DESIGN OF PERFECT RECONSTRUCTION QMF LATTICE WITH SIGNED POWERS-OF-TWO COEFFICIENTS USING CORDIC ALGORITHM

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ABSTRACT

Lattice structure has several advantages over the tapped delay line form, especially for the hardware implementation of general digital filters. It is also efficient for the implementation of Quadrature Mirror Filter (QMF), because the Perfect Reconstruction (PR) is conserved even under the severe coefficient quantization. Moreover, if lattice coefficients are implemented by Signed Powers-of-Two (SPT), the hardware complexity can also be reduced. But the discrete space represented by the SPT is sparse when the number of non-zero bits is small. This paper proposes an orthogonal QMF lattice with SPT coefficients that can provide much denser discrete coefficient space than the conventional structure. For this purpose, we employ CORDIC algorithm that is structurally related with the PR lattice filter with SPT coefficients. The paraunitariness of CORDIC subrotation also continues to hold the PR condition to our wishes. Since the proposed architecture provides denser coefficient space, it shows less coefficient quantization error than the conventional QMF lattice.

1. INTRODUCTION

The Quadrature Mirror Filter (QMF) banks [1] shown in Fig. 1 (a) have a number of applications in digital signal processing. It can be verified that if analysis filters $H_0(z)$, $H_1(z)$, and synthesis filters $F_0(z)$, $F_1(z)$ constitute the paraunitary (PU) filter bank, then the system is a Perfect Reconstruction (PR) system, i.e., $\hat{x}(n)$ is scaled and delayed version of $x(n)$. Analysis bank of Fig. 1 (a) can be implemented in the lattice form shown in (b). In practical hardware implementation of Fig. 1 (b), lattice coefficient α_n needs to be quantized for fixed-point arithmetic. For more efficient implementation, using constrained Signed Powers-of-Two (SPT) coefficients is a good alternative because multiplication can be replaced by simple shift/add operation. The QMF lattice has the advantage that the PU is conserved even under severe coefficient quantization [1]. Lattice coefficient is selected in a set of sums of SPT represented as

$$\alpha_n = \sum_{k=1}^{M} s_{k,n} 2^{-p_{k,n}} \qquad \text{for} \quad n = 0, 1, ..., N,$$
$$s_{k,n} \in \{-1, 1\}, \qquad p_{k,n} \in \{-2, -1, 0, 1, \cdots, B\}. \qquad (1)$$

Each coefficient has $(B+3)$-bit wordlength and M non-zero bits that determine the density of discrete coefficient space, i.e., B and M determine the amount of possible numbers represented by (1). For obtaining $s_{k,n}$ and $p_{k,n}$ which provide a good frequency response, we need to employ an appropriate optimization technique

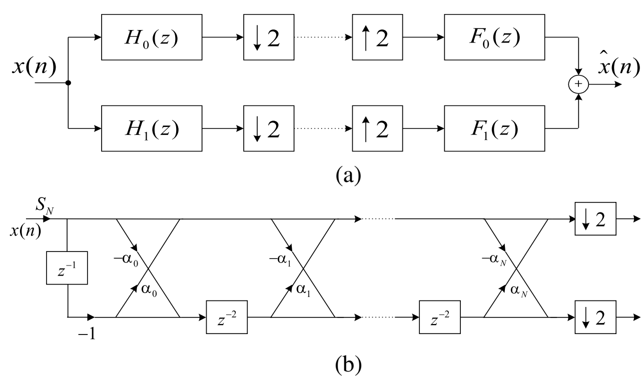


Fig. 1. (a) Two channel QMF bank, (b) Analysis bank of two channel PR lattice filter.

rather than simple rounding of infinite precision coefficients. In the case of FIR filter design, many optimization techniques have been developed based on global or local optimization [3], since Lim and Parker [2] proposed Mixed Integer Linear Programming (MILP) over a discrete SPT coefficient space. But these conventional optimization methods cannot be used for the optimal selection of the lattice coefficient, because lattice coefficient obtained after the quantization of FIR coefficient no longer guarantees the PR condition. The MILP cannot be applied to the lattice structure, also because the FIR coefficient is not a linear combination of lattice coefficients. Thus, the α_n should be directly quantized using local optimization techniques.

Although many papers have discussed the optimization of lattice filter bank with SPT coefficients [4, 5], the modification of discrete coefficients space has rarely been reported in the literature. In this paper, we develop a new structure of multiplierless two-channel PR lattice filter bank that has more dense discrete coefficient space. In other words, we propose a lattice structure that can have larger number of possible coefficients with the given B and M than the conventional methods. As a result, the coefficient quantization error can be largely reduced.

For this purpose, we employ the COordinate Rotation DIgital Computer (CORDIC) which is an effective method for calculating vector rotation [6–8]. More specifically, proposed design provides more dense space based on the following two techniques. The first is to use a cascade of various CORDIC subrotations [8], whereas the conventional method considers only one subrotation. The second is to add rotation angles of $\pi/2 + \theta$ and $\pi/2 - \theta$ with the assumption that θ is a candidate of rotation angle. For the case where

$M = 2$, the proposed method provides about three times as dense distribution as conventional SPT without the expense of additional hardwares. Moreover, if $M = 3$, it provides nine times. The proposed methods continue to hold the PR condition by preserving the unitariness of rotation. Moreover, the angle distribution of the proposed structure is relatively uniform compared to the conventional method. Design examples show that the proposed method can largely reduce the overall quantization error, and improve the filter performance.

2. PR QMF LATTICE WITH SPT

Fig. 1 (a) shows QMF bank that is the two-channel version of the maximally decimated filter bank. Assuming no degradation in channel, the reconstructed signal $\hat{x}(n)$ can be related to $x(n)$ by

$$\begin{aligned} \hat{X}(z) &= T(z)X(z) + A(z)X(-z) \quad \text{where} \\ T(z) &= \frac{1}{2}\big[H_0(z)F_0(z) + H_1(z)F_1(z)\big] \quad \text{distortion term,} \\ A(z) &= \frac{1}{2}\big[H_0(-z)F_0(z) + H_1(-z)F_1(z)\big] \quad \text{aliasing term.} \end{aligned} \tag{2}$$

$T(z)$ must be z^{-2N-1} for no distortion, and $A(z)$ be zero for alias cancellation, and then the PR condition that $\hat{x} = x(n - 2N - 1)$ is satisfied. To satisfy these constraints, low pass filter $H_0(z)$ with order $2N + 1$ needs to be designed satisfying power-symmetry property represented as [1]

$$\widetilde{H}_0(z)H_0(z) + \widetilde{H}_0(-z)H_0(-z) = 1 \tag{3}$$

where $\widetilde{H}(z) = \big[H(z)\big]^*$. Then, the other three filters are designed as follows.

$$\begin{aligned} H_1(z) &= -z^{-N}\widetilde{H}_0(-z), \\ F_0(z) &= 2z^{-N}\widetilde{H}_0(z), \\ F_1(z) &= -2H_0(-z). \end{aligned} \tag{4}$$

It can be seen that analysis filters have the power complementary property given by

$$|H_0(e^{jw})|^2 + |H_1(e^{jw})|^2 = 1. \tag{5}$$

3. RELATIONSHIP BETWEEN CORDIC AND QMF LATTICE

In this section, we show the relationship between CORDIC and orthogonal QMF lattice with SPT coefficients. The CORDIC can compute vector rotation using a series of specific incremental rotation angles, where each rotation is performed by a shift/add operation [6]. The iterative equation of CORDIC relates the the input vector $\mathbf{v}(i)$ to the output $\mathbf{v}(i+1)$ as

$$\begin{aligned} \mathbf{v}(i+1) &= \begin{bmatrix} 1 & \sigma_i 2^{-i} \\ -\sigma_i 2^{-i} & 1 \end{bmatrix} \cdot \mathbf{v}(i), \\ \phi(i+1) &= \phi(i) - \sigma_i a_i \qquad \text{for} \quad i = 0, 1, ..., S-1 \end{aligned} \tag{6}$$

where S is the number of CORDIC subrotations, and σ_i is a sequence of ± 1s that determines the direction of remaining angle.

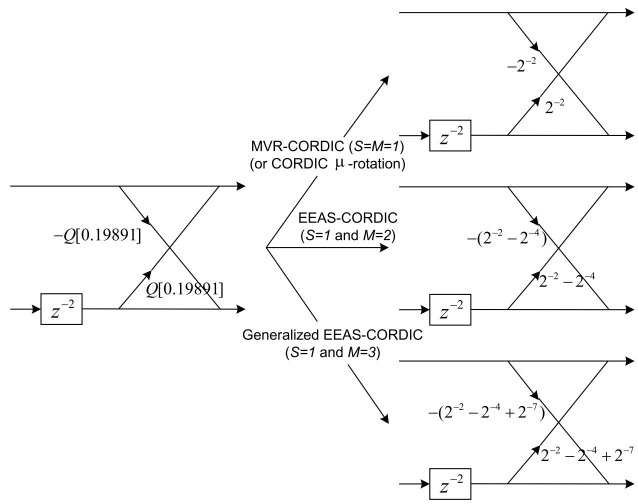


Fig. 2. An example of the relationship between one stage ($\theta = \pi/16$) of the QMF lattice structure and modified CORDIC algorithm.

$\phi(i)$ is the remaining angle after the $(i-1)$-th iteration, and determines the sign of σ_i. The a_i is defined as

$$a_i \triangleq \tan^{-1}(2^{-i}). \tag{7}$$

The subrotation of CORDIC is similar to the one stage of QMF lattice. More specifically, if the subrotation matrix of (6) is replaced by the one stage of QMF lattice, it is equivalent to quantizing α_n into SPT with $M = 1$. For further consideration of relationship between CORDIC and QMF lattice, let us summarize the modified CORDIC algorithms [7]. We are specially interested in Modified Vector Rotational CORDIC (MVR-CORDIC), and Extended Elementary Angle Set CORDIC (EEAS-CORDIC), and generalized EEAS-CORDIC among various CORDIC algorithms. The main difference between the modified algorithm and the conventional scheme is that the elementary rotation angle set is expanded in a more flexible way, while a basic shift/add operation is maintained. More precisely, the number of iterations S is fixed to a predetermined value (usually 2 or 3) that is less than wordlength B, and the elementary angle a_i is modified as follows:

$$\begin{aligned} a_i &\triangleq \tan^{-1}\left(\sum_{k=1}^{M} s_{k,i} 2^{-p_{k,i}}\right) \qquad \text{for} \quad i = 0, 1, ..., S-1, \\ s_{k,i} &\in \{-1, 0, 1\}, \qquad p_{k,i} \in \{0, 1, \cdots, B\}. \end{aligned} \tag{8}$$

Then, modified CORDIC algorithms can be distinguished by parameter M as follows:

$$\begin{aligned} &\text{if } M = 1, && \text{MVR-CORDIC,} \\ &\text{else if } M = 2, && \text{EEAS-CORDIC,} \\ &\text{else if } M \geq 3, && \text{Generalized EEAS-CORDIC.} \end{aligned} \tag{9}$$

In the modified CORDIC, each elementary rotation angle can be performed repeatedly.

The definition of elementary angle of the modified CORDIC is related with the SPT definition of (1). That is, if $M = S = 1$, subrotation R_n of QMF lattice is replaced by MVR-CORDIC which is equivalent to quantizing α_n into SPT where $M = 1$. Else if $M = 2$ and $S = 1$, it can be replaced by EEAS-CORDIC which is equivalent to quantizing α_n into SPT where $M = 2$. As with the EEAS-CORDIC, generalized EEAS-CORDIC where $M \geq 3$ can be similarly compared with the SPTs. For example, when the subrotation angle is $\pi/16$, the elementary angle can be represented as

$$a_i = \frac{\pi}{16} \simeq \begin{cases} \tan^{-1}(2^{-2}) & \text{for MVR,} \\ \tan^{-1}(2^{-2} - 2^{-4}) & \text{for EEAS,} \\ \tan^{-1}(2^{-2} - 2^{-4} + 2^{-7}) & \text{for Gen. EEAS.} \end{cases} \tag{10}$$

Fig. 2 shows an example of the relationship between the QMF lattice structure and the modified CORDIC algorithms.

4. DESIGN OF PROPOSED PR QMF LATTICE

The design problem is to quantize the α_n into a sum of SPT, that is, determine the $s_{k,n}$ and $p_{k,n}$ of (1) that minimize the stopband attenuation. The proposed method increases the number of elements in the discrete coefficient space by using a cascade of the various CORDIC subrotations (or called MAR in [8]) introduced in Section 3, whereas the conventional method considers only single subrotation. In Table 1 (a), the conventional SPT representation ($M = 3$) of the lattice coefficient is shown. Many optimization methods [4,5] have considered only this type of structure that quantizes the α_n into a sum of SPT directly. In Section 3, it can be seen that this prototype[1] is based on the EEAS-CORDIC where $S = 1$ and $M = 3$. Here, we can consider another prototype based on MVR-CORDIC as shown in Table 1 (b). While it uses the same number of SPT or shift/add operations to the case (a), it indeed performs a completely different angle rotation. Since cascaded system of PU building blocks satisfies PU as well, two channel QMF lattice satisfies desirable PR condition [1]. As shown in Table 1 (c), the combination of MVR and EEAS-CORDIC is able to perform completely different rotation. For the case of $M = 3$, the first method provides about six times as dense distribution as the conventional SPT without the expense of additional hardwares. The second method is to insert the angles such as $\pi/2 + \theta$ and $\pi/2 - \theta$ into the coefficient candidate pool assuming that the θ is an element in the candidate set. Thus, following relationships hold:

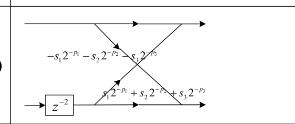
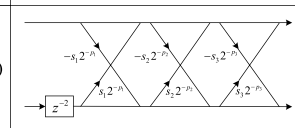
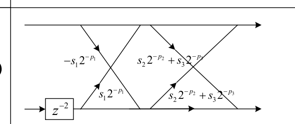
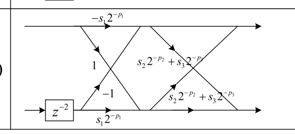
$$\begin{bmatrix} \cos(\frac{\pi}{2} + \theta_n) & \sin(\frac{\pi}{2} + \theta_n) \\ -\sin(\frac{\pi}{2} + \theta_n) & \cos(\frac{\pi}{2} + \theta_n) \end{bmatrix} = -\cos\theta_n \begin{bmatrix} \alpha_n & -1 \\ 1 & \alpha_n \end{bmatrix},$$

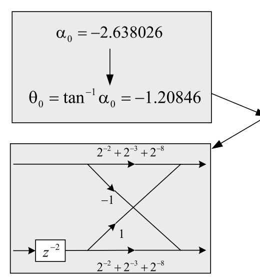
$$\begin{bmatrix} \cos(\frac{\pi}{2} - \theta_n) & \sin(\frac{\pi}{2} - \theta_n) \\ -\sin(\frac{\pi}{2} - \theta_n) & \cos(\frac{\pi}{2} - \theta_n) \end{bmatrix} = \cos\theta_n \begin{bmatrix} \alpha_n & 1 \\ -1 & \alpha_n \end{bmatrix}. \tag{11}$$

This can additionally provide about 1.5 times the size of reachable angle. By using these two ideas at the same time, we can provide about nine times dense distribution compared to the conventional SPT. Table 1 (d) shows the structure that $\pi/2$ is added to the first section (MVR part) of (c). Although it is added to the second section (EEAS part), it performs an equivalent rotation. If $M > 3$, since more diverse kinds of combinations are possible, the number of elements would increase dramatically. In the case of conventional SPT QMF lattice, we search for a similar value to the ideal α_n in the coefficient candidate pool. In other words, it

[1] In this paper, let the word 'prototype' mean the typical SPT lattice structures as shown in Table 1.

Table 1. Prototypes of one stage of QMF lattice in SPT domain for the case of $M = 3$.

	Lattice Structure	Angle	Type
(a)		$\tan^{-1}(s_1 2^{-p_1} + s_2 2^{-p_2} + s_3 2^{-p_3})$	EEAS-CORDIC ($S = 1$, $M = 3$)
(b)		$\tan^{-1}(s_1 2^{-p_1}) + \tan^{-1}(s_2 2^{-p_2}) + \tan^{-1}(s_3 2^{-p_3})$	MVR-CORDIC ($S = 3$, $M = 1$)
(c)		$\tan^{-1}(s_1 2^{-p_1}) + \tan^{-1}(s_2 2^{-p_2} + s_3 2^{-p_3})$	MVR+EEAS CORDIC
(d)		$\frac{\pi}{2} + \tan^{-1}(s_1 2^{-p_1}) + \tan^{-1}(s_2 2^{-p_2} + s_3 2^{-p_3})$	MVR+EEAS CORDIC



Angle	Type	A sum of SPT
…………	…………	…………
−1.20916	211/0	$-2^{2}+2^{-3}-2^{-7}$
−1.20882	121/−1	$2^{-2}+2^{-3}-2^{-7}$
−1.20860	111/−1	$2^{-2}+2^{-3}+2^{-8}$
−1.20811	112/−1	$2^{-1}-2^{-3}+2^{-8}$
−1.20681	111/0	$-2^{1}-2^{-1}-2^{-3}$
−1.20591	112/0	$-2^{1}-2^{-1}-2^{-6}$
−1.20536	123/0	$-2^{2}+2^{-3}-2^{-8}$
…………	…………	…………

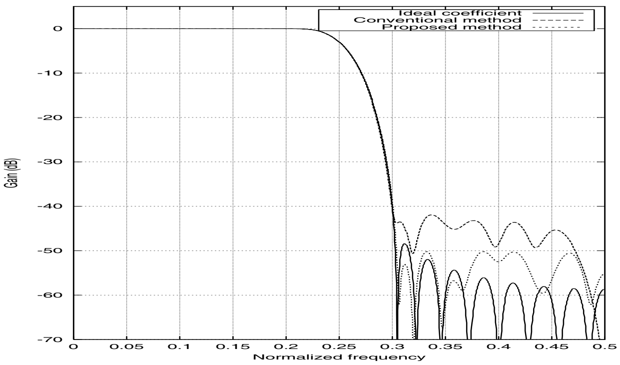
Fig. 3. Angle quantization example.

is possible to quantize α_n directly into SPT representation. However, in our design, it would be impossible to directly search for the value of $[\alpha_n]_Q$[2] because the $[\alpha_n]_Q$ corresponding to the cases such as MVR or mixed (MVR+EEAS) CORDIC shown in Table 1 (b), (c), or (d) cannot be seen easily. Wu *et al.* [7, 8] proposed an angle quantization approach that performs the quantization process on the angle domain. A simple angle quantization example is demonstrated in Fig. 3. In this figure, the coefficient candidates are stored in the type of the angle, and α_n which we aim to quantize is also mapped into the angle domain using $tangent$ function in order to proceed all search process in the angle domain. The 'type' can be used to distinguish various prototypes which can be represented as a sum of SPT. The system designer is able to use his/her own type to distinguish the various prototypes. If the SPT coefficient candidates in the angle domain are constructed, we can perform the optimization process.

5. DESIGN EXAMPLE

$32F$ filter bank denoted in [1] is examined to show the improvement of the proposed design. The quantized coefficient's wordlength

[2] $[\ \cdot\ ]_Q$ represents quantization operator.



n	Type	α_n
1	112/−1	$2^{-2}-2^{-5}+2^{-7}$
2	211/1	$-2^{2}+2^{0}-2^{-3}$
3	211/0	$-2^{-1}-2^{-2}+2^{-6}$
4	121/0	$2^{-1}+2^{-4}-2^{-6}$
5	121/−1	$2^{1}+2^{-4}+2^{-4}$
6	111/1	$-2^{2}+2^{-1}+2^{-2}$
7	111/−1	$2^{2}+2^{-2}+2^{-5}$
8	211/1	$-2^{2}-2^{-4}-2^{-7}$
9	112/0	$-2^{-3}-2^{-6}+2^{-7}$
10	121/0	$2^{-1}-2^{-1}+2^{-3}$
11	121/0	$2^{0}-2^{0}-2^{-3}$
12	121/0	$2^{1}-2^{1}+2^{-2}$
13	211/0	$-2^{-6}-2^{-7}-2^{-8}$
14	123/0	$2^{-7}+2^{-8}+2^{-8}$
15	211/0	$-2^{2}+2^{2}-2^{-3}$
16	211/0	$-2^{-1}+2^{-1}+2^{-8}$

Fig. 4. Frequency response and SPT coefficients for the $32F$ filter bank in [1].

is 11 bits ($B = 8$) and $M = 3$. The local optimization process of [3, 5] is employed to minimize the maximum ripple of stopband. Since the normalized frequency response of the original impulse response $h(n)$ will be changed by a constant multiplication, we can obviate the scaling process. The frequency responses of low pass filters for the infinite precision coefficient design, conventional direct quantization, and proposed design are illustrated in Fig. 4. The ideal coefficient design results in maximum stopband attenuation of 48.4 dB. The stopband attenuation of proposed design is 50.2 dB which is 8.3 dB higher than the conventional design of 41.9 dB. The maximum ripple of proposed design is less than that of the ideal design since they minimize different objective functions. The SPT coefficient values are listed in Fig. 4. In this figure, 'Type' column assigns a proper prototype to each SPT coefficient. The complexity of proposed filter bank is almost the same as the conventional one in that the stages of two filter banks can be implemented by 6 shifter and 6 adder. Table 2 shows the performance comparison between conventional SPT design and proposed design of various low pass filters denoted in [1]. It can be observed that the proposed design reduces the stopband ripple more than 3 dB in most cases.

6. CONCLUSIONS

A structure for the PR lattice filter bank with SPT coefficients has been proposed based on the various CORDIC algorithms. The proposed structure is based on two main ideas: the first is to use the cascade of CORDIC algorithms and the second is to add $\pm\pi/2$ to the rotation angle. The proposed method can provide more possible elements in the discrete coefficient space than the conventional SPT, and thus largely reduces the maximum ripple of the stopband. This method can also be used to the similar applications such as design and implementation of linear phase QMF lattice or orthogonal wavelets.

Table 2. Performance comparison between the ideal coefficient design, conventional SPT design, and proposed design (dB).

Filter # [1]	Ideal	Conv.	Prop.	M
$8A$	40.97	34.18	40.24	2
$16B$	51.42	47.79	50.82	3
$24F$	38.37	34.15	39.37	2
$32C$	57.26	46.36	53.68	3
$32E$	25.04	28.76	28.90	2
$48E$	31.61	31.24	33.41	2
$64E$	39.39	37.28	41.03	3

7. REFERENCES

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