ON THE DESIGN OF FIR OPTIMUM ORTHONORMAL FILTER BANKS

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ABSTRACT

The problem of designing optimum filter banks for different applications is a popular research subject. It has also been shown that principal component filter banks (PCFB) are the optimum filter bank for many application. Existing methods to design FIR PCFBs are based on designing energy compaction filters. In this work we concentrate on designing FIR PCFB with the same frequency response as the ideal one. The presented approach results in filter banks with a very good approximation of ideal PCFB, as verified by simulations.

1. INTRODUCTION

Filter banks have been successfully applied in many signal processing applications such as multiresolution signal representation, noise suppression, signal compression and transmission. The problem of finding optimum filter banks, especially in the class of orthonormal filter banks, has been considered by many researchers. Although, at the beginning, filter banks were optimized for each application individually, later, by defining principal component filter banks, it was found that the optimum solution for most applications is the same.

Principal component filter bank (*PCFB*) was defined as the optimum solution of multiresolution signal decomposition [1]. Using majorization and convex theories [2, 3], the optimality of PCFB for other applications such as subband coding, discrete multitone modulation and white-noise suppression was shown. Therefore, the problem of optimizing orthonormal filter banks have been unified to a single problem of finding PCFB and its existence issues.

Because of the close relationship between energy compaction filters and PCFB, almost all of the existing algorithms to design PCFB relies on designing energy compaction filter and completing it to an orthonormal filter bank. This approach to design filter bank is suboptimum in the sense that if there is no PCFB, it may not lead to a globally optimal solution and the resulting filter banks may perform even worse than the ordinary ones.

In this paper, we concentrate on the problem of designing FIR optimum filter banks. The proposed algorithm is based on an approximation of ideal (infinite length) PCFB with FIR filter bank such that they have nearly the same frequency responses and therefore, similar subband energies. As a result, the approximated FIR PCFB will have almost the same performance as an ideal PCFB, for most of the applications. Although, the designed filter bank is not a globally optimal solution, but its performance will be near optimum as verified by simulations.

This work was supported by Iran Telecommunication Research Center under the grant number 500/4044.

2. A REVIEW ON PCFB

Consider the M-band uniform orthonormal filter bank(figure 1). The principal component filter bank is defined as the optimum filter bank for multiresolution signal decomposition [1], i.e. for every K, $1 \leq K \leq M$, by keeping the first K subband signals, the reconstructed signal will be as similar as possible to the input signal, in the mean square sense. In other words, in an arbitrary class of orthonormal filter banks (class $\mathcal{C}$), PCFB maximizes the following quantity among all other filter banks in $\mathcal{C}$ for all K, $1 \leq K \leq M-1$:

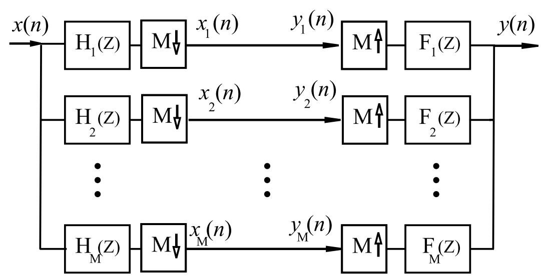


Fig. 1. A uniform M-band filter bank

$$\sum_{i=1}^{K} \sigma_i^2 \tag{1}$$

where σ_i^2 is the variance of the ith subbband signal ($x_i(n)$).

As it is obvious from the definition of PCFB, the first band of PCFB has maximum subband variance (besides the Nyquist(M) property due to the orthonormality). Such a filter is known as energy compaction (EC) filter. Similarly, the second subband signal has the most *available* energy and so forth. This is the base of the sequential search algorithm to find PCFB in an arbitrary class $\mathcal{C}$ of orthonormal filter banks, as follows [4]:

- Find all filter banks in $\mathcal{C}$ such that their first subband's variance is maximum and denote it by $\mathcal{C}_1$.
- For $2 \leq i \leq M$:

  Among all filter banks in class $\mathcal{C}_{i-1}$, find filter banks whose ith subband variance is maximum and construct the class $\mathcal{C}_i$ by all these filter banks.
- Finally, $\mathcal{C}_M$ will contain the desired filter banks

If PCFB exists, this algorithm will find it, else it converges to a suboptimum solution. The existence of PCFB in the classes of orthonormal transforms and infinite length filter banks has previously been shown. Indeed, the ideal PCFB has the extreme performance achievable among *all* orthonormal filter banks.

Designing ideal PCFB consists of sequential design of ideal EC filters which results in non-overlapping break-wall Nyquist(M) filters (and hence, they constitute an orthonormal filter bank). The details of algorithm can be found e.g. in [5].

An example of ideal PCFB for three channel case is shown in figure 2.

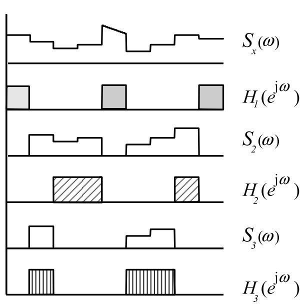


Fig. 2. Ideal three channel PCFB for PSD $S_x(\omega)$

Since it is not practical to use ideal filters, we developed a practical algorithm to find PCFB in the class of FIR filter banks. Although the sequential search algorithm may be used, due to the nonlinear constraints and cost function, it is computationally expensive and numerically unstable.

3. DESIGNING FIR PCFB

As mentioned before, principal component filter banks have a close relationship to the energy compaction filters, and

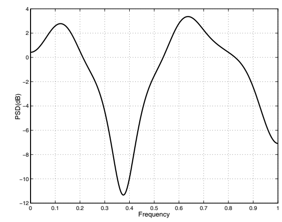
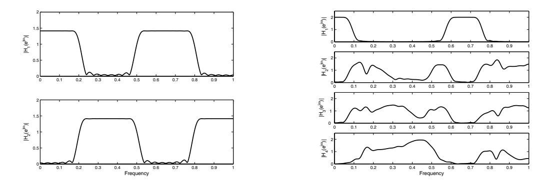


Fig. 3. The PSD used in simulations



(a) (b)

Fig. 4. Designing PCFB using method in [6],(a) M=2, L=24 and (b) M=4, L=16

their design includes designing EC filters. Many algorithms have been proposed to design EC filters, such as lattice parametrization, linear programming (LP) [6] , windowing method [7], etc. Two approaches which are simple enough and result in good solutions are linear programming and windowing.

Since, the variance of the output of filter $h(n)$ is a linear function of the product filter and imposing the Nyquist(M) constraint on the product filter is easier, both LP and windowing methods, first design the product filter and obtain the EC FIR filter using factorization of the product filter. In the LP method, the product filter is directly determined from the optimization of coefficients but in windowing method, it is derived by windowing the ideal EC (product) filter in the frequency domain.

After designing the first band, the remaining filters are found by completing the orthonormal filter bank. A simple and useful approach is based on lattice decomposition of EC filter and completing, i.e. the designed EC filter is decomposed to lattice structure and the remaining free parameters are determined using KLT [6].

In all these approaches, designing the first band nearly consumes all free parameters and only M-1 degrees of freedom remain to design the second band and finally, for the last filter, no free parameter remains. This may cause inappropriate frequency shape of the resulting filter bank (e.g. too much frequency overlap between bands, see figure 4(b)) and in the cases where PCFB does not exist, may result in filter banks with poor performance even compared with ordinary filter banks. Some design examples based on the above method for a signal with PSD in figure 3 are shown in figures 4(a) and (b).

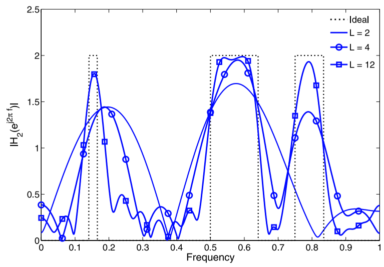
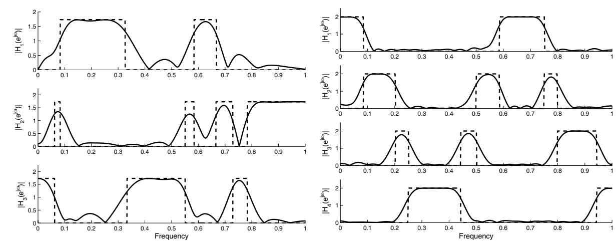


Fig. 5. The frequency responses of the second band of 4-band PCFB using sequential search method.



(a) (b)

Fig. 6. Designed PCFBs (a) M=3, L=16 (b) M=4, L=16 (solid line: Designed, dotted: Ideal PCFB)

4. PROPOSED METHOD TO DESIGN NEARLY OPTIMUM FIR FILTER BANKS

As justified in [7], to design EC filters, both windowing method and linear programming (the exact solution of EC filter design) have nearly the same results , i.e. the EC FIR filter will have nearly the same frequency response as the ideal EC filter (Its similarity increases by increasing the length of filter). We can expect that if we design FIR PCFB, it will have nearly the same response as the ideal PCFB. This suggestion is verified through simulations where the other bands of FIR (near) PCFB are designed and compared with the ideal one. In figure 4, the second band of FIR (near) PCFB is shown for various lengths. [1] As obvious from figure, by increasing the length of filter, it converges to the second band of ideal PCFB (dotted line).

As a result, instead of designing EC filters and completing it to an orthonormal filter bank, we suggest to approximate the ideal PCFB with an FIR orthonormal filter bank. Although this method will not result in exact PCFB, the frequency responses of filters will be nearly the same as the ideal ones and therefore, the subband variances will be nearly equal to those of ideal PCFB. So, with the approximated ideal filter bank, the performance of the system will be near the ideal case.

Besides, since the design parameters can be distributed among all filters arbitrarily, it can be expected that the designed filters have good frequency responses (less frequency overlap of different filters), compared to the methods based on filter bank completion.

The design algorithm can be divided into two main parts:

1. *Perfect reconstruction and orthonormality*: Although the lattice parametrization satisfies the perfect reconstruction and orthonormality constraints inherently, but it has a nonlinear relation to the frequency responses of filter banks, resulting in a nonlinear optimization problem.Here, we used a time-domain approach to force the constraints [8].

[1]Here, the direct optimization of the second band to maximize the output variance with the orthonormality constraint is employed.

2. *Cost function*: The cost function is based on the frequency responses of filters:

$$\mathcal{J} = \sum_{i=1}^{M} \alpha_i \int_{\omega\in\Omega_{p,i}} \left(|H_i(e^{j\omega})| - \sqrt{M}\right)^2 \, d\omega + \sum_{i=1}^{M} \beta_i \int_{\omega\in\Omega_{s,i}} |H_i(e^{j\omega})|^2 \, d\omega \qquad (2)$$

   where $\Omega_{p,i}$ and $\Omega_{s,i}$ are respectively the passband and stopband frequency regions of the ith ideal PCFB filter α_i and β_i are some real positive constants, weighting the importance of filters.

Therefore, the optimization problem will be minimizing $\mathcal{J}$ (equation 2) subject to the orthonormality constraint. Some design examples for the PSD in figure 3 are shown in figures 6(a) and (b).

To increase the convergence speed of the algorithm, choosing suitable initial filters is necessary. A good choice is to select the initial filters such that they have nearly the same frequency response as the ideal case. To do so, the windowing method [7] can be extended to design the ith energy compaction filter (for ith subband filter), i.e. the ith subband filter is derived using windowing the ith (product) filter of ideal PCFB. This results in a set of M Nyquist(M) filters which do not necessarily construct an orthonormal filter bank, but for long filters the error will be small. Therefore, an optimization is necessary to bring the orthonormality constraint to the filter bank.

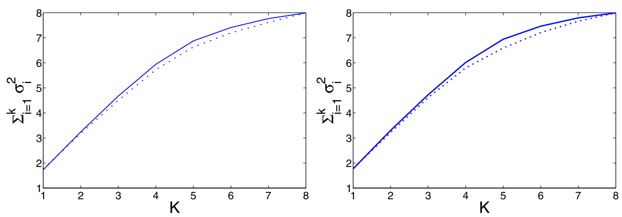
5. PERFORMANCE EVALUATION

We designed PCFB for different number of bands and lengths and compared the subbands' variances with methods based on EC design and filter bank completion [6], and the ideal case. Besides, to have a subjective comparison, we also computed $\prod \sigma_i^2$ which is related to the subband coding gain.

For two channel case, the second band is uniquely determined from the first filter and the algorithm in [6] results in

Table 1. Subband variances, $M = 8$

Algorithm	L	σ_1^2	σ_2^2	σ_3^2	σ_4^2	σ_5^2	σ_6^2	σ_7^2	σ_8^2	$\Pi\sigma_i^2$
[6]	4	1.766	1.423	1.312	1.229	0.904	0.559	0.422	0.380	0.927
Proposed	4	1.731	1.514	1.429	1.268	0.936	0.533	0.357	0.227	0.192
[6]	8	1.776	1.446	1.392	1.183	0.798	0.612	0.455	0.335	0.315
Proposed	8	1.768	1.536	1.423	1.288	0.929	0.521	0.330	0.202	0.161
Ideal	∞	1.786	1.546	1.428	1.305	0.917	0.513	0.315	0.185	0.141



(a) (b)

Fig. 7. Cumulative sum of subband variances for M=8 and L=4 (a) and L=8 (b), proposed: solid, [6]: dotted.

Table 2. Subband variances, $M = 4$

Algorithm	L	σ_1^2	σ_2^2	σ_3^2	σ_4^2	$\Pi\sigma_i^2$
[6]	16	1.646	1.020	0.765	0.567	0.728
Proposed	16	1.635	1.369	0.717	0.277	0.444
[6]	24	1.648	1.099	0.699	0.552	0.699
Proposed	24	1.640	1.370	0.721	0.267	0.432
Ideal	∞	1.647	1.382	0.717	0.252	0.411

the exact solution of PCFB. We designed nearly orthonormal (the error is negligible) 4 and 8 channel filter banks with different lengths for PSD in figure 3 and compared the subband variances (tables 1 and 2). Also, the cumulative sums of subband variances ($\sum_{i=1}^{k} \sigma_i^2$) are shown in figure 5.

As verified by simulations, although the first band in our method is not exact EC filter, the subband variances are closer to the ideal PCFB than methods based on EC design and filter bank completion, indeed, the subband variances of our method *nearly* majorizes the ones resulting from EC filter design (figure 5). Indeed, by letting the first band to violate the energy compaction criterion slightly (less than 5%), it is possible to design the other bands more appropriately and obtain a *well-behaved* filter bank, which resembles the ideal PCFB.

6. CONCLUSION

In this paper, we have introduced a new approach to design nearly optimum FIR filter banks. Since, among all orthonormal filter banks, infinite length PCFB is the optimum filter bank for most applications, and for long enough filters, the FIR PCFB tends to the ideal case, we suggested to design FIR orthonoraml filter banks such that the frequency responses of analysis filters become nearly the same as ideal PCFB. Therefore, their subband variances will be nearly equal and the designed filter bank will have a near-optimum performance as verified by simulations.

7. REFERENCES

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