CHANNEL ESTIMATION USING TIME-FREQUENCY TECHNIQUES

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ABSTRACT

We propose the use of time-varying signals and time-frequency techniques to estimate fast fading channels with unknown state information. Specifically, we apply the reassigned spectrogram to estimate multiple sinusoids resulting from transmitting and processing two linear frequency-modulated (FM) chirp pilots. The estimation procedure is complicated by the need to pair sinusoids resulting from the same multipath. This problem is overcome using a single nonlinear FM chirp pilot with power characteristics. The channel estimation simplifies to estimating linear FM chirps which we perform using the modified matching pursuit decomposition algorithm. Simulations demonstrate the increased estimation performance when pilot signals with nonlinear structures are used instead of linear structured ones.

1. INTRODUCTION

Time-varying (TV) signals such as linear or nonlinear frequency-modulated (FM) chirps have spectral properties that vary with time. They are bandwidth-efficient signals that are robust to interference and system distortions. Linear FM (LFM) chirps have been extensively used in wireless communication applications [1]. For example, they have been used for equalization [2], for combating interference [3], for multipath diversity [4], and for modulation in multiple access schemes [5].

Many techniques have been introduced in the literature for estimating channel parameters based on both pilot sequences as well as blind techniques. Recently, LFM chirp pilots have been used for estimating the parameters of TV channels in [6]. The proposed technique reduced the estimation of the channel parameters to the estimation of the parameters of multiple sinusoidal signals.

In this paper, we extend the work in [6] by providing an implementation of the method using the periodogram [7] for slow varying multipath channels and using time-frequency (TF) techniques such as the reassigned spectrogram [8] for fast varying TV channels. We also propose a new TV channel estimation approach based on power FM chirp (PFM) signals [9]. The nonlinear instantaneous frequency (IF) nature of these signals in the TF plane allows for reduced complexity channel estimation. This follows since we need to estimate the parameters of L LFM chirp signals instead of $2L$ sinusoids without having to pair parameters resulting from the same paths. We perform the estimation using the modified matching pursuit decomposition [10, 11] that is well-matched to LFM chirps.

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2. TIME-VARYING CHANNEL MODEL

Fast fading wireless channels can be represented using the TV impulse response $h(t,\tau)$ [12]. Specifically, the resulting output signal is given by $r(t) = \int h(t,\tau)s(t-\tau)d\tau + w(t)$ where $s(t)$ is the transmitted signal and $w(t)$ is additive white Gaussian noise (AWGN). The impulse response can be modeled as

$$h(t,\tau) = \sum_{l=0}^{L-1} h_l \, e^{j2\pi\lambda_l t} \, \delta(\tau - \tau_l) \tag{1}$$

to demonstrate the amplitude fading coefficient $h_l = A_l e^{j\varphi_l}$, multipath delay τ_l and Doppler shift λ_l, $l = 0, \ldots, L-1$, of the resulting L multipath components. Here, $\delta(\cdot)$ is the Dirac delta function. The output can thus be expressed as

$$r(t) = \sum_{l=0}^{L-1} h_l \, e^{j2\pi\lambda_l t} s(t-\tau_l) + w(t)\,. \tag{2}$$

Although the commonly used tapped delay line model uses discretized time shifts $\tau_l = l/W$ (where W is the signal bandwidth) [12], we consider the more general case of real-valued τ_l.

Note that, for slow varying channels, we can assume that the Doppler shifts in (1) are negligible and the impulse response can simplify to $h(t,\tau) = \sum_{l=0}^{L-1} h_l \, \delta(\tau - \tau_l)$.

3. LINEAR FM BASED ESTIMATION

3.1. Estimation Algorithm

The algorithm presented in [6] aims to estimate the channel parameters τ_l, λ_l and h_l in (1) of a TV channel as follows. Let $s_p^{(i)}(t) = e^{j2\pi c_p^{(i)} t^2}$, for $i = 1, 2$, be two LFM chirp pilots transmitted in two consecutive symbol durations (during which the channel is assumed constant). We multiply the ith received signal $r_p^{(i)}(t)$ (the signal in (2) with $s(t)$ replaced with $s_p^i(t)$) with $s_p^{(i)*}(t)$, $i = 1, 2$, to obtain

$$y_i(t) = r_p^{(i)}(t)s_p^{(i)*}(t) = \sum_{l=0}^{L-1} \tilde{h}_l^{(i)} e^{j2\pi f_l^{(i)} t} + \tilde{w}_i(t)\,, \quad (3)$$

where $\tilde{w}_i(t) = w_i(t)s_p^{(i)*}(t)$ and $\tilde{h}_l^{(i)} = h_l e^{j2\pi c_p^{(i)}\tau_l^2}$. Equation (3) yields two sets of L sinusoids each with frequencies $f_l^{(i)} = \lambda_l - 2c_p^{(i)}\tau_l$ for $i = 1, 2$ and $l = 0, \ldots, L-1$. By solving the equations of the frequencies, we can obtain the estimated time delay and Doppler shift of the lth path as [6]

$$\begin{aligned} \hat{\tau}_l &= (f_l^{(1)} - f_l^{(2)})/(2(c_p^{(2)} - c_p^{(1)})) \\ \hat{\lambda}_l &= (f_l^{(1)}c_p^{(2)} - f_l^{(2)}c_p^{(1)})/(c_p^{(2)} - c_p^{(1)})\,. \end{aligned} \quad (4)$$

Note that we can choose the FM rates of the pilots to be $c_p^{(1)} = B/(2T) = -c_p^{(2)}$, without loss of generality, where T is the pilot duration and B is the available bandwidth. The estimated amplitude $\tilde{h}_l$ of the sinusoids can yield an estimate of the fading coefficients h_l.

3.2. Implementation Using the Reassigned Spectrogram

The above procedure is described in [6], however no implementation algorithm is given for estimating the $2L$ sinusoids in (3). And, although there exist many spectral estimation techniques [7], this particular problem is complicated by the fact that although we use the index l to represent the lth path, we do not have a way of correctly pairing the frequencies $f_l^{(1)}$ and $f_l^{(2)}$ to guarantee that they both resulted from the same lth path. To achieve this, we use the fact that the corresponding two sinusoids with frequencies $f_l^{(1)}$ and $f_l^{(2)}$ have the same starting time τ_l in the TF plane. Thus, we propose to use a very highly localized TF representation, the reassigned spectrogram (RSP) [8], to achieve the pairing of the frequencies as follows.

We first compute the RSPs $R_{y_i}(t, f)$ of $y_i(t)$ for $i = 1, 2$, in (3). Each RSP consists of L highly concentrated lines in the TF plane that are centered around the L sinusoidal frequencies and start at the time delay τ_l of the lth path. To obtain the estimated frequencies, we calculate the line integral of $R_{y_i}(t, f)$ along the horizontal time axis, which results in $V_{y_i}(f) = \frac{1}{T}\int_0^T R_{y_i}(t, f)dt$. By finding the location of the L peak values of $V_{y_i}(f)$, we can obtain L frequencies that we label $\Upsilon_{l'}^{(i)}$, for $l' = 0, \ldots L-1$. We then use these frequencies to compute $R_{y_i}(t, \Upsilon_{l'}^{(i)})$ from $R_{y_i}(t, f)$; this is a time function for each frequency $\Upsilon_{l'}^{(i)}$. The starting times of $R_{y_i}(t, \Upsilon_{l'}^{(i)})$ correspond to the different time delays that we are trying to estimate. Since the two sinusoids in $y_1(t)$ and $y_2(t)$ corresponding to the same path of the channel have the same starting time and the same ending time T (but not the same frequency), they also have the same period $T_{l'}$. Therefore, we can match the two sinusoids into a pair according to finding $T_{l'}^{(1)}$ and $T_{l'}^{(2)}$ that are closest in value. For each $R_{y_i}(t, \Upsilon_{l'}^{(i)})$, $i = 1, 2$ and $l' = 0, \ldots, L-1$, we set up a threshold $\mathrm{th}_{l'}^{(i)} = \frac{1}{T}\int_0^T R_{y_i}(t, \Upsilon_{l'}^{(i)})dt$. Then, the total time duration of $R_{y_i}(t, \Upsilon_{l'}^{(i)})$ that exceeds the threshold $\mathrm{th}_{l'}^{(i)}$ can be an approximate estimate of $T_{l'}^{(i)}$. To realize the matching, we arrange $T_{l'}^{(1)}$, $l' = 0, \ldots, L-1$ in decreasing order of magnitude, and repeat the same arrangement for $T_{l'}^{(2)}$. We then relabel the reordered sets as $\hat{T}_l^{(i)}$, $i = 1, 2$, $l = 0, \ldots, L-1$ to correspond to the lth path. We also reorder the two sets of frequencies $\Upsilon_{l'}^{(i)}$ to follow the order of $\hat{T}_l^{(i)}$, and relabel the reordered frequencies as $\hat{f}_l^{(i)}$, $i = 1, 2$, $l = 0, \ldots, L-1$. At last, we can calculate the estimates of the time delays and Doppler shifts the lth path according to (4) using the new ordered set of frequencies $\hat{f}_l^{(i)}$, $i = 1, 2$, $l = 0, \ldots, L-1$.

The fading coefficients for the lth path can be obtained using these estimated values and maximum likelihood estimation techniques.

3.3. Slowly varying multipath channel

When the channel is slow varying, there is no need to estimate Doppler shifts. Thus, the estimation algorithm only requires one LFM chirp pilot with FM rate c_p in (3) to estimate the resulting lth multipath delay parameter as $\hat{\tau}_l = -f_l/(2c_p)$. As a result, the implementation procedure does not have a pairing problem, and the estimation of the parameters of the L sinusoids can be performed using the periodogram as we presented in [4].

4. NONLINEAR FM BASED ESTIMATION

As described above, to estimate the parameters of a TV channel with L paths, we require two LFM chirp pilots and the estimation of $2L$ sinusoids, further complicated by the need to pair sinusoids with the same starting times. Here, we propose to use a single nonlinear PFM pilot signal to estimate the channel more efficiently.

4.1. Estimation Algorithm

The PFM chirp, $s_p(t) = e^{j2\pi\beta_p t^3}$, $t \in [0, T)$, is a TV signal with FM rate β_p and IF $f_{IF}(t) = 3\beta_p t^2$ that corresponds to a parabola in the TF plane. We choose the FM rate as $\beta_p = B/(3T^2)$ to make full use of the available bandwidth B. After transmitting a single PFM pilot, we multiply the received signal $r_p(t)$ in (2) with $s_p^*(t)$ to obtain a linear combination of L LFM chirp signals,

$$y(t) = r_p(t)s_p^*(t) = \sum_{l=0}^{L-1} \gamma_l\, e^{j2\pi c_l t^2}\, e^{j2\pi f_l t} + \tilde{w}(t)\,, \quad (5)$$

with LFM rates c_l, frequency shifts f_l and amplitudes γ_l given by

$$c_l = -3\beta_p\tau_l,\ f_l = \lambda_l + 3\beta_p\tau_l^2,\ \gamma_l = h_l e^{-j2\pi\beta_p\tau_l^3} \tag{6}$$

and $\tilde{w}(t) = w(t)e^{-j2\pi\beta_p t^3}$. Thus, to estimate the channel, it is sufficient to estimate the LFM parameters in (6) since

$$\hat{\tau}_l = -\frac{\hat{c}_l}{3\beta_p},\ \hat{\lambda}_l = \hat{f}_l + \frac{\hat{c}_l^2}{3\beta_p},\ \ \hat{h}_l = \hat{\gamma}_l e^{j2\pi\beta_p\hat{\tau}_l^3}\,. \tag{7}$$

4.2. Algorithm Implementation

To estimate the L LFM chirp parameters in (5) and (6), we use the modified matching pursuit (MMP) technique [10, 11]. This is an iterative algorithm that can be used to expand any signal into a linear weighted and parsimonious sum of waveforms that are taken from a redundant dictionary that is built to match the signal. For this application, we form our dictionary using a basic LFM chirp signal, $q_\alpha(t) = e^{j2\pi\alpha t^2}$, which we translate in the TF plane to allow for all possible chirp signals. Note that to limit our search of LFM chirps, we limit our translations based on assumed knowledge of the channel's maximum time delay and Doppler shift.

The basic steps of the algorithm are described next. As the duration of $s_p(t)$ is T and the lth received path in (1) is delayed by τ_l, we can rewrite the product signal in (5) as $y(t) = \sum_{l=0}^{L-1} \gamma_l\, e^{j2\pi c_l t^2} e^{j2\pi f_l t}[u(t-\tau_l) - u(t-T)] + \tilde{w}(t)$ where $u(t)$ is the unit step function. Thus, the lth LFM chirp is defined for $t \in (\tau_l, T)$. Then, we form the dictionary by transforming the basic dictionary element as

$$g(t;\underline{\theta}) = (\mathcal{H}_\tau \mathcal{F}_f \mathcal{G}_c\, q_\alpha(t)) = \frac{1}{T-\tau} e^{j2\pi(\alpha+c)t^2}\, e^{j2\pi f t} z(t)$$

where $z(t) = u(t-\tau) - u(t-T)$, $\underline{\theta} = [c, f, \tau]$, $c, f \in \mathbb{R}$ and $\tau \in \mathbb{R}^+$. The operator $(\mathcal{G}_c\, q)(t) = q(t)e^{j2\pi c t^2}$ results in a shift c of the FM rate, $(\mathcal{F}_f\, q)(t) = q(t)e^{j2\pi f t}$ causes a frequency shift f, and $(\mathcal{H}_\tau\, q)(t) = \frac{1}{T-\tau} q(t)z(t)$ causes a windowing within $(T-\tau)$. The energy of $g(t;\underline{\theta})$ is restricted to be unity for every $\underline{\theta}$ to ensure energy preservations [10].

The MMP first projects $y(t) \stackrel{\triangle}{=} (\mathcal{R}_0 y)(t)$ onto each element of the dictionary, and selects $g(t;\underline{\theta}_0)$ which satisfies

$$|\langle y, g(\underline{\theta}_0)\rangle| = \max_{\underline{\theta}} |\langle y, g(\underline{\theta})\rangle| \tag{8}$$

where $\langle y, g(\underline{\theta})\rangle = \int_\tau^T y(t) g^*(t;\underline{\theta}) dt$ and $\underline{\theta}_0$ is the parameter vector of the first selected element. Equation (8) ensures that the signal component $g(t;\underline{\theta}_0)$ with the highest energy is extracted first. This results in $y(t) = \rho_0 g(t;\underline{\theta}_0) + (\mathcal{R}_1\, y)(t)$ with the expansion coefficient $\rho_0 = \langle y, g(\underline{\theta}_0)\rangle$. For the lth iteration, we compute $\rho_l = \langle \mathcal{R}_l\, y, g(\underline{\theta}_l)\rangle$ where $(\mathcal{R}_l\, y)(t) = y(t) - \sum_{l'=0}^{l-1} \rho_{l'}\, g(t;\underline{\theta}_{l'})$. As we are only estimating L LFM chirps, we run only L MMP iterations to obtain

$$y(t) = \sum_{l=0}^{L-1} \rho_l\, g(t;\underline{\theta}_l) + (\mathcal{R}_L\, y)(t)\,. \tag{9}$$

Thus, $\underline{\theta}_l = [\hat{c}_l, \hat{f}_l, \hat{\tau}_l]$ provides the estimates of the parameters of one of the L multipaths. It will correspond to the parameters of the lth path if we reorder $\underline{\theta}_l$ and ρ_l according to increasing values of $\hat{\tau}_l$. Finally, ρ_l is directly the estimate $\hat{\gamma}_l$ from which we can obtain $\hat{h}_l$ using (7).

The effectiveness of the MMP algorithm is demonstrated next using a PFM chirp pilot with duration $T = 1$ ms and a TV channel with maximum Doppler shift $f_m = 1$ kHz and $L = 3$ multipath components with corresponding time delays $\tau_0 = 0.2$ ms, $\tau_1 = 0.5$ ms and $\tau_2 = 0.7$ ms. Using (9), $y(t)$ in (5) can be decomposed into $L = 3$ LFM chirps that can be represented in the TF plane using the TF representation (TFR)

$$T_y(t,f) = \sum_{l=0}^{L-1} |\rho_l|^2\, \mathrm{WD}_{g(\theta_l)}(t,f)\,. \tag{10}$$

This TFR consists of weighted Wigner distributions (WD), $\mathrm{WD}_{g(\theta_l)}(t,f) = \int g(t+t'/2;\theta_l)\, g^*(t-t'/2;\theta_l) e^{-j2\pi t' f} dt'$ [9], of the decomposed elements in (9) [10, 11]. As the WD of an LFM chirp is a line with slope proportional to its FM rate in the TF plane, the resulting TFR in (10) is depicted in Fig. 1(a) showing the three LFM chirps in $y(t)$ with different slopes and starting points.

5. SIMULATIONS

To demonstrate the performance of the two estimation techniques, we simulate a TV wireless channel with a varying number L of paths using the filtered Gaussian noise model for the fading coefficients, minimum time delay difference of 0.1 ms and maximum Doppler shift of 1 kHz. We considered a single user transmission using LFM modulation that was designed as in [4] to yield multipath diversity. We use symbols with duration $T = 1$ ms and transmission bandwidth $B = 100$ kHz. We ran 1,000 Monte Carlo simulations and transmitted 100 information bits for each simulation. We consider bit error rate (BER) performance for different average signal-to-noise ratios (SNRs) per bit.

In Fig. 1(b), we use two LFM pilots every[1] 11 transmission symbols and the RSP algorithm in Section 3 to estimate the channel parameters. The poor BER performance observed can be due to the pairing of the estimated sinusoids corresponding to the same path. To improve this performance, we estimate the same channel using a single PFM pilot every 11 transmission symbols following the

[1]The number 11 was chosen arbitrarily. Better performance is expected if the pilots are transmitted more often.

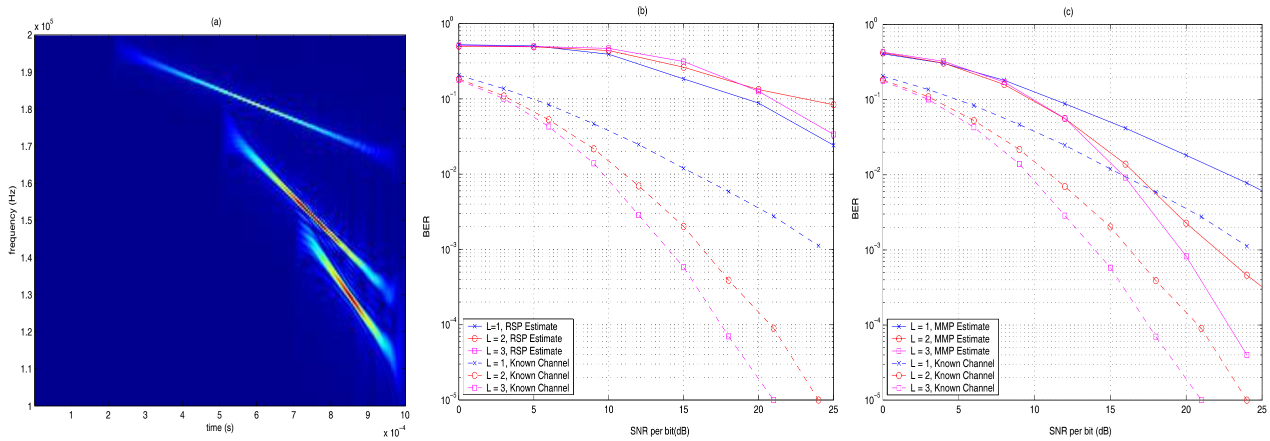

Fig. 1. (a) The MMP TFR in (10) of $y(t)$ in (5) for $L = 3$. Performance of the LFM chirp modulation with (b) two LFM pilots and RSP estimation, and (c) one PFM pilot and MMP estimation of a TV channel.

MMP estimation method proposed in Section 4. As it can be observed in Fig. 1(c), the BER performance has increased from the corresponding one in Fig. 1(b). For example, to achieve a BER performance of 10^{-2} for $L = 2$, the MMP estimation method requires about 5.5 dB more SNR than when the channel is known, whereas the RSP estimation technique cannot achieve this BER performance when the SNR is lower than 25 dB.

6. CONCLUSION

In this paper, we presented an implementation approach for the TV channel estimation algorithm in [6] using the reassigned spectrogram TF representation. The algorithm estimates sinusoidal parameters and benefits from the high localization properties of the reassigned spectrogram. However, the estimation performance is limited by the need to pair sinusoids that originate from the same path. We proposed a new TV channel estimation algorithm by transmitting a nonlinear power FM chirp pilot instead of two linear FM chirp pilots. The channel estimation simplifies to estimating the parameters of L LFM chirps instead of $2L$ sinusoids without requiring pairing. The LFM estimation was successfully performed using the modified matching pursuit as we demonstrated with simulations. In order to reduce the computation of this iterative technique, we assume knowledge of the channel multipath and Doppler spread.

7. REFERENCES

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