SIMPLIFIED DESIGN OF LOW-DELAY OVERSAMPLED NPR GDFT FILTERBANKS

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ABSTRACT

We propose an efficient algorithm for designing the prototype filters of oversampled, near perfect reconstruction (NPR), GDFT modulated, biorthogonal filterbanks with arbitrary delay. Given the analysis prototype, we show that the minimization of the stopband energy of the synthesis prototype, subject to NPR constraints on the frequency response of the distortion transfer function, can be expressed as a convex optimization problem. Our algorithm consists of initialization with the prototype of an orthogonal filterbank and then successive optimization of the synthesis and analysis prototypes. We compare our algorithm with previous methods and give several design examples.

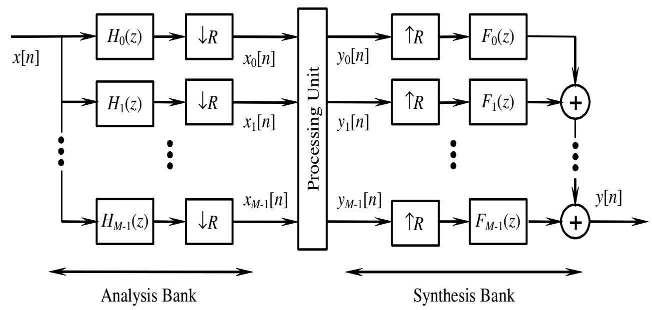

Fig. 1. M-channel filterbank.

1. INTRODUCTION

Recent studies [1, 2, 3] on subband adaptive filtering have shown that good performances and flexibility are obtained by using oversampled nearly perfect reconstruction filterbanks. Moreover, a low implementation complexity is ensured by uniform filterbanks whose filters are obtained by (complex) modulation from a single prototype; the filters have complex coefficients, but the prototype is real. In this paper, we give an efficient design algorithm for such filterbanks.

A general filterbank (FB) structure is presented in Figure 1. The FB is oversampled when the down-sampling factor R is smaller than the number of channels M. The subband signals $x_k[n]$, $k = 0 : M-1$, are processed by adaptive filtering or other type of algorithm; however, since here we are interested in a general design method, we can ignore the subband processing and assume that $y_k[n] = x_k[n]$. We assume that all analysis filters $H_k(z)$ and synthesis filters $F_k(z)$, $k = 0 : M-1$, have complex coefficients. Ideally, the passband of a filter has a width of $2\pi/M$; more precisely, the passband of $H_k(z)$ or $F_k(z)$ covers the interval $[2k\pi/M, 2(k+1)\pi/M]$, as shown in Figure 2b.

If the input signal is real, then only the first $M/2$ channels of the FB are necessary. In this case, the FB processes the frequencies between 0 and π; those from $-\pi$ to 0 are discarded; no information is lost, but the subband signals are complex. (Real subband signals could be used, but with a down-sampling factor R twice smaller, as shown in [4].)

If the output signal is a delayed version of the input one, i.e. $y[n] = x[n-D]$, where D is a positive integer, then the filterbank is perfect reconstruction (PR); theory of oversampled modulated PR FBs is discussed in [5, 4, 6]. Since subband processing changes the subband signals, near PR (NPR) filterbanks are more interesting for practical purposes; in NPR filterbanks, $y[n]$ approximates $x[n-D]$, in a sense that will be detailed later.

We consider generalized DFT (GDFT) modulated filterbanks, whose filters have the impulse responses

$$\begin{aligned} h_k[n] &= h[n]\mathrm{e}^{j\pi(2k+1)(n-D/2)/M}, \\ f_k[n] &= f[n]\mathrm{e}^{j\pi(2k+1)(n-D/2)/M}, \end{aligned} \tag{1}$$

where $h[n]$ and $f[n]$ are the impulse responses of the analysis and, respectively, the synthesis prototypes. The prototypes are FIR filters of degrees N_h and N_f, whose transfer functions are denoted $H(z)$ and $F(z)$, respectively. The idealized magnitude responses of the analysis prototype and filters are given in Figure 2. For the synthesis bank, the responses are similar. We treat the general case of biorthogonal filterbanks, in which the only relation between $H(z)$ and $F(z)$ is that imposed by the NPR condition, as detailed later in Section 2. Orthogonal GDFT filterbanks, in which $N_h = N_f$ and $f_k[n] = h_k^*[N_h - n]$, have a fixed delay, i.e. $D = N_h$. The only way to obtain simultaneously low delay and good filtering properties is to use biorthogonal filterbanks. Design methods for oversampled NPR orthogonal FBs are given in [7] (and [5, 4] in the PR case). For oversampled NPR biorthogonal FBs, design algorithms are presented in [3, 6]; we will explain later the differences between these methods and the one we propose here.

The input-output relation for the FB from Figure 1 is

$$Y(z) = T_0(z)X(z) + \sum_{\ell=1}^{R-1} T_\ell(z)X(z\mathrm{e}^{-j2\pi\ell/R}), \tag{2}$$

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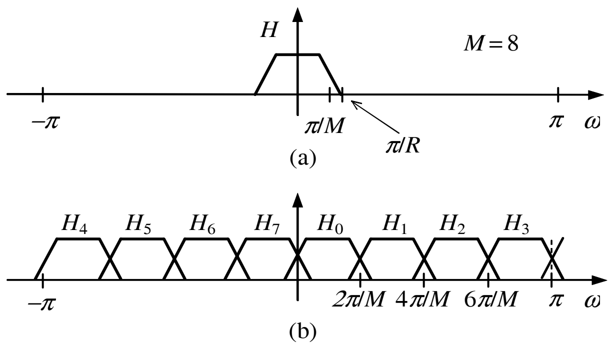

Fig. 2. Magnitude response of an eight-channel GDFT filterbank: a) prototype filter, b) analysis filters.

where

$$T_0(z) = \frac{1}{R} \sum_{k=0}^{M-1} H_k(z) F_k(z) \tag{3}$$

is the distortion transfer function and

$$T_\ell(z) = \frac{1}{R} \sum_{k=0}^{M-1} H_k(z \mathrm{e}^{-j2\pi\ell/R}) F_k(z), \tag{4}$$

for $\ell = 1 : R-1$ are called aliasing transfer functions.

2. SIMPLIFIED DESIGN PRINCIPLE

To obtain oversampled NPR FBs, we impose two conditions. The first is that the prototype filters, $H(z)$ and $F(z)$, have a magnitude response that is very small outside the baseband $[0, \pi/R]$ as suggested in Figure 2a, i.e.

$$|H(\mathrm{e}^{j\omega})| \leq \delta_s, \;\; |F(\mathrm{e}^{j\omega})| \leq \delta_s, \;\; \text{for } \pi/R \leq \omega \leq \pi. \tag{5}$$

It can be proved that the aliasing transfer functions (4) are bounded in the sense that

$$|T_\ell(\mathrm{e}^{j\omega})| \leq \delta_a, \tag{6}$$

where $\delta_a \leq \kappa\delta_s$, with κ a constant proportional with M/R. With prototypes obeying to (5), the second NPR condition is on the distortion transfer function (3), in the form

$$|T_0(\mathrm{e}^{j\omega}) - \mathrm{e}^{-jD\omega}| \leq \delta_d, \tag{7}$$

where δ_d is a conveniently chosen constant. Assuming $|X(\mathrm{e}^{j\omega})| \leq 1$, then if $\delta_s \ll \delta_d$, satisfying (5) and (7) leads to the NPR condition

$$|Y(\mathrm{e}^{j\omega}) - \mathrm{e}^{-jD\omega} X(\mathrm{e}^{j\omega})| \leq \tilde{\delta}_d, \tag{8}$$

where $\tilde{\delta}_d > \delta_d$, but practically $\tilde{\delta}_d \approx \delta_d$. A similar design principle was used for orthogonal FBs in [7, 8]; there, relation (7) had the specific form of a power complementarity constraint. Here, we generalize the principle to biorthogonal FBs.

The next result shows that the distortion transfer function has a very simple form in the case of GDFT FB. The proof is omitted; it consists of straightforward manipulations of the basic relations (1) and (3).

Theorem 1 *Consider the M-channel GDFT FB with filters defined as in (1). Let $\boldsymbol{h}$, $\boldsymbol{f}$ be the vectors containing the coefficients of the analysis and synthesis prototypes, respectively (of length N_h+1 and N_f+1, respectively). Then, the distortion transfer function (3) has the expression*

$$T_0(z) = \frac{M}{R} \sum_{i,\; 0 \leq D+iM \leq N_t} (-1)^i (\boldsymbol{h}^T \boldsymbol{\Psi}_{D+iM} \boldsymbol{f}) z^{-(D+iM)}, \tag{9}$$

where $N_t = N_h + N_f$ is the order of the distortion transfer function and $\boldsymbol{\Psi}_n$ is the elementary Hankel matrix having ones on the anti-diagonal n and zeros elsewhere (anti-diagonals are counted from zero, starting from the upper-left corner).

Remark: There are at most $(N_t+1)/M$ nonzero coefficients in the sum from (9), i.e. a very small number; this is a very favorable feature in reducing the complexity of optimization algorithms.

3. PROPOSED DESIGN ALGORITHMS

The prototypes of the GDFT FB are optimized by minimizing their stopband energy, which has favorable effects on aliasing. A good stopband ripple δ_s, as defined in (5), is obtained as a side effect by taking a stopband edge ω_s slightly smaller than π/R, namely

$$\omega_s = (1+\rho)\pi/M, \tag{10}$$

with ρ slightly smaller than $M/R - 1$. The stopband energy (of the analysis prototype) is

$$E_s = \int_{\omega_s}^{\pi} |H(\mathrm{e}^{j\omega})|^2 d\omega = \boldsymbol{h}^T \boldsymbol{\Phi} \boldsymbol{h}, \tag{11}$$

where $\boldsymbol{\Phi}$ is a positive definite Toeplitz matrix with the element ϕ_n on diagonal n defined by

$$\phi_n = \begin{cases} \pi - \omega_s, & \text{if } n = 0, \\ -\frac{\sin(\pi\omega_s n)}{\pi n}, & \text{otherwise.} \end{cases} \tag{12}$$

We implement the simplified NPR constraint (7), using the expression (9) of the distortion transfer function, in the form

$$\left\| \frac{M}{R} \begin{bmatrix} \boldsymbol{h}^T \boldsymbol{C}(\omega) \boldsymbol{f} \\ \boldsymbol{h}^T \boldsymbol{S}(\omega) \boldsymbol{f} \end{bmatrix} - \begin{bmatrix} \cos\omega D \\ \sin\omega D \end{bmatrix} \right\| \leq \delta_d, \;\; \text{for } \omega \in \Omega, \tag{13}$$

where Ω is a discrete grid of frequencies and

$$\begin{aligned} \boldsymbol{C}(\omega) &= \textstyle\sum_i (-1)^i \boldsymbol{\Psi}_{D+iM} \cos\omega(D+iM), \\ \boldsymbol{S}(\omega) &= \textstyle\sum_i (-1)^i \boldsymbol{\Psi}_{D+iM} \sin\omega(D+iM). \end{aligned}$$

3.1. Orthogonal FB

In an orthogonal FB, the synthesis prototype vector $\boldsymbol{f}$ contains the coefficients of the analysis prototype $\boldsymbol{h}$, in reverse order. Then, the next equality holds,

$$\boldsymbol{h}^T \boldsymbol{\Psi}_n \boldsymbol{f} = \boldsymbol{h}^T \boldsymbol{\Theta}_n \boldsymbol{h},$$

where $\boldsymbol{\Theta}_n$ is the elementary Toeplitz matrix with ones on the diagonal n and zeros elsewhere. The problem of minimizing the energy (11) subject to the NPR constraints (13) is equivalent to a convex optimization problem, in terms of the coefficients of the product filter $H(z)H(z^{-1})$; see [9] for details on this type of transformation, that leads to a semidefinite programming (SDP) problem. The same idea has been used in [7], but with a time-domain constraint on the coefficients of the distortion transfer function (9). We remark that each frequency domain constraint of type (13) has a second order cone (SOC) form; SOC is a particular case of SDP, with faster implementation.

Table 1. Outline of proposed algorithm for designing oversampled NPR biorthogonal filterbanks.

Input: M - the number of channels, D - FB delay, R - the downsampling factor, ω_s - the stopband edge, δ_d - the maximum NPR error from (7), N_h, N_f - the orders of the analysis and synthesis prototypes, N_0 - the order of the analysis prototype for the first iteration.

1. Design oversampled NPR orthogonal FB, with prototype $H(z)$ of order N_0, as described in Section 3.1.
2. Solve the SOC problem (15) for $F(z)$ of order N_f, using $H(z)$ computed at step 1.
3. Reversing the roles of $\boldsymbol{h}$ and $\boldsymbol{f}$, solve the SOC problem (15) for $H(z)$ of order N_h, using $F(z)$ computed at step 2.

3.2. Biorthogonal FB

Let us assume for the beginning that the analysis prototype $H(z)$ is given. We next show that the optimization of the synthesis prototype has a convex formulation, as a SOC problem. Minimizing the stopband energy of $F(z)$ subject to the NPR conditions (13), i.e.

$$\begin{array}{ll} \min_f & \boldsymbol{f}^T \boldsymbol{\Phi} \boldsymbol{f} \\ \text{subject to} & (13) \end{array} \tag{14}$$

can be expressed in the form

$$\begin{array}{ll} \min\limits_{f,\alpha} & \alpha \\ \text{s.t.} & \|\boldsymbol{\Phi}^{1/2}\boldsymbol{f}\| \le \alpha \\ & \left\| \dfrac{M}{R} \begin{bmatrix} \boldsymbol{c}^T(\omega) \\ \boldsymbol{s}^T(\omega) \end{bmatrix} \boldsymbol{f} - \begin{bmatrix} \cos\omega D \\ \sin\omega D \end{bmatrix} \right\| \le \delta_d, \quad \text{for } \omega \in \Omega \end{array} \tag{15}$$

where $\boldsymbol{c}(\omega) = \boldsymbol{C}(\omega)\boldsymbol{h}$ and $\boldsymbol{s}(\omega) = \boldsymbol{S}(\omega)\boldsymbol{h}$ are constant vectors for a given frequency ω. The optimization problem (15) is a typical SOC problem. Since this is a convex optimization problem, the solution is unique.

Certainly, if $F(z)$ is given, then $H(z)$ can be optimized similarly. We propose the following idea for the design of a biorthogonal FB. First, an orthogonal FB is designed, using the algorithm suggested in Section 3.1. The obtained prototype, $H(z)$ is used as analysis prototype and a synthesis prototype $F(z)$ is computed by solving the SOC problem (15). Finally, a new analysis prototype is obtained solving the problem similar to (15) for given $F(z)$ and unknown $H(z)$. Theoretically, this procedure could be continued, but we have noticed that usually no significant improvement is obtained. The algorithm is described formally in Table 1. We note that the initial prototype (for the orthogonal FB) may have an order N_0 different from N_h; in our experiments, we have remarked that it is beneficial to take $N_0 \le N_h$, $N_0 \le N_f$. This three-step algorithm is useful especially when we desire prototypes $H(z)$ and $F(z)$ with similar orders and performance. Otherwise, using only the first two steps of the algorithm (with $N_0 = N_h$) gives also good results.

Our algorithm differs from those in [6, 7] in several features. We offer the possibility of controlling the NPR error δ_d at all frequencies (in [6, 7], the synthesis prototype is optimized using a composite criterion, combining stopband energy and an integral NPR error). Moreover, designing the initial analysis prototype as part of an orthogonal FB appears to be more meaningful than the use of the Remez algorithm in [6] or the approximate linear phase filter from [7]. Finally, we remark that our algorithm imposes no apriori relation between the delay D and the orders N_h, N_f.

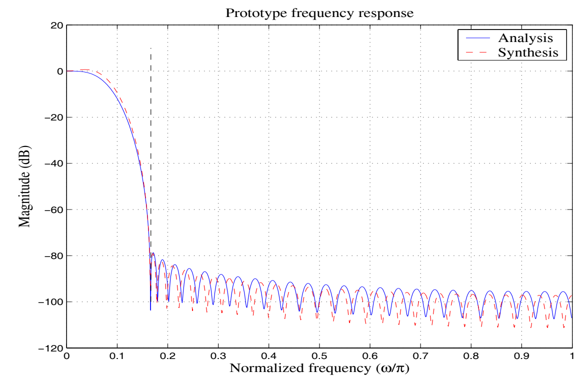

Fig. 3. Frequency responses of prototypes in Example 1.

4. DESIGN EXAMPLES

The algorithm described in the previous section has been implemented using the SDP library SeDuMi [10]. We give here three examples of design.

Example 1. The design data are taken as in the first example of NPR FB from [6], i.e. $M = 16$ (8 channels are used for real signals), $R = 6$, $N_h = 53$, $N_f = 59$. We take the delay $D = 40$ (in [6], the delay appears to be 59). The stopband edge ω_s from (10) is defined by $\rho = 1.6$ (value slightly smaller than $M/R-1 = 1.667$). The distortion transfer function bound from (7) is $\delta_d = 0.01$; in [6], where this parameter is not directly controlled, the amplitude distortion of the FB is more than 0.04. With $N_0 = 51$, we have obtained the prototypes whose frequency responses are shown in Figure 3. The dashed line marks the frequency π/R, i.e. the edge of the baseband. The attenuations outside the baseband are $A_h = 78.7$dB for the analysis prototype and $A_f = 79.1$dB for the synthesis prototype. In [6], these values are less than 60dB (the analysis prototype is an equiripple filter), so our FB is clearly better.

Example 2. The complex filterbank has $M = 64$ channels (only 32 used for real signals). The delay is $D = 80$. The downsampling factor R is 16. Accordingly, the stopband edge ω_s is defined by $\rho = 2.9$ (while $M/R - 1 = 3$). The distortion transfer function bound is $\delta_d = 0.003$ (about 50dB). We want prototypes that have similar orders ($|N_h - N_f| \le 4$) and at least 60dB stopband attenuation (i.e. $\delta_s = 0.001$ in (5)).

To obtain such prototypes, we have run our algorithm for different orders N_0, N_h and N_f. We report the FBs with smallest order satisfying the requirements. We have obtained $N_h = 96$, $N_f = 94$ (with $N_0 = 76$). The frequency responses of the analysis and synthesis prototypes are shown in Figure 4. The attenuations outside the baseband are at least $A_h = 60.5$dB and $A_f = 60.0$dB.

The initial order N_0 is not a critical parameter. We show in Table 2 the values of the attenuations A_h and A_f and also of the stopband energies E_h and E_f (for the analysis and synthesis prototypes, respectively), as well as their averages, for $N_h = 96$, $N_f = 94$ and different values of N_0. It is clear that the averages are approximately constant; hence, the parameter N_0 is interesting

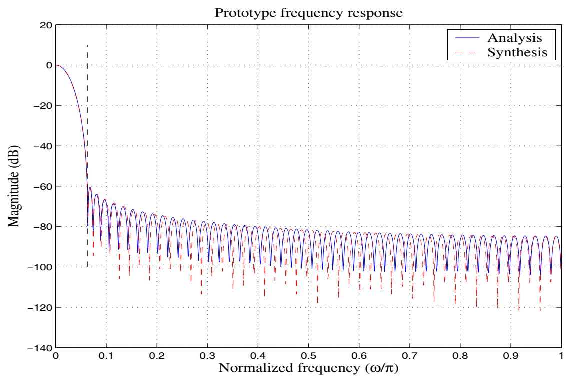

Fig. 4. Frequency responses of prototypes in Example 2.

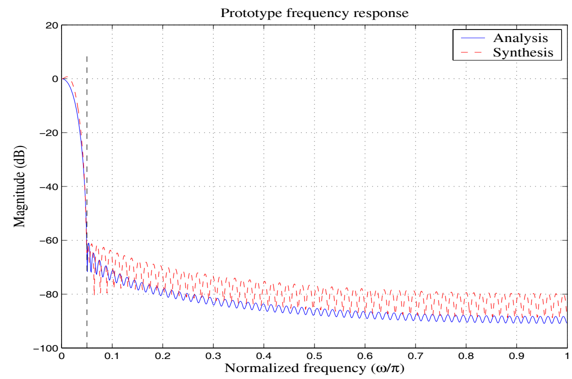

Fig. 5. Frequency responses of prototypes in Example 3.

Table 2. Attenuations and stopband energies for prototypes obtained with different initial orders N_0 (with $N_h = 96$, $N_f = 94$).

N_0	A_h	A_f	$\frac{A_h+A_f}{2}$	E_h	E_f	$\frac{E_h+E_f}{2}$
64	58.37	63.10	60.73	3.1e-8	0.7e-8	1.93e-8
68	59.24	60.88	60.06	2.4e-8	1.3e-8	1.89e-8
72	59.99	60.45	60.22	2.0e-8	1.6e-8	1.81e-8
76	60.50	60.02	60.26	1.8e-8	1.9e-8	1.81e-8
80	60.72	59.88	60.30	1.6e-8	2.0e-8	1.82e-8
84	60.65	59.93	60.29	1.7e-8	1.9e-8	1.82e-8
88	60.72	59.87	60.30	1.6e-8	2.0e-8	1.82e-8
92	60.86	59.73	60.30	1.6e-8	2.1e-8	1.84e-8

only if one desires to match the orders and performances of the two prototypes. The design time for each such FB is about 6 seconds on a Pentium III PC at 1GHz.

Example 3. The only parameters changing values with respect to Example 2 are $R = 20$ and $\rho = 2.1$ (while $M/R - 1 = 2.2$). The frequency responses of the prototypes are shown in Figure 5. The orders are $N_h = 130$ and $N_f = 134$ (with $N_0 = 124$). The attenuations are $A_h = 61.0$dB, $A_f = 61.3$dB and the stopband energies $E_h = 1.17$e-8 and $E_f = 5.72$e-8. The execution time for obtaining the FB is about 17 seconds.

5. CONCLUSIONS

We have proposed an algorithm for the design of oversampled NPR GDFT modulated filterbanks. The algorithm consists of solving an SDP problem for designing the prototype of an orthogonal FB (used as initialization) and then solving two SOC problems (15), for obtaining first the synthesis, then the analysis prototype; each time, the other prototype is the one given by the solution of the previous problem. The algorithm not only gives good FBs, but allows obtaining analysis and synthesis prototypes with similar degrees and stopband attenuations (or energies).

Continuing the current work, we will study methods for designing *nonuniform* low-delay oversampled FBs.

6. REFERENCES

[1] M. Harteneck, S. Weiss, and R.W. Stewart, "Design of Near Perfect Reconstruction Oversampled Filter Banks for Subband Adaptive Filters," *IEEE Trans. Circ.Syst. II*, vol. 46, no. 8, pp. 1081–1085, Aug. 1999.

[2] J.P. Reilly, M. Wilbur, M. Seibert, and N. Ahmadvand, "The Complex Subband Decomposition and its Applications to the Decimation of Large Adaptive Filtering Problems," *IEEE Trans. Signal Proc.*, vol. 50, no. 11, pp. 2730–2473, Nov. 2002.

[3] J.M. de Haan, N. Grbic, I. Claesson, and S.E. Nordholm, "Filter Bank Design for Subband Adaptive Microphone Array," *IEEE Trans. Speech Audio Proc.*, vol. 11, no. 1, pp. 14–23, Jan. 2003.

[4] H. Bölcskei and F. Hlawatsch, "Oversampled Cosine Modulated Filter Banks with Perfect Reconstruction," *IEEE Trans. Circ.Syst. II*, vol. 45, no. 8, pp. 1057–1071, Aug. 1998.

[5] J. Kliewer and A. Mertins, "Oversampled Cosine-Modulated Filter Banks with Arbitrary System Delay," *IEEE Trans. Signal Proc.*, vol. 46, no. 4, pp. 941–955, April 1998.

[6] K. Eneman and M. Moonen, "DFT Modulated Filter Bank Design for Oversampled Subband Systems," *Signal Processing*, vol. 81, pp. 1947–1973, 2001.

[7] M.R. Wilbur, T.N. Davidson, and J.P. Reilly, "Efficient Design of Oversampled NPR GDFT Filterbanks," *IEEE Trans. Signal Proc.*, vol. 52, no. 7, pp. 1947–1963, July 2004.

[8] Z. Cvetkovic and J.D. Johnston, "Nonuniform Oversampled Filter Banks for Audio Signal Processing," *IEEE Trans. Speech Audio Proc.*, vol. 11, no. 5, pp. 393–399, Sept. 2003.

[9] B. Dumitrescu, I. Tăbuş, and P. Stoica, "On the Parameterization of Positive Real Sequences and MA Parameter Estimation," *IEEE Trans. Signal Proc.*, vol. 49, no. 11, pp. 2630–2639, Nov. 2001.

[10] J.F. Sturm, "Using SeDuMi, a Matlab Toolbox for Optimization over Symmetric Cones," *Optimization Methods and Software*, vol. 11-12, pp. 625–653, 1999, http://fewcal.kub.nl/sturm/software/sedumi.html.