MODULATED HYBRID FILTER BANKS FOR DATA CONVERSION

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ABSTRACT

A hybrid filter bank requiring only lowpass analog filters is described. This architecture is suitable for A/D and D/A conversion of wideband signals. The structure uses identical pairs of lowpass filter and mixer in each to achieve the perfect reconstruction of the continuous input signal.

I. INTRODUCTION

Emergence of new standards and products is more than ever calling for data converters with higher conversion rates. In telecommunications, the advent of ultra-wideband (UBW) is expected to further strengthen the demand for high-speed low-power data converters. The most popular method for increasing the speed of the state-of-the art Nyquist-rate data converters is time-interleaving. However, time-interleaving is sensitive to gain mismatch between channels, offset and clock skew [1]. Parallel architectures based on discrete-time filter banks were shown to be less sensitive to these effects [2]. However, the discrete-time analog filters in such a structure need to be implemented using switched-capacitor circuits which are very noisy and power hungry. To overcome these shortcomings, hybrid filter banks (HFB) composed of continuous-time analog analysis filters and digital synthesis filters were considered [3], [4]. The main issue in HFB's is the inaccuracy of analog filters which requires tuning or calibration [5]. Such adjustments become even more complex when one notes that the analog filters are not identical. Here, we present hybrid filter banks in which mixer/lowpass filter pairs replace the bandpass filters. In contrast with digital modulated filter banks where the transfer function of bandpass filters is shifted along the frequency axis [6], this structure, called modulated HFB (MHFB), shifts the signal spectrum by means of mixers. Two different MHFB's will be considered: complex MHFB where modulating signals are complex exponentials and cosine MHFB where the modulating signals are sinusoids. In both cases, the simplicity of the system makes the design and adjustment easier and cheaper. The proposed architecture is particularly attractive where speed is the primary requirement while a low or moderate accuracy is adequate. UWB is an example of such applications.

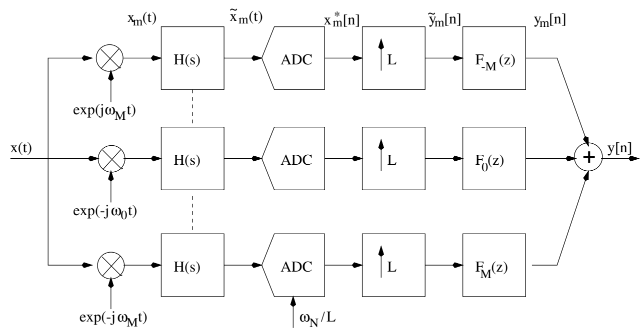

Fig. 1. Complex exponential MHFB for A/D conversion.

II. COMPLEX EXPONENTIAL MODULATED HYBRID FILTER BANK

The complex exponential modulated HFB (MHFB) is shown in Fig. 1. The input signal propagates through $L = 2M + 1$ channels, each one including an analog mixer, a lowpass analysis filter $H(s)$, an ADC, a digital expander and a digital synthesis filter. The mixers shift the spectrum of the input signal by ω_m, $m = -M, \cdots, 0, \cdots, M$ along the frequency axis by mixing the input signal with a complex exponential. The modulating frequencies ω_m are uniformly distributed over the signal bandwidth, i.e., $\omega_m = 2\pi m/L$, $m = -M, \cdots, 0, \cdots, M$. The frequency-translated signals are subsequently lowpass filtered and converted to digital signals. The sampling period is normalized to $T = 1$, and the input signal $x(t)$ is supposed to be bandlimited to $\omega \in (-\pi, \pi)$. This implies a normalized the Nyquist rate of $\omega_N = 2\pi$. Aliasing occurs at the output of the ADC's because they are operated at $1/LT = 1/L$ which is smaller than the Nyquist rate. The digital filters are selected such that any aliasing and distortion caused by undersampling the signal will be minimized. Consequently, the effective conversion rate of the system will correspond to the Nyquist rate.

Mixing the input signal in channel m with $\exp(-j\omega_m t)$ cause its Fourier transform, $X(\omega)$, to be shifted by ω_m in only one direction along the frequency axis, i.e., $X_m(\omega) = X(\omega + \omega_m)$. Subsequently, each ADC captures the portion $\widetilde{X}_m(\omega) = X_m(\omega) H(\omega)$ of the input signal which falls within the bandwidth of $H(s)$.

The periodic spectrum of the sampled signal $x_m^*(t)$ is given by [6]

$$X_m^*\left(e^{j\omega}\right) = \frac{1}{L}\sum_{n=-\infty}^{+\infty} X\left(\frac{\omega}{L} - \frac{2\pi n}{L} + \omega_m\right) H\left(\frac{\omega}{L} - \frac{2\pi n}{L}\right)$$

We choose the frequency shifts ω_m to be uniformly distributed over the signal bandwidth, i.e., $\omega_m = 2\pi m/L,\ m = -M, \cdots, M$. Then, making a change of variables $m-n \rightarrow n$, we get

$$X_m^*\left(e^{j\omega}\right) = \frac{1}{L}\sum_{n=-\infty}^{+\infty} X\left(\frac{\omega}{L} + \frac{2\pi n}{L}\right) H\left(\frac{\omega - 2\pi m + 2\pi n}{L}\right)$$

The above manipulation is essential in the concept of MHFB because its signifies that frequency translation of $X(\omega)$ by ω_m is equivalent to frequency translation of $H(\omega)$ by the same amount. The output of each expander is related to its input by $\widetilde{Y}_m\left(e^{j\omega}\right) = X_m^*\left(e^{j\omega L}\right)$. Therefore,

$$\widetilde{Y}_m\left(e^{j\omega}\right) = \frac{1}{L}\sum_{n=-\infty}^{+\infty} X\left(\omega + \frac{2\pi n}{L}\right) H\left(\omega - \frac{2\pi m}{L} + \frac{2\pi n}{L}\right)$$

Now, we can define

$$H_m(\omega) = H\left(\omega - \frac{2\pi m}{L}\right)$$

The output signal is obtained by summing the output of all channels

$$\begin{aligned} Y\left(e^{j\omega}\right) = \frac{1}{L}\sum_{m=-M}^{M}\sum_{n=-\infty}^{+\infty} X\left(\omega + \frac{2\pi n}{L}\right) \\ \times H_m\left(\omega + \frac{2\pi n}{L}\right) F_m\left(e^{j\omega}\right) \end{aligned}$$

Since the signal is bandlimited to $\omega \in (-\pi, \pi)$, summation on n can be limited to $n = -(M-1), \cdots, M-1$. Now, we decompose the output signal as $Y\left(e^{j\omega}\right) = X(\omega) D(\omega) + A(\omega)$ where $D(\omega)$ represents distortion and $A(\omega)$ aliasing. To achieve perfect reconstruction of the input signal, the distortion term is allowed to be a delay d and the aliasing must be canceled. These conditions lead to a set of L equations having L unknown functions $F_m\left(e^{j\omega}\right)$. Note that this set of equations has to be solved over the signal bandwidth, i.e., $\omega \in (-\pi, \pi)$. Hence, we can simplify the procedure by dividing the signal bandwidth into L sub-intervals $I_k : \omega \in ((k-1/2)\, 2\pi/L, (k+1/2)\, 2\pi/L)$, $k = -M, \cdots, M$ and solve the set of equations for each interval. On interval I_0, we need to cancel the aliasing terms $n = -M, \cdots, -1, 1, \cdots, M$ and make the term $n = 0$ equal to a delay.

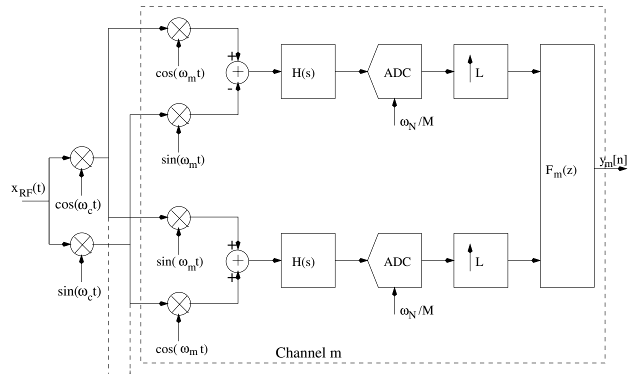

Fig. 2. MHFB for radio heterodyne receivers.

Similarly, on an arbitrary interval I_k we have to cancel the aliasing terms $n = -(M+k), \cdots, -1, 1, \cdots, M-k$ and make the term $n = 0$ equal to a delay. This procedure leads to the following matrix equation for the sub-interval I_k

$$\mathbf{H}\left(\omega - \frac{2\pi k}{L}\right)\mathbf{F}(\omega) = \begin{bmatrix} 0 \\ \vdots \\ Le^{-jd\omega} \\ \vdots \\ 0 \end{bmatrix} \begin{array}{l} \\ \\ \leftarrow (M+k+1)th \text{ row} \\ \\ \\ \end{array} \tag{1}$$

where

$$\mathbf{H}(\omega) = \begin{bmatrix} H(\omega) & \cdots & H\left(\omega - \frac{4\pi M}{L}\right) \\ \vdots & \cdots & \vdots \\ H\left(\omega + \frac{4\pi M}{L}\right) & \cdots & H(\omega) \end{bmatrix}$$

$$\mathbf{F}(\omega) = \begin{bmatrix} F_{-M}\left(e^{j\omega}\right) \\ \vdots \\ F_M\left(e^{j\omega}\right) \end{bmatrix}$$

Frequency translation due to multiplication by a complex exponential occurs in homodyne receivers and other communication systems. In a radio architecture, the RF signal is usually downconverted using a pair of mixers and in-quadrature sinusoids, e.g., $\cos\omega_c t$ and $\sin\omega_c t$. This is equivalent to multiplying the RF signal by $\exp(j\omega_c t)$. If the MHFB is used in a receiver, downconversion and frequency translation can be combined, as shown in Fig. 2. Note that the first group of mixers represent a classical quadrature downconversion and they can be shared between the channels.

III. COSINE MODULATED HYBRID FILTER BANK

The MHFB described in the previous section required complex signal processing. Now, we present a two-channel cosine MHFB, as shown in Fig. 3. In this system, the input signal $x(t)$ is multiplied in channel m by $\cos \omega_m t$. This operation shifts the spectrum of the input signal, $X(\omega)$, by $\pm\omega_m$, i.e., $X_m(\omega) = [X(\omega+\omega_m) + X(\omega-\omega_m)]/2$, where $\omega_m = 2\pi m/2,\ m = 0, 1$. Following a procedure similar to the above, we obtain the output as

$$Y\left(e^{j\omega}\right) = \frac{1}{2} \sum_{n=-\infty}^{+\infty} X(\omega+\pi n) \left[H_0(\omega+\pi n) F_0\left(e^{j\omega}\right) + H_1(\omega+\pi n) F_1\left(e^{j\omega}\right)\right]$$

where

$$H_m(\omega) = \frac{1}{2}\left[H(\omega-\pi m) + H(\omega+\pi m)\right]$$

Note that in contrast with the complex MHFB, the transfer function of the lowpass filter is now shifted in two opposite directions. Once again we can separate the aliasing and distortion terms as $Y\left(e^{j\omega}\right) = X(\omega) D(\omega) + A(\omega)$ and divide the signal bandwidth into sub-intervals $I_k:\ \omega \in (-\pi+\pi k, -\pi+\pi(k+1)),\quad k = 0, 1$. Then, we require that the distortion term be a delay of the form $2e^{-jd\omega}$ the aliasing term be canceled. This leads us to the following set of equation equations on each sub-interval I_k

$$\mathbf{H}(\omega-\pi k)\,\mathbf{F}(\omega) = \begin{bmatrix} 2e^{-jd\omega} \\ 0 \end{bmatrix} \leftarrow (k+1)th \text{ row} \quad (2)$$

where

$$\mathbf{F}(\omega) = \begin{bmatrix} F_0\left(e^{j\omega}\right) \\ F_1\left(e^{j\omega}\right) \end{bmatrix}$$

$$\mathbf{H}(\omega) = \frac{1}{2}\begin{bmatrix} H(\omega)+H(\omega) & H(\omega+\pi)+H(\omega-\pi) \\ H(\omega+\pi)+H(\omega+\pi) & H(\omega)+H(\omega+2\pi) \end{bmatrix}$$

Note that aliasing in this system occurs right after downconversion because the mixers shift the signal spectrum in both directions by an amount $\omega_m < \omega_N$. However, the synthesis filter can still be selected such that aliasing and distortion be reduced to the desired value. For RF signals separate cosine MHFB's can be used in each path after quadrature downconversion. In this case, the analog filters $H(s)$ fulfill the double task of eliminating the high-frequency component as well as splitting the spectrum of the downconverted component.

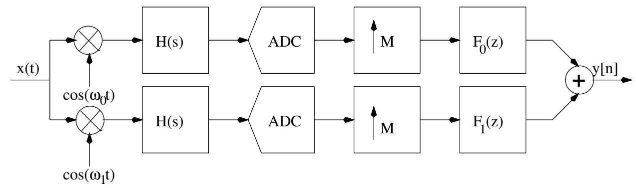

Fig. 3. Cosine MHFB for A/D conversion.

IV. OPTIMIZATION OF SYNTHESIS FILTERS

Digital filters derived from solving (1) or (2) represent ideal transfer functions which would guarantee the perfect reconstruction of the input signal. However, these frequency responses are generally unrealizable and can just serve as initial guess for an optimization process. Optimization leads to realizable digital filters allowing for signal reconstruction within a desired accuracy ε. Obviously, the accuracy improves as the order of the filters increases. The design and optimization algorithm can be summarized in the following steps: 1. Select the analog filter $H(\omega)$. 2. Obtain the ideal transfer function of the digital filters by solving (1) or (2). 3. Select the order of the digital filters. 4. Design digital filters approximating the ideal characteristics. 5. Change the coefficients of the digital filters so as to minimize $A(\omega)$ under the constraint $\left|D(\omega) - e^{-jd\omega}\right| < \varepsilon$. 6. Go to steps 2 or 1 if the requirements are not met.

V. DESIGN EXAMPLES

The above algorithm was implemented in Matlab to design several exponential and cosine MHFB's. The command "fir2" was employed in step 4 and the command "fgoalattain" in the optimization toolbox was used to solve the minimization under constraint problem involved in step 5. The algorithm usually converged in a few tens of iterations. The first example, shown in Fig.4, is a 3-channel ($L = 3$) exponential MHFB which uses third-order Butterworth analog analysis filters and 32nd-order FIR synthesis filters. The optimization goal was to preserve only the magnitude of the input signal [7]. Optimization did not include the edges of the bandwidth in order to improve the results. Frequency response of the filters is plotted in Fig. 5. The asymmetric frequency response of $F_{-1}\left(e^{j\omega}\right)$ and $F_1\left(e^{j\omega}\right)$ indicates that they are complex filters. The second example, shown in Fig. 6, is a 2-channel ($M = 2$) cosine MHFB with fourth-order Butterworth analog analysis filters and 32nd-order FIR synthesis filters. The optimization goal was to preserve both magnitude and phase of the input signal. The frequency response of the filters is plotted in Fig. 7. The symmetric frequency response of the digital filters indicates that they are real filters. Both examples achieve -55 dB aliasing attenuation while the magnitude variation is kept less than 0.1 dB.

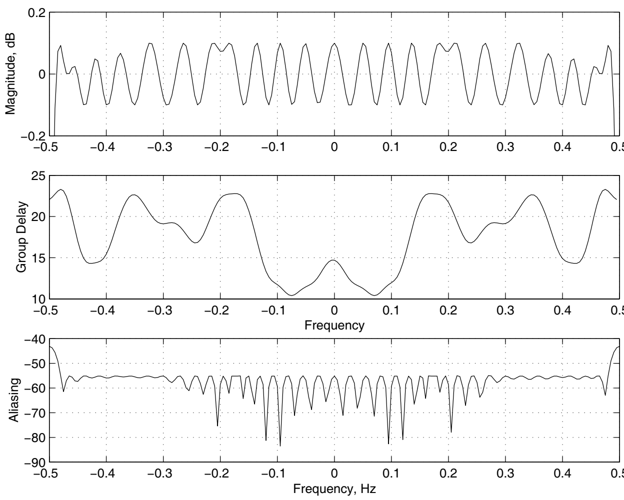

Fig. 4. Magnitude, group delay and aliasing.

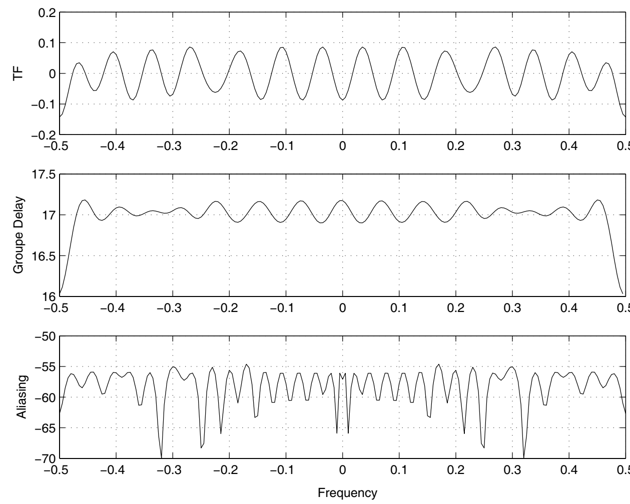

Fig. 6. Magnitude, group delay and aliasing.

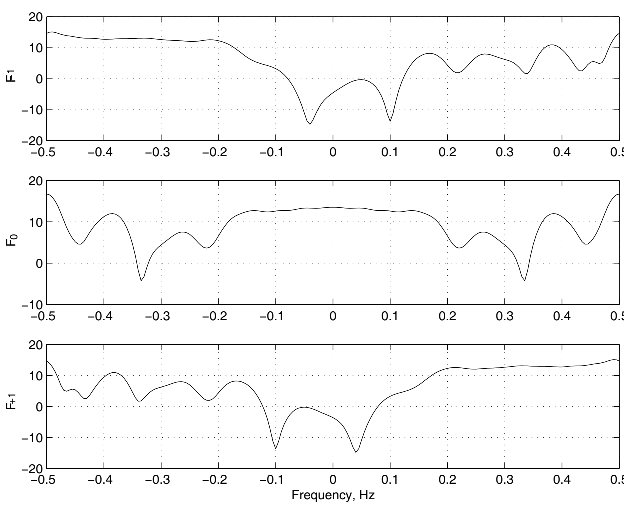

Fig. 5. Frequency response of digital filters, EXP-MHFB.

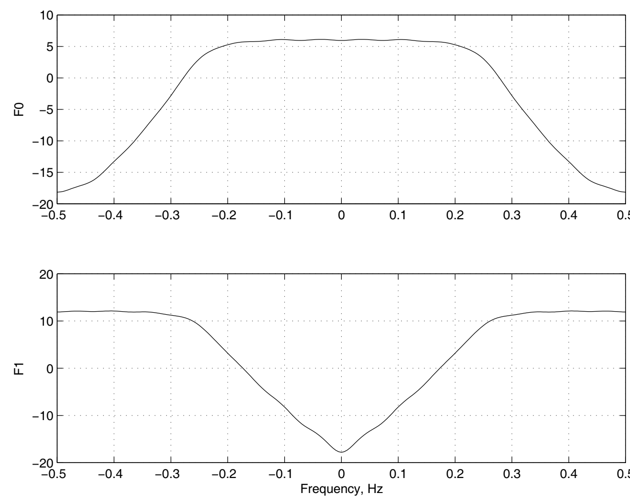

Fig. 7. Frequency response of digital filters, COS-MHFB.

VI. CONCLUSION

We presented a hybrid filter bank which allows for reconstruction of analog signals in the digital domain with requiring bandpass filters. The fact that all the channels of the MHFB are identical greatly simplifies the tuning of analog filters. A similar architecture can be used for D/A conversion. The proposed structure is attractive where speed is the primary requirement while a low or moderate accuracy is adequate. UWB is an example of such applications.

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