DESPECKLING SAR IMAGES IN THE UNDECIMATED WAVELET DOMAIN: A MAP APPROACH

Fabrizio Argenti, Nicola Rovai, Luciano Alparone

Universitá di Firenze
Dipartimento di Elettronica e Telecomunicazioni
Via Santa Marta, 3, 50139 Firenze, Italy
e-mail: argenti@lenst.det.unifi.it

ABSTRACT

A method to despeckle SAR images based on the *maximum a posteriori* (MAP) estimation strategy in the undecimated wavelet domain is proposed. The method uses the assumption that the wavelet coefficients probability density functions (PDFs) are generalized Gaussians. The parameters of such distributions are computed by using the moments and the cumulants of the PDFs of the processes that constitute the SAR image, i.e., radar reflectivity and speckle noise. Experimental results demonstrate that the theory of MAP filtering can be successfully applied to SAR images represented in the shift-invariant wavelet domain.

1. INTRODUCTION

SAR images are usually modeled as affected by a purely multiplicative (or *fully developed*) speckle noise. This fact introduces significant difficulties in designing effective despeckling algorithms. A possible approach to despeckling is represented by *linear minimum mean square error* (LMMSE) estimation [1]. LMMSE is the optimum statistical operator when the involved signals are Gaussian, which is not the case for the processes that form a SAR image. *Maximum a posteriori* (MAP) estimates exploit the knowledge we have about statistical distributions of SAR images. An example of such technique is given in [2]. A major breakthrough in the field of despeckling is given by multiresolution processing. LMMSE and MAP approaches to SAR images despeckling in the undecimated wavelet domain have been proposed in [3] and in [4][5], respectively. The MAP criterion needs the knowledge of the probability density functions (PDFs) of the wavelet coefficients of the SAR image. The wavelet coefficients have been assumed to follow a Pearson system of distributions and a Normal inverse Gaussian distribution in [4] and in [5], respectively. Both these distributions are characterized by four parameters and are able to describe asymmetries in the histograms of the modeled variables.

In this paper, a new MAP approach to despeckling in the undecimated wavelet domain is proposed. We assume that the wavelet coefficients obey a generalized Gaussian (GG) law. GG PDFs have been commonly used to *globally* - i.e., referring to the whole subband - model the histograms of wavelet coefficients. In this paper, we conjecture that the GG assumption for wavelet coefficients holds also *locally*, with space-varying parameters. We will show how to derive such parameters based on: 1) the moments and the cumulants of the GG distributions of the wavelet coefficients, and 2) the knowledge of the PDF of the speckle-free reflectivity and of the speckle noise. The main features of the proposed MAP approach as well as some experimental results will be illustrated in the following.

2. SIGNAL MODEL OF SAR IMAGES

For the sake of clarity, we will describe the method assuming the signals as 1-D sequences. The extension to the 2-D case is straightforward. In the fully developed speckle model, the multiplicatively corrupted backscattered signal can be expressed as

$$\begin{aligned} g(n) &= f(n)\cdot u(n) = f(n) + f(n)\cdot[u(n)-1] \\ &= f(n) + f(n)\cdot u'(n) = f(n) + v(n), \end{aligned} \tag{1}$$

in which $g(n)$ and $f(n)$ are the observed noisy signal and the noise-free reflectivity, respectively, at pixel position n, and u is the fading variable modeled as a stationary random process independent of f, with $E[u(n)] = 1$. The random process $u'(n) \triangleq u(n) - 1$ is zero-mean. All the processes are assumed as *intensity* signals. The term $v(n) = f(n)\cdot u'(n)$ represents an additive, zero-mean, signal-dependent, nonstationary noise term.

3. THE UNDECIMATED WAVELET TRANSFORM

The wavelet transform provides a multiresolution representation of continuous and discrete-time signals and images [6]. For discrete-time signals, the wavelet decomposition is implemented by filtering the input signal with a lowpass filter $H_0(z)$ and a highpass filter $H_1(z)$ and downsampling the outputs by a factor 2. Applying the same decomposition to the lowpass channel output yields a two-level wavelet transform: such scheme can be iterated in a dyadic fashion to generate a multilevel decomposition. The synthesis of the signal is obtained with a scheme symmetrical to that of the analysis stage, i.e., by upsampling the coefficients of the decomposition and by lowpass and highpass filtering. It can be shown that if we omit the downsamplers from the analysis stage and the upsamplers from the synthesis stage, then perfect reconstruction can still be achieved [4][3]. We will refer to this decomposition as the *undecimated wavelet transform*. We will use the notation $A_f^{[j]}(n)$ and $W_f^{[j]}(n)$ to denote the *approximation* and the *detail* (or *wavelet*) coefficients, respectively, at the jth level of the decomposition of the signal f, whereas n is the spatial index. It can be easily shown [4][3] that the undecimated approximation and wavelet coefficients can be obtained by filtering the original signal by means of the following *equivalent filters*

$$\begin{array}{l} H_{eq,l}^{[j]}(z) = \prod_{m=0}^{j-1} H_0(z^{2^m}), \\ H_{eq,h}^{[j]}(z) = \left[\prod_{m=0}^{j-2} H_0(z^{2^m})\right] \cdot H_1(z^{2^{j-1}}). \end{array} \quad (2)$$

The aim of this study is decomposing a SAR image with an undecimated wavelet transform, estimating the noise-free reflectivity coefficients by using the MAP criterion, and reconstructing the denoised image by synthesizing the despeckled wavelet coefficients.

4. MAP FILTER IN THE WAVELET DOMAIN

Since the wavelet transform is linear, from equation (1) we have

$$\begin{array}{c} A_g^{[j]}(n) = A_f^{[j]}(n) + A_v^{[j]}(n), \\ W_g^{[j]}(n) = W_f^{[j]}(n) + W_v^{[j]}(n). \end{array} \quad (3)$$

In the following, we will consider only the wavelet coefficients, i.e., the second relationship in equation (3). To simplify the notation and whenever it does not create ambiguity, we will drop the superscript $[j]$ as well as the spatial index n. The *a posteriori* probability density function of the noise-free reflectivity wavelet coefficients conditional to the observed signal wavelet coefficients is $p(W_f|W_g)$. Hence, by applying Bayes rule, we have

$$\begin{array}{ll} p_{W_F|W_G}(W_f|W_g) & = \frac{p_{W_G|W_F}(W_g|W_f) p_{W_F}(W_f)}{p_{W_G}(W_g)} \\ & = \frac{p_{W_V|W_F}(W_g - W_f|W_f) p_{W_F}(W_f)}{p_{W_G}(g)}. \end{array} \quad (4)$$

The MAP estimate of the process W_f coincides with

$$\begin{array}{ll} \widehat{W}_f & = \text{argmax}_{W_f}\, p_{W_F|W_G}(W_f|W_g) \\ & = \text{argmax}_{W_f}\, p_{W_V|W_F}(W_g - W_f|W_f) p_{W_F}(W_f). \end{array} \quad (5)$$

By taking the logarithm, we have that $\widehat{W}_f$ is the solution of the equation

$$\frac{d}{dW_f}\left\{\ln p_{W_V|W_F}(W_g - W_f|W_f) + \ln p_{W_F}(W_f)\right\} = 0. \quad (6)$$

Finding a solution to equation (6) needs modeling the PDFs of the wavelet coefficients of the speckle-free reflectivity f and of the additive signal-dependent noise v.

5. REFLECTIVITY AND SPECKLE PDFS

5.1. Reflectivity and Speckle PDFs in the spatial domain

In this section, we consider the most commonly used PDFs to model the processes u, f, and g [7]. The variable u accounts for intensity speckle noise. If L-look processing, i.e., averaging L independent observations of the same area, is performed, then the process u is modeled as a $\Gamma(L, L)$ distribution, given by

$$p_U(u) = \frac{L^L}{\Gamma(L)} u^{L-1} e^{-uL}. \quad (7)$$

As to the PDF of the reflectivity f and of the observed process g, several distributions have been proposed based on the nature of the sounded terrain. In *heterogeneous* areas, the reflectivity of an L-look SAR image is assumed to follow a $\Gamma(\upsilon, \lambda)$ distribution, that is

$$\Gamma_X(\upsilon, \lambda) = \frac{\lambda^{\upsilon}}{\Gamma(\upsilon)} x^{\upsilon-1} e^{-\lambda x}, \quad (8)$$

where υ and λ are parameters. The former is given by $\upsilon = \mu_f^2/\sigma_f^2 = 1/C_f^2$ ($C_f = \sigma_f/\mu_f$) and measures the degree of homogeneity within the image (C_f is also referred to as *coefficient of variation*), while the latter is given by $\lambda = \upsilon/\mu_f$, so that the PDF of f can be written as

$$p_F(f) = \frac{\upsilon^{\upsilon}}{\mu_f^{\upsilon}\Gamma(\upsilon)} f^{\upsilon-1} e^{-\upsilon f/\mu_f}. \quad (9)$$

In this case, the observed image g is distributed as a K PDF. In *extremely heterogeneous* areas and nearby *point targets*, the speckle does not obey to a fully developed model. To avoid blurring of strong textures and preserve point targets as they are acquired no filtering is performed for these classes of backscatter, so that we will be mainly interested to the class of Gamma-distributed reflectivity.

5.2. Wavelet coefficients PDFs

Since the birth of the wavelet recursive algorithm by Mallat [6] a Generalized Gaussian (GG) PDF has been used to model image wavelet coefficients. The GG PDF depends only on two parameters and is characterized by being symmetric around the origin. Its expression is given by

$$p_X(x) = \left[\frac{\nu \cdot \eta(\nu, \sigma)}{2 \cdot \Gamma(1/\nu)}\right] e^{-[\eta(\nu,\sigma)\cdot|x|]^{\nu}}, \quad (10)$$

where σ is the standard deviation of the distribution, ν is the *shape factor*, and $\eta(\nu, \sigma) = 1/\sigma\sqrt{\Gamma(3/\nu)/\Gamma(1/\nu)}$.

In the following, a GG distribution is used to fit the wavelet coefficients data. An important consideration regards the context, either global or local, of validity of the model. In several studies, e.g., [6], an unique, or *global*, set of parameters is used to model the coefficients of a whole subband. In some applications, such as image compression, this is recommended not to increase too much the amount of side information. In denoising applications, in which the overhead is not a problem, the *local* estimation of the parameters allows the estimator to follow the nonstationary characteristics of the image, at the expense of an increase of the computational burden to derive pixel-wise parameters. In the following, we will make the following *conjecture*: *The wavelet coefficients obey to a GG law whose parameters locally vary.*

The next section is dedicated to the estimation of the (spatially varying) parameters (i.e., the *standard deviation* σ and the *shape factor* ν) of the GG distributions involved in the MAP equation (6), namely, the PDFs of the wavelet coefficients of the reflectivity f and of the additive signal-dependent noise term $v = f \cdot u'$.

6. ESTIMATION OF THE WAVELET COEFFICIENTS PDF PARAMETERS

6.1. Variance estimation

The variance of W_f and W_v is estimated by using the model of the signal (1) and some observable variables that can be computed from g. The complete procedure is described in [3]; only the final results are recalled here. From the signal model we have $v = f \cdot u'$, where $E[u'] = 0$. The undecimated wavelet coefficients of a signal are obtained by filtering it with the equivalent filter $h_{eq,h}^{[j]}(n)$, which will be denoted as $h(n)$ for the sake of simplicity. The mean of W_v is given by $E\left[W_v(n)\right] = \sum_i h(i)$

$\cdot E\left[f(n-i)\right] E\left[u'(n-i)\right] = 0$, where we used the fact that u' is independent of f. Hence, we have $E[W_f] = E[W_g]$, so that $E[W_f]$ can be computed from the observable variable W_g. In the filter implementation, ensemble averages are substituted by spatial averages. It can be shown [3] that the variance of W_g is given by

$$\sigma^2_{W_f}(n) = \sigma^2_{W_g}(n) - E\left[W_v^2(n)\right], \qquad (11)$$

where $E\left[W_v^2(n)\right]$ is given by

$$E\left[W_v^2(n)\right] = \frac{\sigma^2_{u'}}{1+\sigma^2_{u'}} \sum_i h(i)^2 E[g(n-i)^2]. \qquad (12)$$

By using (12) into (11), the quantity $\sigma^2_{W_f}(n)$ can be estimated from the space-varying observable variables $E[g^2(n)]$ and $E[W_g^2(n)]$, substituted by spatial averages in filter implementation.

6.2. Shape factor estimation

Several methods have been devised to estimate the shape factor of a GG distribution, see, e.g., [6], where the observation of the variable to be fitted is needed. In this paper, we will consider a method that relies only on the knowledge of the moments of the GG distribution. Since the second and fourth moment of the GG PDF are given by

$$\mu_X^{[2]} = \frac{\Gamma(3/\nu)}{\Gamma(1/\nu)\eta^2(\nu,\sigma)}, \quad \mu_X^{[4]} = \frac{\Gamma(5/\nu)}{\Gamma(1/\nu)\eta^4(\nu,\sigma)}, \qquad (13)$$

the following expression can be written

$$\frac{\mu_X^{[2]}}{\sqrt{\mu_X^{[4]}}} = \frac{\Gamma(3/\nu)}{\sqrt{\Gamma(1/\nu)\Gamma(5/\nu)}}. \qquad (14)$$

If the left hand side of (14) could be somehow estimated, this expression would yield a nonlinear equation whose solution is the estimate of ν. The *cumulants* of the PDF will be used to derive the second and fourth order moments of the desired processes, i.e., the undecimated wavelet coefficients of the reflectivity f and of the additive signal-dependent noise term v.

6.2.1. *Cumulants and moments of LTI filtered processes*

Consider a random process $x(n)$ and let $h(n)$ represent the impulse response of the equivalent filter used to achieve the wavelet coefficients in a given subband. Hence, we have

$$y \triangleq W_x(n) = \sum_{k=0}^{N-1} h(k)x(n-k) = \sum_{k=0}^{N-1} y_k, \qquad (15)$$

where $y_k \triangleq h(k)x(n-k)$. In [4], the moment generating function and the second moment generating function of a random variable are considered. They are defined by $\Phi_X(s) = E_X[e^{sx}]$ and $\Psi_X(s) = ln(\Phi_X(s))$, respectively [8]. The mth-order cumulants of the random variable X are defined as

$$\kappa_X^{[m]} = \frac{d^m}{ds^m}\Psi_X(s)\bigg|_{s=0}. \qquad (16)$$

In [4], by using the hypotheses that: 1) $x(n)$ are independent variables, and 2) they are equally distributed, an expression between the cumulants of the variables y and $x(n)$ is obtained. Here, we will use only the former hypothesis. Under this hypothesis, we can write

$$\begin{aligned}\Phi_Y(s) &= E_{Y_0}[e^{sy_0}] \cdot E_{Y_1}[e^{sy_1}] \cdot \ldots \cdot E_{Y_{N-1}}[e^{sy_{N-1}}] \\ &= \Phi_{Y_0}(s) \cdot \Phi_{Y_1}(s) \cdot \ldots \cdot \Phi_{Y_{N-1}}(s)\end{aligned} \qquad (17)$$

and then, since $y_k = h(k)x(n-k)$, we have that $\Phi_Y(s) = \prod_{k=0}^{N-1} \Phi_{X(n-k)}(h(k)s)$. Taking the logarithm of both sides of this expression yields

$$\Psi_Y(s) = \sum_{k=0}^{N-1} \Psi_{X(n-k)}(h(k)s) \qquad (18)$$

Differentiating m times and using (16), we have

$$\kappa_Y^{[m]} = \sum_{k=0}^{N-1} \kappa_{X(n-k)}^{[m]} h(k)^m = \kappa_{X(n)}^{[m]} * h(n)^m. \qquad (19)$$

Thanks to the relations that exist between moments and cumulants [8], the desired moments (the second and the fourth) of the variables of interest are given by:

$$\begin{aligned}\mu_Y^{[2]} &= \kappa_Y^{[2]} + (\kappa_Y^{[1]})^2 \\ \mu_Y^{[4]} &= \kappa_Y^{[4]} + 4\kappa_Y^{[3]}\kappa_Y^{[1]} + 3(\kappa_Y^{[2]})^2 + 6\kappa_Y^{[2]}(\kappa_Y^{[1]})^2 + (\kappa_Y^{[1]})^4\end{aligned} \qquad (20)$$

The problem of computing the mth-order moment ($m = 2$ or m=4) of the wavelet coefficients PDFs is transformed into the following procedure: 1) compute the cumulants, up to the mth-order, of the process that must be decomposed by the wavelet transform, 2) obtain the cumulants of the wavelet coefficients process by filtering the cumulant sequence using the impulse response $h(n)^m$, and 3) compute the mth-order moment of the wavelet coefficients process by using (20). Hence, the problem becomes that of computing the cumulants of the involved PDFs.

6.2.2. *Cumulants of the reflectivity f*

The reflectivity f is assumed to follow the distribution in (9). Its cumulants can be analytically derived and are given by [8]

$$\kappa_F^{[m]} = \frac{\mu_f^m}{\upsilon^{m-1}}(m-1)! = \kappa_F^{[m]}(\mu_f, \sigma_f^2), \qquad (21)$$

where we recall that $\upsilon = \mu_f^2/\sigma_f^2$. As can be seen, the cumulants depend on the (space-varying) mean and variance of $f(n)$, computable from observable variables as shown in section 6.1.

6.2.3. *Cumulants of the additive noise term $v = f \cdot u'$*

For the additive noise term v, a more complex strategy must be devised. The cumulants of the product of two random variables can not be easily derived. By assuming that f and u' are statistically independent, the expression of the moments of v is instead immediate and is given by

$$\mu_V^{[m]} = \mu_F^{[m]}\mu_{U'}^{[m]}. \qquad (22)$$

Since the cumulants of a random variable Y can be computed from its moments (the inverse of what presented in (20)) [8], the problem becomes that of computing the moments of f and u'. The moments of the Γ-distributed reflectivity are given by [8]

$$\mu_F^{[m]} = \frac{\Gamma(\upsilon+m)}{\Gamma(\upsilon)}\left(\frac{\mu_f}{\upsilon}\right)^m. \qquad (23)$$

The moments of the process $u' = u - 1$ are given by

$$\mu_{U'}^{[m]} = E[(u-1)^m] = \sum_{k=0}^{m} \begin{pmatrix} m \\ k \end{pmatrix} (-1)^k \mu_U^{[m-k]}, \qquad (24)$$

in which the moments of the process u, which is $\Gamma(L, L)$ distributed, are given by [8]

$$\mu_U^{[m]} = \frac{\Gamma(L+m)}{\Gamma(L)} \frac{1}{L^m} \qquad (25)$$

The following relationships between cumulants and moments [8], finally, yield the desired cumulants

$$\begin{aligned} \kappa_V^{[2]} &= \mu_V^{[2]} - (\mu_V^{[1]})^2 \\ \kappa_V^{[4]} &= \mu_V^{[4]} - 4\mu_V^{[3]}\mu_V^{[1]} - 3(\mu_V^{[2]})^2 + 12\mu_V^{[2]}(\mu_V^{[1]})^2 - 6(\mu_V^{[1]})^4 \end{aligned} \qquad (26)$$

7. SOLUTION OF THE MAP EQUATION

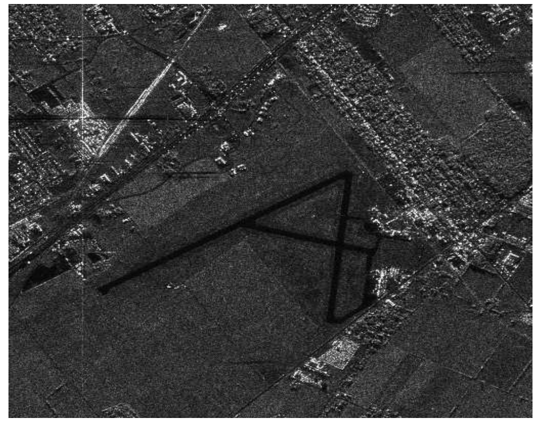
By taking the derivative of the GG PDFs given in (10), the MAP equation in (6) becomes

$$\begin{aligned} &\nu_{W_V|W_F} \cdot \eta(\nu_{W_V|W_F}, \sigma_{W_V|W_F})^{\nu_{W_V|W_F}} \\ &\cdot |W_g - W_f|^{(\nu_{W_V|W_F}-1)} \mathrm{sgn}(W_g - W_f) \\ &= \nu_{W_F} \eta(\nu_{W_F}, \sigma_{W_F})^{\nu_{W_F}} |W_f|^{(\nu_{W_F}-1)} \mathrm{sgn}(W_f) \end{aligned} \qquad (27)$$

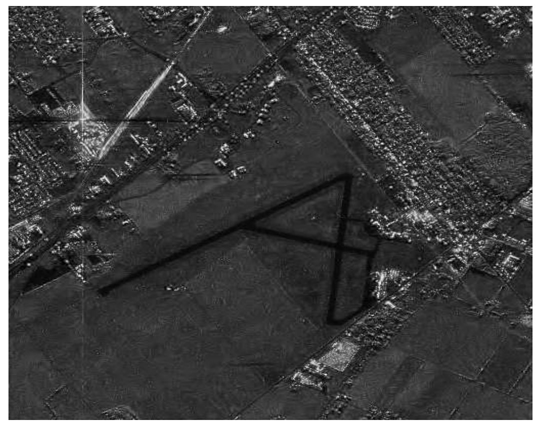
In this equation, the quantities $\nu_{W_V|W_F}$, $\eta(\nu_{W_V|W_F}, \sigma_{W_V|W_F})$, ν_F, and $\eta(\nu_{W_F}, \sigma_{W_F})$ are estimated as described in Section 6, whereas W_g is the observed wavelet coefficient. Solving the equation with respect to W_f yields the desired MAP estimation $\widehat{W}_f$. The solution is obtained numerically.

8. EXPERIMENTAL RESULTS

An example of MAP filtering is shown in Fig. 1. A 4-look SAR image has been processed by means of the MAP estimator described in the previous sections. The 8-taps Daubechies' orthogonal wavelet decomposition has been used. A comparison with the result obtained from the wavelet-based LMMSE approach [3] (not shown here due to the lack of space) has also been performed. The MAP and LMMSE methods yield comparable results in homogeneous areas, whereas the MAP estimator slightly surpasses the LMMSE one in heterogeneous areas. This justifies the fact that if there is some knowledge about the distributions of the involved processes, then MAP filtering works better than the LMMSE one.



(a)



(b)

Fig. 1. (a) Original "airport" SAR image (4-look); (b) MAP filtered image.

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