DESIGN OF GENERALIZED LINEAR-PHASE PARAUNITARY FILTER BANKS WITH ZEROS AT MIRROR FREQUENCIES

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ABSTRACT

In this paper, we present a novel lattice structure and design of the linear phase M-channel paraunitary filter banks whose all filters have zeros at the mirror frequencies except for its passband. This structure is implemented by replacing the first block of the lattice structure with two type II DCTs. With this structure, the filter banks are implemented with high speed and is designed without taking account of the stopband attenuation. Finnaly we show some examples to validate the proposed method.

1. INTRODUCTION

Filter banks (FBs) have found many applications such as in speech, audio and video compression, statistical signal processing, discrete multitone modulation and channel equalization[1]- [5]. Fig. 1 shows an M-channel maximally decimated filter bank, where $H_k(z)$ and $F_k(z)$ are the k-th (for $k = 0 \cdots M-1$) analysis filter and the synthesis filter, respectively. The analysis and synthesis filters are represented by using the polyphase matrix $\mathbf{E}(z)$ and $\mathbf{R}(z)$, respectively.

$$\begin{aligned} &[H_0(z)\ H_1(z) \cdots H_{M-1}(z)]^T = \mathbf{E}(z^M)\mathbf{e}(z)^T \\ &[F_0(z)\ F_1(z) \cdots F_{M-1}(z)] = \mathbf{e}(z)\mathbf{R}(z^M) \\ &\mathbf{e}(z) = [1\ z^{-1}\ \cdots\ z^{-(M-1)}] \end{aligned} \tag{1}$$

When $\mathbf{E}(z)\mathbf{E}^\dagger(z^{-1}) = \mathbf{I}$ and $\mathbf{R}(z) = \mathbf{E}^\dagger(z^{-1})$, where $.^\dagger$ stands for the conjugate transpose, the filter banks are called paraunitary filter banks(PUFBs). PUFBs are efficiently designed and implemented by the lattice factorization.

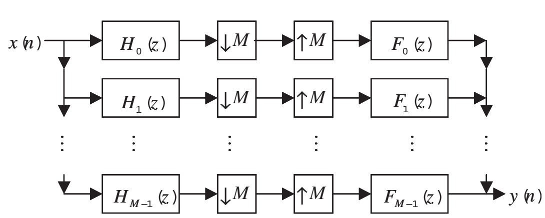

Fig. 1. M-channel filter bank

Recently, the lattice structure with some degrees of regularity has been proposed[2], which is important in image coding. Extending this idea, the linear phase paraunitary filter banks whose all filters have zeros at mirror frequencies except for its passband are proposed in this paper. This structure is implemented by replacing the first block of the conventional lattice structure with two type II DCTs. With this method, the filter bank is designed without taking account of the stopband attenuation, since the stopband of each analysis filter is formed automatically,

The filter banks using M points DCT has also been proposed in[5]. However the proposed filter banks use two $M/2$ points DCTs with parallel connection, its implementation is faster than the conventional one.

2. FILTER BANKS

The paraunitary filter banks can be implemented by the lattice structure that consists of the product of several othogonal blocks. In this paper, we assume that the filter length is KM.

The polyphase matrix $\mathbf{E}(z)$ of the paraunitary filter bank is expressed by[1][2]

$$\mathbf{E}(z) = \mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z)\cdots\mathbf{G}_1(z)\mathbf{E}_0 \tag{2}$$

$\mathbf{E}_0$ is called first block. $\mathbf{G}_i(z)$ is called building block and expressed by

$$\mathbf{G}_i(z) = \mathbf{\Gamma}_i\mathbf{W}\mathbf{\Lambda}(z)\mathbf{W}\mathbf{\Gamma}_i^T,\ \ \mathbf{E}_0 = \mathbf{\Gamma}_0\mathbf{W}\tilde{\mathbf{I}} \tag{3}$$

where,

$$\begin{aligned} &\mathbf{\Gamma}_i = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_i \end{bmatrix},\ \ \mathbf{\Gamma}_0 = \begin{bmatrix} \mathbf{U}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_0 \end{bmatrix} \\ &\mathbf{W} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{I} & \mathbf{I} \\ \mathbf{I} & -\mathbf{I} \end{bmatrix},\ \ \mathbf{\Lambda}_k(z) = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & z^{-1}\mathbf{I} \end{bmatrix} \\ &\tilde{\mathbf{I}} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{bmatrix} \end{aligned}$$

where $\mathbf{U}_0,\ \mathbf{V}_i(i = 0, 1, \cdots, K-1)$ are orthogonal matrices with size $M/2$.

3. FILTER BANKS WITH ZEROS AT MIRROR FREQUENCIES

In this section, we consider the linear phase paraunitary filter banks whose all filters have zeros at mirror frequencies except for its pass band.

First we change the order of the analysis filters as follows.

$$\begin{aligned}\mathbf{H}(z) &= \mathbf{E}(z^M)\mathbf{e}(z) \\ &= [H_0(z)\ H_2(z)\cdots H_{M-2}(z) \\ &\quad H_{M-1}(z)\ H_{M-3}(z)\cdots H_1(z)]^T\end{aligned} \tag{4}$$

Note that the lower half matrix with odd index start from the high-pass filter $H_{M-1}(z)$ when M is even.

When the lowpass filter $H_0(z)$ has zeros at mirror frequencies $\omega = 2\pi k/M (k \neq 0)$, its condition using the polyphase matrix is expressed by

$$\begin{aligned}&[H_0(e^{j0})\ H_0(e^{j\frac{2\pi}{M}})\cdots H_0(e^{j\frac{2\pi(M-1)}{M}})] \\ &\qquad = [1\ 0\ \cdots 0]\mathbf{E}(1)\mathbf{DFT}\end{aligned} \tag{5}$$

where

$$\begin{aligned}[\mathbf{DFT}]_{k,\ell} &= \frac{1}{\sqrt{M}}e^{-j\frac{2\pi}{M}k\ell} \\ & \quad k,\ell = 0,1,\cdots,M-1\end{aligned}$$

We set $\mathbf{E}(1)$ by substituting $z = e^{j\frac{2\pi k}{M}}$ into $\mathbf{E}(z^M)$. Furthermore,

$$\begin{aligned}\mathbf{E}(1) &= \mathbf{G}_{K-1}(1)\mathbf{G}_{K-2}(1)\cdots\mathbf{G}_1(1)\mathbf{E}_0 \\ &= \mathbf{E}_0 = \mathbf{\Gamma}_0\mathbf{W}\tilde{\mathbf{I}}\end{aligned} \tag{6}$$

Therefore we only consider the first block even if the filters have long length.

Next, we consider the case that all analysis filters have zeros at mirror frequencies except for its passband. Since the first block is a real matrix , and its upper half is symmetry and the lower half is anti-symmetry, its condition is expressed by

$$\begin{aligned}&\begin{bmatrix} H_0(e^{j0}) & H_0(e^{j\frac{2\pi}{M}}) & \cdots & H_0(e^{j\frac{2\pi(M-1)}{M}}) \\ H_2(e^{j0}) & H_2(e^{j\frac{2\pi}{M}}) & \cdots & H_2(e^{j\frac{2\pi(M-1)}{M}}) \\ \vdots & \vdots & \ddots & \vdots \\ H_1(e^{j0}) & H_1(e^{j\frac{2\pi}{M}}) & \cdots & H_1(e^{j\frac{2\pi(M-1)}{M}}) \end{bmatrix} \\ &= \mathbf{E}(1)\mathbf{DFT} = \mathbf{A} = \begin{bmatrix} \mathbf{A}_0 & \mathbf{A}_1 \\ -j\mathbf{A}_1 & j\mathbf{A}_0 \end{bmatrix}\mathbf{S}\end{aligned} \tag{7}$$

where the matrix $\mathbf{S}$ defines the phase term of a linear phase filter with length M and $[\mathbf{S}]_{kk} = e^{-j\frac{M-1}{2}\frac{2\pi k}{M}} = (-1)^k e^{j\frac{\pi k}{M}}$. Also

$$\mathbf{A}_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} \sqrt{2} & 0 & \cdots & 0 \\ 0 & a_1 & & \\ \vdots & & \ddots & \\ 0 & & & a_{\frac{M}{2}-1} \end{bmatrix} \tag{8}$$

$$\mathbf{A}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & & & a_1 \\ \vdots & & \iddots & \\ 0 & a_{\frac{M}{2}-1} & & 0 \end{bmatrix} \tag{9}$$

Because $\mathbf{A}$ is a orthogonal matrix, $a_i = \pm 1$, Note that the rows of $\mathbf{A}$ correspond to the filters and the columns correspond to the mirror frequencies. For example, when $M = 8$, the component $[\mathbf{A}]_{3,3}$ corresponds to $H_6(e^{j\frac{3\pi}{4}})$. Thus if the filter banks satisfy the above equation, the resulting filter banks have zeros at the mirror frequencies.

Furthermore, since the filter banks have real coefficients, their frequency responses are symmetry and we consider the range from 0 to π, that is from 0-th to $(M/2)$-th columns. Therefore (7) is rewritten as

$$\begin{aligned}&\begin{bmatrix} H_0(e^{j0}) & H_0(e^{j\frac{2\pi}{M}}) & \cdots & H_0(e^{j\pi}) \\ H_2(e^{j0}) & H_2(e^{j\frac{2\pi}{M}}) & \cdots & H_2(e^{j\pi}) \\ \vdots & \vdots & \ddots & \vdots \\ H_1(e^{j0}) & H_1(e^{j\frac{2\pi}{M}}) & \cdots & H_1(e^{j\frac{2\pi(M-1)}{M}}) \end{bmatrix} \\ &= \mathbf{E}(1)\mathbf{D\bar{F}T} = \begin{bmatrix} \hat{\mathbf{A}}_0 \\ -j\hat{\mathbf{A}}_1 \end{bmatrix}\bar{\mathbf{S}}\end{aligned} \tag{10}$$

where $\mathbf{D\bar{F}T}$ is $\mathbf{DFT}$ from 0-th to $(M/2)$-th columns and $\bar{\mathbf{S}}$ is the left upper matrix with size $M/2+1$ of $\mathbf{S}$. From (7) and (10), $\hat{\mathbf{A}}_0$ and $\hat{\mathbf{A}}_1$ are rewritten by

$$\hat{\mathbf{A}}_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} \sqrt{2} & 0 & \cdots & & 0 \\ 0 & 1 & & & \vdots \\ \vdots & & \ddots & & \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix} \tag{11}$$

$$\hat{\mathbf{A}}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & \cdots & & 0 & \sqrt{2} \\ \vdots & 0 & \cdots & 1 & 0 \\ & \vdots & & 0 & \vdots \\ 0 & 1 & 0 & \cdots & 0 \end{bmatrix} \tag{12}$$

In practice, the components of the above matrices are ± 1. However the sign is negated by the sign of $\bar{\mathbf{S}}$. As result, $\bar{\mathbf{S}}$ is the diagonal matrix with $[\bar{\mathbf{S}}]_{k,k} = e^{j\frac{\pi k}{M}} (k = 0,1,\cdots,\frac{M}{2})$.

Next, in order to obtain $\mathbf{U}_0$ and $\mathbf{V}_0$, $\mathbf{D\bar{F}T}$ is represented by

$$\begin{aligned}&\mathbf{D\bar{F}T} = \begin{bmatrix} \mathbf{Q} \\ \mathbf{QD} \end{bmatrix} \\ &[\mathbf{Q}]_{k,\ell} = \frac{1}{\sqrt{M}}e^{j\frac{2\pi k\ell}{M}} \quad \begin{cases} k = 0,\cdots,\frac{M}{2}-1 \\ \ell = 0,\cdots,\frac{M}{2} \end{cases} \\ &\mathbf{D} = diag([1\ \cdots -1\ \cdots\ 1\ -1\ 1])\end{aligned}$$

Based on these equations, (10) is rewritten as

$$\begin{aligned}&\mathbf{E}(1)\mathbf{D\bar{F}T} = \mathbf{E}_0\mathbf{D\bar{F}T} = \mathbf{\Gamma}_0\mathbf{W}\tilde{\mathbf{I}}\begin{bmatrix} \mathbf{Q} \\ \mathbf{QD} \end{bmatrix} \\ &= \mathbf{\Gamma}_0\begin{bmatrix} \mathbf{Q}+\mathbf{JQD} \\ \mathbf{Q}-\mathbf{JQD} \end{bmatrix} = \mathbf{\Gamma}_0\begin{bmatrix} \mathbf{R}_0 \\ \mathbf{R}_1 \end{bmatrix}\end{aligned} \tag{13}$$

where

$$\begin{aligned}&[\mathbf{R}_0]_{k,\ell} \\ &= \frac{1}{\sqrt{M}}\left\{e^{-j\frac{2\pi}{M}k\ell} + e^{-j\frac{2\pi}{M}(\frac{M}{2}-1-k)\ell}(-1)^\ell\right\} \\ &= \frac{2}{\sqrt{M}}e^{j\frac{\pi}{M}\ell}\cos\{\frac{\pi}{M}(2k+1)\ell\}\end{aligned} \tag{14}$$

On the above equation, $[\mathbf{R}]_{k,M} = 0$ and the matrix from 0-th and $(M/2-1)$-th columns of $\mathbf{R}_0$ is identical to type III DCT($\mathbf{DCT}$) with $\frac{M}{2}$ points except for the normalized constant and phase term[7].

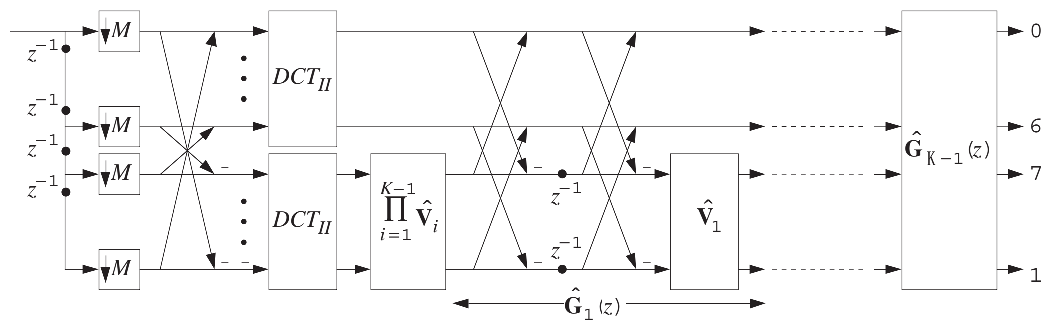

Fig. 2. The lattice structure of the proposed method

$\mathbf{R}_1$ is also rewritten as

$$\begin{aligned}
&[\mathbf{R}_1]_{k,\ell} \\
&= \frac{1}{\sqrt{M}}\left\{e^{-j\frac{2\pi}{M}k\ell} - e^{-j\frac{2\pi}{M}(\frac{M}{2}-1-k)\ell}(-1)^{\ell}\right\} \\
&= \frac{2}{\sqrt{M}}e^{j\frac{\pi}{M}\ell}\sin\{\frac{\pi}{M}(2k+1)\ell\} \qquad (15)
\end{aligned}$$

From (13),

$$\begin{aligned}
\mathbf{U}_0\mathbf{R}_0 &= \frac{1}{\sqrt{2}}\mathbf{U}_0[\mathbf{DCT}_{\mathrm{III}}\ \mathbf{0}] \\
&\quad \cdot \begin{bmatrix} \sqrt{2} & & & 0 \\ & 1 & & \vdots \\ & & \ddots & \vdots \\ & & & 1 \end{bmatrix} \\
&= \hat{\mathbf{A}}_0
\end{aligned}$$

and we get

$$\mathbf{U}_0 = \mathbf{DCT}_{\mathrm{III}}^T = \mathbf{DCT}_{\mathrm{II}} \qquad (16)$$

Next, considering the lower half of (13), $\mathbf{V}_0$ has to satisfy:

$$\mathbf{V}_0\mathbf{R}_1 = \hat{\mathbf{A}}_1$$

Then we multiply $\mathbf{J}$ to the above equation from right

$$\mathbf{V}_0\mathbf{R}_1\mathbf{J} = \hat{\mathbf{A}}_0$$

Ignoring the phase term,

$$\begin{aligned}
&[\mathbf{R}_1\mathbf{J}]_{k,\ell} \\
&\quad = \frac{2}{\sqrt{M}}\sin\{\frac{\pi}{M}(2k+1)(\frac{M}{2}-\ell)\} \\
&\quad = \frac{2}{\sqrt{M}}(-1)^k\cos\{\frac{\pi}{M}(2k+1)\ell\}
\end{aligned}$$

This is identical to $\mathbf{R}_0$ except for the sign. Therefore

$$\begin{aligned}
\mathbf{V}_0\mathbf{R}_1\mathbf{J} &= \frac{1}{\sqrt{2}}\mathbf{V}_0[\mathbf{D}\cdot\mathbf{DCT}_{\mathrm{III}}\ \mathbf{0}] \\
&\quad \cdot \begin{bmatrix} \sqrt{2} & & & 0 \\ & 1 & & \vdots \\ & & \ddots & \vdots \\ & & & 1 \end{bmatrix} \\
&= \hat{\mathbf{A}}_0
\end{aligned}$$

As result,

$$\mathbf{V}_0 = \mathbf{DCT}_{\mathrm{II}}\cdot\mathbf{D} \qquad (17)$$

Finally the first block of the filter banks is represented by

$$\boldsymbol{\Gamma} = \begin{bmatrix} \mathbf{DCT}_{\mathrm{II}} & \mathbf{0} \\ \mathbf{0} & \mathbf{DCT}_{\mathrm{II}}\cdot\mathbf{D} \end{bmatrix} \qquad (18)$$

Thus the filter banks whose all filters have zeros at mirror frequencies are implemented by setting the first block as two type II DCT with size $M/2$. Fig.2 shows the proposed lattice structure.

4. EXAMPLE

In general, the filter banks are designed by minimizing the objective function which consists of a linear combination of the coding gain, DC leakage and stopband attenuation. However the proposed filter banks automatically form the stopband and its DC leakage is also zero. Therefore we optimize only the coding gain without taking account of the stopband.

We designed the filter banks with 8 and 16 channels which optimize only the coding gain. Fig.3 shows the magnitude response and impulse response of the filter bank with $M = 8, L = 16$. Fig.4 shows the magnitude response of the filter bank with $M = 16, L = 32$. As shown these figures, the designed filter banks have better attenuation than that of the conventional method and all filters have zeros at the mirror frequency except for the passband. Table 1 shows the comparison of the coding gain and the number of free parameters between the proposed and the conventional method with a reguralarity. The proposed method has comparable coding gain with the conventional one inspite of less free parameters.

Table 1. Comparison of Coding Gain and the number of free parameters

		Conventional[4]		Proposed	
M	K	CG	FP	CG	FP
8	2	9.2685	18	9.2663	6
8	3	9.3802	24	9.3747	12
8	4	9.4564	30	9.4532	18
16	2	9.7701	84	9.8102	28

Table 2. Comparison of PSNR in image coding

	Lena			Girl			Barb			Camera		
[bpp]	1/8	1/4	1/2	1/8	1/4	1/2	1/8	1/4	1/2	1/8	1/4	1/2
Conventional	30.07	33.25	36.54	31.92	34.10	35.99	26.25	29.31	33.36	24.15	26.65	29.91
Proposed	30.07	33.25	36.52	31.90	34.10	36.00	26.20	29.24	33.30	24.16	26.68	29.85

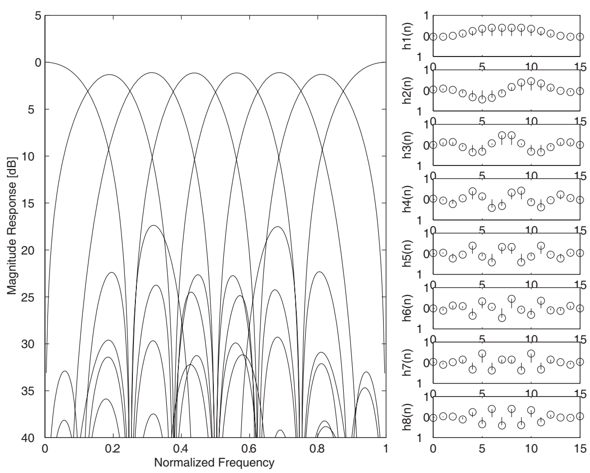

Fig. 3. The magnitude and impulse response when M=8, L=16

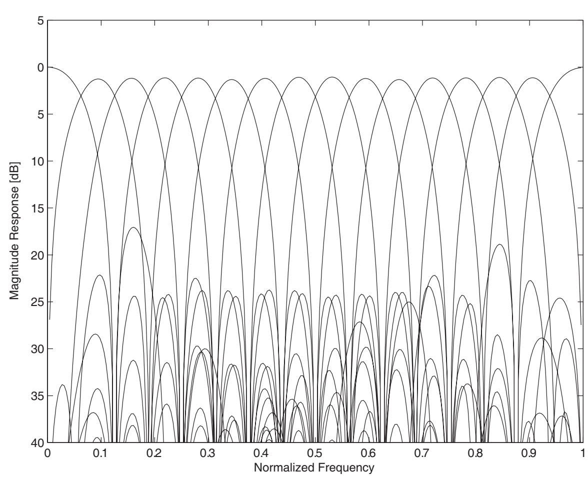

Fig. 4. The magnitude response when M=16, L=32

Next, we apply the designed filter banks to image coding application. Table 2 shows the PSNR comparison of the proposed and conventional method in various images. From these results, the proposed method has comparable coding performance with the conventional method and the high speed computation is possible.

5. CONCLUSION

In this paper, we proposed a new structure and design of the linear phase paraunitary filter banks with zeros at mirror frequencies. This structure replace only first block of the conventional lattice structure by two type II DCT with $\frac{M}{2}$ points, which has high speed computation. The proposed method has comparable coding gain and coding performance in image coding with the conventional method inspite of less free parameters.

6. REFERENCES

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