the signals are zero mean wide sense stationary (WSS) processes, and $w_k(n)$ is the zero mean white Gauss noise (WGN) which is uncorrelated to the signals and has identical variance σ^2 in each sensor. Then the output covariance matrix is given by

$$R_{xx} = \mathbf{A}R_{ss}\mathbf{A}^{\mathrm{H}} + \sigma^2 I_K, \tag{5}$$

where R_{ss} denotes the source covariance matrix and I_K is a $K \times K$ identity matrix, and superscript H denotes conjugate transpose.

By selecting $d \leq \lambda/2$ in the array, the aliasing of the spatial spectrum can be prevented.

3. SUBBAND-BASED ESPRIT

The idea of SB-ESPRIT is to first partition the measured data into several subbands and then apply the ESPRIT algorithm to each subband. For the convenience of analysis, we define two matrices $\mathbf{H}$ and $\mathbf{G}$, both of dimension $N_f \times K$, to filter the measured data, described in (2), into a low frequency subband and a high frequency subband, where $N_f = \lfloor (K+N_d)/2 \rfloor - 1$ with N_d denoting the length of filter and $\lfloor y \rfloor$ denoting the largest integer not exceeding y. The output of the two filters are formulated by

$$\mathbf{x}_h(n) = \mathbf{H}\mathbf{A}\mathbf{s}(n) + \mathbf{w}_h(n) \tag{6}$$

and

$$\mathbf{x}_g(n) = \mathbf{G}\mathbf{A}\mathbf{s}(n) + \mathbf{w}_g(n). \tag{7}$$

where $\mathbf{x}_h(n)$ and $\mathbf{x}_g(n)$, both of $N_f \times 1$, are referred to as the low frequency and high frequency data vectors. Each of them can be used to compose two subarrays. Take $\mathbf{x}_h(n)$ for example. We choose its 1st to $(N_f - 1)$th rows to form the vector $\mathbf{x}_h^1(n)$ corresponding to the first subarray, whereas its 2nd to N_fth rows are used to form vector $\mathbf{x}_h^2(n)$ corresponding to the second subarray.

For simplicity, we choose Haar wavelets as the analysis filters and assume K is even so that $K/2$ is an integer. The 1-level lowpass and highpass filtering matrices $\mathbf{H}$ and $\mathbf{G}$ are given respectively by

$$\mathbf{H} = c \cdot \begin{bmatrix} \mathcal{J}_2 & O_2 & \cdots & O_2 \\ O_2 & \mathcal{J}_2 & \cdots & O_2 \\ \vdots & \vdots & \ddots & \vdots \\ O_2 & O_2 & \cdots & \mathcal{J}_2 \end{bmatrix} \in \mathcal{R}^{K/2\times K} \tag{8}$$

and

$$\mathbf{G} = c \cdot \begin{bmatrix} \mathcal{K}_2 & O_2 & \cdots & O_2 \\ O_2 & \mathcal{K}_2 & \cdots & O_2 \\ \vdots & \vdots & \ddots & \vdots \\ O_2 & O_2 & \cdots & \mathcal{K}_2 \end{bmatrix} \in \mathcal{R}^{K/2\times K}, \tag{9}$$

where $\mathcal{J}_2 = [1, 1]$, $\mathcal{K}_2 = [1, -1]$, $O_2 = [0, 0]$, and $c = 1/\sqrt{2}$.

Next we exploit the rotational invariance between subarrays. Denote

$$\tilde{\mathbf{A}} = \mathbf{H}\mathbf{A} = \begin{bmatrix} \tilde{\mathbf{a}}_1 & \tilde{\mathbf{a}}_2 & \cdots & \tilde{\mathbf{a}}_D \end{bmatrix} \tag{10}$$

as the $K/2 \times D$ mixing matrix at the high frequency subband, where $\tilde{\mathbf{a}}_i = \left(1 + \mathrm{e}^{-j\omega_i}\right)\left[1, \mathrm{e}^{-j2\omega_i}, \cdots, \mathrm{e}^{-j(K-2)\omega_i}\right]^{\mathrm{T}}$. Then, (6) can be rewritten as

$$\mathbf{x}_h(n) = \tilde{\mathbf{A}}\mathbf{s}(n) + \mathbf{w}_h(n). \tag{11}$$

If the first subarray consists of the 1st to the $(K/2 - 1)$th sensors and the second subarray consists of the 2nd to the $K/2$th sensors, the mixing matrices $\tilde{\mathbf{A}}_1$ and $\tilde{\mathbf{A}}_2$ of two subarrays can then be related by a diagonal matrix Φ, i.e., $\tilde{\mathbf{A}}_2 = \tilde{\mathbf{A}}_1\Phi$, where Φ is the rotational invariance in the subband signals, given by

$$\Phi = \mathrm{diag}\{\mathrm{e}^{-j2\omega_1}, \mathrm{e}^{-j2\omega_2}, \cdots, \mathrm{e}^{-j2\omega_D}\}. \tag{12}$$

The rotational invariance property between two subarrays is evident because they undergo the same lowpass filtering. Same can be said for the subarrays corresponding to the high frequency subband. By exploiting the diagonal elements of Φ using conventional ESPRIT, we can obtain the spatial frequency $\tilde{\omega}_{l,m}$ in the subbands without knowing the mixing matrix $\tilde{\mathbf{A}}_1$, where $\tilde{\omega}_{l,m}$ means the subband frequency obtained at the lth-level and mth-node. So the validity of SB-ESPRIT is shown with a 1-level Haar wavelet decomposition. The extension to an *any-level any-type* wavelet decomposition is straightforward. Note that the subband frequency is amplified in Φ, which accords with the superiority of frequency widening.

To obtain the fullband frequencies, we need to map the frequencies from subbands back to the fullband. We map the frequencies as follows

$$\omega_{fb} = \begin{cases} \dfrac{\tilde{\omega}_{l,m} + (m-1)\pi\mathrm{sgn}(\tilde{\omega}_{l,m})}{2^l}, & m = 1, 3, 5, \cdots \\ \dfrac{\tilde{\omega}_{l,m} - m\pi\mathrm{sgn}(\tilde{\omega}_{l,m})}{2^l}, & m = 2, 4, 6, \cdots \end{cases} \tag{13}$$

where $\text{sgn}(\tilde{\omega}_{l,m})$ denotes the sign of $\tilde{\omega}_{l,m}$. Then it is easy to obtain the DOAs from

$$\theta_i = \arcsin\left\{\frac{\omega_{i,fb} \cdot c}{\omega_0 \cdot d}\right\} = \arcsin\left\{\omega_{i,fb} \cdot \frac{\lambda}{d}\right\}. \quad (14)$$

4. PROPOSED NON-MAPPING BACK APPROACH

Because the spatial frequency spacing is widened in each-level decomposition [8], we need to map the frequency in the subband back to the fullband by using (13). Here, we propose a pre-expansion method to circumvent this procedure. Take the Haar wavelet as an example, the distance between sensors is doubled after each level of decomposition, which is the key principle for multiresolution estimation of the signal [9]. It is the spatial widening property that requires the mapping back manipulation (13). The idea of pre-expansion method is to modify the received matrix before it is input to the wavelet filters. The pre-expansion matrix $\mathbf{P}$ is defined as following,

$$\mathbf{P} = \begin{bmatrix} 1 & & \\ & \tilde{\mathbf{P}} & \\ & & 1 \end{bmatrix} \in \mathcal{R}^{K_p \times K} \quad (15)$$

where $K_p = 2(K-1)$ is the number of virtual expanded array and $\tilde{\mathbf{P}}$ is given by

$$\tilde{\mathbf{P}} = \begin{bmatrix} \mathcal{J}_2 & O_2 & \cdots & O_2 \\ O_2 & \mathcal{J}_2 & \cdots & O_2 \\ \vdots & \vdots & \ddots & \vdots \\ O_2 & O_2 & \cdots & \mathcal{J}_2 \end{bmatrix}^{\mathrm{T}} \in \mathcal{R}^{(2K-4)\times(K-2)}.$$

Expanding (2) with (15) yields

$$\mathbf{x}_p(n) = \mathbf{P}\mathbf{A}\mathbf{s}(n) + \mathbf{w}_p(n). \quad (16)$$

It is easy to verify that $\mathbf{w}_p(n)$ is also WGN with variance σ^2. SB-ESPRIT can also be applied to the expansion matrix, while we should prove the rotational variance after expanding and filtering the matrix.

Suppose we filter $\mathbf{x}_p(n)$ with a lowpass Harr filter $\mathbf{H} \in \mathcal{R}^{(K-1)\times 2(K-1)}$ as in (6). We obtain

$$\mathbf{x}_{ph}(n) = \check{\mathbf{A}}\mathbf{s}(n) + \mathbf{w}_{ph}(n) \quad (17)$$

where $\check{\mathbf{A}} = \mathbf{H}\mathbf{P}\mathbf{A}$ denotes the $(K-1)\times D$ subband matrix

$$\check{\mathbf{A}} = \begin{bmatrix} \check{\mathbf{a}}_1 & \check{\mathbf{a}}_2 & \cdots & \check{\mathbf{a}}_D \end{bmatrix} \quad (18)$$

with $\check{\mathbf{a}}_i = [1 + \mathrm{e}^{-j\omega_i}, \mathrm{e}^{-j\omega_i} + \mathrm{e}^{-j2\omega_i}, \cdots, \mathrm{e}^{-j(K-2)\omega_i} + \mathrm{e}^{-j(K-1)\omega_i}]^{\mathrm{T}}$.

Let the first subarray consist of the 1st to $(K-2)$th sensors and the second subarray consist of those from 2nd to $(K-1)$th. The rotational invariance still holds between the subband mixing matrices $\check{\mathbf{A}}_1$ and $\check{\mathbf{A}}_2$, given by

$$\check{\Phi} = \text{diag}\{\mathrm{e}^{-j\omega_1}, \mathrm{e}^{-j\omega_2}, \cdots, \mathrm{e}^{-j\omega_D}\} \quad (19)$$

which is the same to the conventional ESPRIT and does not require mapping back manipulation after the subband frequency is estimated. Our proposed non-mapping back method can also be easily implemented by adding a step called *pre-expansion* before applying SB-ESPRIT algorithm.

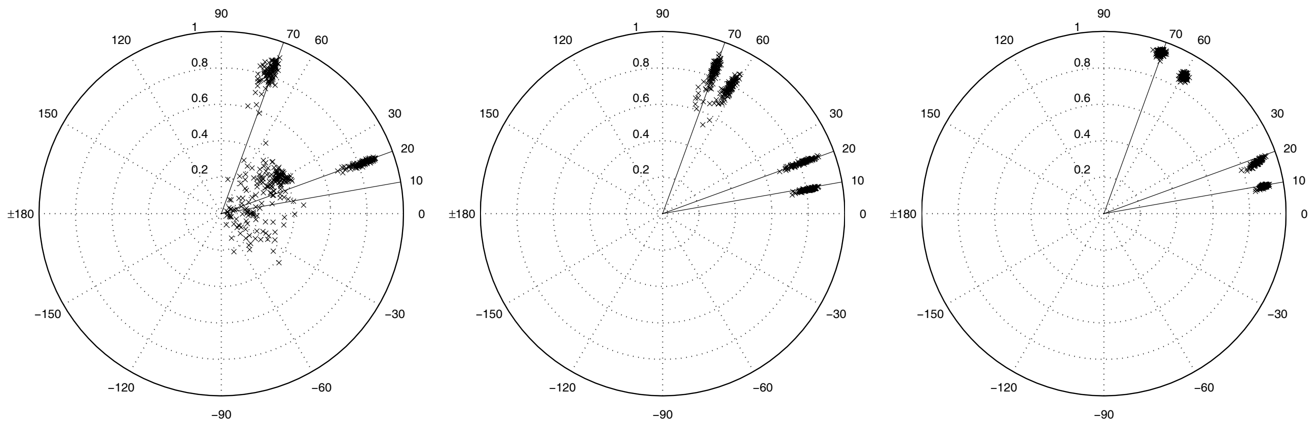
The applicability of SB-ESPRIT in coherent signals scenario is not to decorrelate the coherent signals by averaging the subarray autocorrelation matrix, but to decompose them into different subbands. Given ideal bandpass filters and appropriate division schedule, decorrelation is realized.

5. SIMULATIONS

In this section, we provide computer simulations to evaluate and compare the performance of conventional ESPRIT, subband-based ESPRIT (SB-ESPRIT), and modified SB-ESPRIT. Both simulations are carried out for a ULA of $K = 32$ isotropic sensors with half wavelength $(d = \lambda/2)$ interelement spacing. Four sources emit narrowband signals with the same power from $10°$, $20°$, $60°$ and $70°$. Among them, the two signals from $10°$ and $60°$ are coherent. $N = 100$ snapshots are taken and the Haar wavelet packets are chosen as the subband decomposition filters. We use the Monte-Carlo method to obtain 100 independent runs for each example. Fig. 1 shows the performance at a low SNR $= -13$ dB. It is evident from Fig. 1(a) that the conventional ESPRIT fails to resolve the coherent signals from $10°$ and $60°$, whereas Fig. 1(b) and Fig. 1(c) show that both SB-ESPRIT and our proposed method resolve the four sources. It is also noted that the output power of modified SB-ESPRIT is obvious stronger than that of SB-ESPRIT. Fig. 2 shows the resulting root mean square error (RMSE) of the estimated DOAs as a function of SNR. SB-ESPRIT and modified SB-ESPRIT algorithm outperform ESPRIT especially in low SNR for their ability of dividing coherent signals into different subbands. The RMSE of the subband-based methods, which include the SB-ESPRIT and our proposed method, is significantly lower than that of conventional ESPRIT, especially when the SNR is high. In this case, the performance of modified SB-ESPRIT and that of SB-ESPRIT become very close.

6. CONCLUDING REMARKS

In this paper, a modified subband-based ESPRIT (SB-ESPRIT) algorithm has been proposed which does not require the mapping back manipulation after the bearings are estimated. The modified SB-ESPRIT estimates the spatial frequencies by pre-expanding the array and then applying



(a) ESPRIT (b) SB-ESPRIT (c) Modified SB-ESPRIT

Fig. 1. DOA Estimation for two coherent signals from $10°$ and $60°$, and two incoherent signals from $20°$ and $70°$ with SNR $= -10$ dB and 100 trial runs.

SB-ESPRIT to the expanded data matrices. As the rotational invariance still holds, an ESPRIT estimation of each subband yields the fullband bearings. Simulation results have shown that the modified SB-ESPRIT outperforms both conventional ESPRIT and SB-ESPRIT, especially in low SNR scenarios. The root mean square error of the proposed method is much lower than that of ESPRIT and is close to SB-ESPRIT. The output energy of modified SB-ESPRIT is also enhanced.

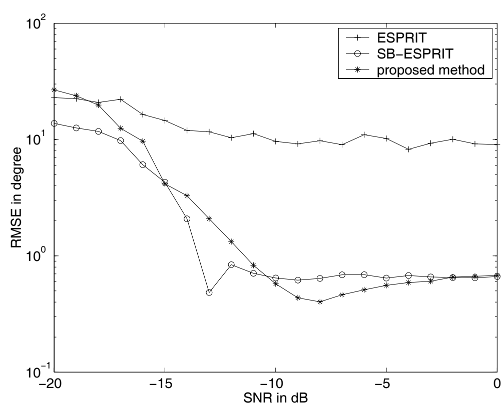


Fig. 2. RMSE of the estimated DOAs as a function of SNR for $\theta = 10°, 20°, 60°$, and $70°$.

7. REFERENCES

[1] Y. B. Xue, J. K. Wang, and Z. G. Liu, "Wavelet packets-based direction-of-arrival estimation," *Proc. IEEE ICASSP*, pp. 505–508, Montreal, Canada, May 2004.

[2] Y. B. Xue and J. K. Wang, "A novel subband-based ESPRIT algorithm", *Proc. Int. Symp. Ant. Propag.*, vol. 2, pp. 933–936, Sendai, Japan, Aug. 2004.

[3] R. H. Roy, "ESPRIT-Estimation of signal parameters via rotational invariance technique," *Ph.D thesis*, Stanford University, 1987.

[4] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propag.*, vol. AP-34, no. 3, pp. 276–280, March 1986.

[5] S. Rao and W. A. Pearlman, "Analysis of linear prediction, coding, and spectral estimation from subbands," *IEEE Trans. Infor. Theor.*, vol. IT-42, no. 4, pp. 1160–1178, April 1996.

[6] K. M. Buckley and X. L. Xu, "Spatial-spectrum estimation in a location sector, *IEEE Trans. Acoust. Speech Signal Proc.*, vol. 38, no. 11, pp. 1842–1852, Nov. 1990.

[7] G. Xu, S. D. Silverstein, R. H. Roy and T. Kailath, "Beamspace ESPRIT," *IEEE Trans. Signal Proces.,* vol. 42, no. 2, pp. 349–356, Feb. 1994.

[8] A. Tknacenko and P. P. Vaidyanathan, "The role of filter banks in sinusoidal frequency estimation," *J. Franklin Inst.*, vol. 338, no. 5, pp. 517–547, Aug. 2001.

[9] W. Xu and W. K. Stewart, "Multiresolution-signal direction-of-arrival estimation: a wavelets approach," *IEEE Signal Proces. Letters,* vol. 7, no. 3, pp. 66–68, March 2000.