CAPON'S TIME-FREQUENCY REPRESENTATION WITH NONSTATIONARY AR AUTOCORRELATION

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ABSTRACT

In this paper, a novel approach for spectral analysis of nonstationary signals is presented. For this purpose, the Capon's Time Frequency Representation (CTFR) is employed. It is shown that the CTFR is an upper bound on the range of non-unique solutions for power estimation of a complex sinusoid contaminated with unknown noise. A new local autocorrelation function using a Nonstationary Auto-Regressive (NAR) model is defined and used in the CTFR. This method efficiently models the autocorrelations of NAR processes. Synthetic signals are generated in order to illustrate the superiority of the CTFR with NAR model in comparison to other methods.

1. INTRODUCTION

Spectral estimation of stationary signals has been extensively investigated; however, in many applications the signal are nonstationary therefore appropriate analysis of nonstationary signals is required. In order to derive a Time Frequency Representation (TFR), Özgen [1] filtered the nonstationary signal with an adaptive, time-varying filter centered at the frequency of interest. The filter's coefficients were chosen to minimize the power of the filter output, which can be viewed as the power of the analyzed signal in the frequency of interest for each time instant, i.e. the Capon's TFR (CTFR). The CTFR is an extension of the Capon's spectral estimation [2] to nonstationary signals. It is a nonnegative method which suppresses the cross terms of spectrum analysis of nonstationary signals. In general, it produces higher time-frequency resolution than the Short-Time Fourier Transform (STFT) with the same window length. The STFT is the standard method for analyzing nonstationary signals.

Another basic approach for analyzing nonstationary signals is the Wigner-Ville Distribution (WVD) [3]. The WVD is a pure function of the signal, it does not involve an auxiliary and arbitrary window and it produces a real TFR of the analyzed signal.

Computation of the CTFR requires a time-dependent autocorrelation matrix, which can be estimated in different methods. The most common and intuitive one is a short-time autocorrelation that assumes stationarity over short-time durations. Kayhan et al. [4] proposed a time dependent autocorrelation matrix based on the Data Adaptive Evolutionary (DAE) spectrum. They modeled the signal in the frequency of interest as a sinusoid with time-varying amplitude, which is expressed as a linear combination of orthonormal basis functions. The time-varying amplitude was estimated and substituted in the expectation to yield the autocorrelation matrix. In [5] a consistent unbiased estimator for the correlation function of nonstationary Gaussian process was developed.

In this paper, a Nonstationary Auto-Regressive (NAR) autocorrelation is defined. The NAR model [6],[7] is a parametric approach for characterizing nonstationary signals. In this method, the signal is modeled by an Auto-Regressive (AR) model, where the coefficients are allowed to vary with time as a linear combination of known basis functions. We use the NAR parameters to extract the time dependent NAR-autocorrelation matrix, which is used to obtain the CTFR.

The paper is organized as follows. A new interpretation for the CTFR is given in Section 2. Section 3 presents the NAR autocorrelation. Section 4 presents simulation result of CTFR using NAR autocorrelation matrix. Our conclusions appear in Section 5.

2. CAPON'S TIME-FREQUENCY REPRESENTATION

We develop a new interpretation for the CTFR. The nonstationary signal is modeled as a complex sinusoid in the frequency of interest with time-varying complex amplitude. This amplitude is estimated using a Best Linear Unbiased Estimator (BLUE) for each instant *n*. Based on this model we show that the CTFR is an upper bound on estimating the square absolute amplitude value in an unknown noise environment at a given frequency and at a given instant n. The signal is modeled as follows,

$$y[n] = A_c[n, f_0]\exp(j2\pi f_0 n) + u[n],\ \ 0 \le n \le N-1, \quad (1)$$

in which the nonstationary signal $y[n]$ is modeled in the frequency of interest, f_0, and $A_c[n, f_0]$ is the time-varying complex amplitude, which is considered deterministic unknown. The zero-mean modeling error $u[n]$, includes all frequency components of $y[n]$ different from the frequency of interest. We will estimate the complex amplitude using a p-order linear filter:

$$\hat{A}_c[n, f_0] = \mathbf{w}^H[n]\mathbf{y}[n], \quad (2)$$

where $\mathbf{y}[n]=\left[y[n], y[n-1],\ldots, y[n-p]\right]^T$ contains the present and the last p samples of the process $y[n]$, $\mathbf{w}[n]=[w_0[n], w_1[n],\ldots, w_p[n]]^T$ is the time-varying weight vector, and $(\cdot)^H$, $(\cdot)^T$ denote the conjugate transpose and the transpose operations, respectively. Assuming that the amplitude is constant over p+1 samples, Eq. (1) can be written in vector notation as,

$$\mathbf{y}[n] = A_c[n, f_0]\mathbf{e}[n, f_0] + \mathbf{u}[n] \;, \tag{3}$$

where $\mathbf{e}[n,f_0]=[\exp(-j2\pi f_0 n),\ldots,\exp(-j2\pi f_0(n-p))]^H$ and $\mathbf{u}[n]=\left[u[n], u[n-1],\ldots, u[n-p]\right]^T$.

The unbiased constraint for estimating $A_c[n, f_0]$ is obtained by inserting (3) into (2) and taking the expectation,

$$\mathbf{w}^H[n]\mathbf{e}[n, f_0] = 1 \,. \tag{4}$$

The variance of the filter output from (2) is given by

$$\mathrm{var}\left(\hat{A}_c[n, f_0]\right) = \mathbf{w}^H[n]\mathbf{R}_{\mathbf{uu}}[n]\mathbf{w}[n] \;, \tag{5}$$

where $\mathbf{R}_{\mathbf{uu}}[n]=E\left[\mathbf{u}[n]\mathbf{u}^H[n]\right]$ is the autocorrelation matrix of the modeling error and $E[\cdot]$ denotes the expectation operation. In order to obtain the BLUE for $A_c[n, f_0]$, we will minimize (5) with respect to $\mathbf{w}[n]$ subject to the constraint in (4). This minimization yields,

$$\hat{\mathbf{w}}[n] = \frac{\mathbf{R}_{\mathbf{uu}}^{-1}[n]\mathbf{e}[n,f_0]}{\mathbf{e}^H[f_0]\mathbf{R}_{\mathbf{uu}}^{-1}[n]\mathbf{e}[f_0]} \;, \tag{6a}$$

and the estimated amplitude minimum variance is

$$\mathrm{var}(\hat{A}_c[n, f_0])_{\min} = \frac{1}{\mathbf{e}^H[f_0]\mathbf{R}_{\mathbf{uu}}^{-1}[n]\mathbf{e}[f_0]} \,. \tag{6b}$$

The vector $\mathbf{e}[f_0]$ is no longer a function of time since, $\mathbf{e}[f_0] \triangleq \mathbf{e}[n, f_0]\exp(-j2\pi f_0 n) = \mathbf{e}[0, f_0]$. If we insert (6a) into (2) with an arbitrary frequency f and take the square absolute value we obtain a new periodogram-like TFR.

$$\hat{P}_u[n, f] = \left|\hat{A}_c[n, f]\right|^2 = \left|\frac{\mathbf{e}^H[n,f]\mathbf{R}_{\mathbf{uu}}^{-1}[n]\mathbf{y}[n]}{\mathbf{e}^H[f]\mathbf{R}_{\mathbf{uu}}^{-1}[n]\mathbf{e}[f]}\right|^2 \,. \tag{7a}$$

The difficulty in estimating the $\hat{P}_u[n, f]$ lays in the noise autocorrelation matrix estimation, which models the signal in frequencies different from the frequency of interest and without proper assumptions, cannot be estimated. The resemblances of $\hat{P}_u[n, f]$ to the periodogram is illustrated if the noise is assumed to be white, for which (7a) reduces to

$$\hat{P}_u[n, f] = \frac{1}{p+1}\left|\mathbf{e}^H[f]\mathbf{y}[n]\right|^2 \;, \tag{7b}$$

which forms a TFR based on the periodogram. In order to obtain (7b), the periodogram of the last p+1 samples is calculated for each time instant n.

For unknown noise autocorrelation, we can use Marzetta's interpretation [8] for the Capon's spectral estimator and generalize it to the nonstationary case to yield the CTFR. The autocorrelation matrix of $\mathbf{y}[n]$, defined in (3), is

$$\mathbf{R}_{\mathbf{yy}}[n] = E[\mathbf{y}[n]\mathbf{y}^H[n]] = \left|A_c[n, f]\right|^2 \mathbf{e}[f]\mathbf{e}^H[f] + \mathbf{R}_{\mathbf{uu}}[n] \,. \tag{8a}$$

We used in (8a) an arbitrary frequency f. Let $\hat{\mathbf{R}}_{\mathbf{yy}}[n]$, $\hat{\mathbf{R}}_{\mathbf{uu}}[n]$ and $\left|\hat{A}_c[n, f]\right|$ denotes the estimation of $\mathbf{R}_{\mathbf{yy}}[n]$, $\mathbf{R}_{\mathbf{uu}}[n]$ and $\left|A_c[n, f]\right|$, respectively. Then an equivalent model to (8a) can be stated as

$$\hat{\mathbf{R}}_{\mathbf{yy}}[n] = \left|\hat{A}_c[n, f]\right|^2 \mathbf{e}[f]\mathbf{e}^H[f] + \hat{\mathbf{R}}_{\mathbf{uu}}[n] \,. \tag{8b}$$

If the estimate of the sinusoid amplitude under unknown noise conditions is required, then the decomposition of the autocorrelation matrix is non-unique. There is a range of values for $\left|\hat{A}_c[n, f]\right|$ and $\hat{\mathbf{R}}_{\mathbf{uu}}[n]$ such that (8b) is satisfied. An upper bound on $\left|\hat{A}_c[n, f]\right|$ can be obtained using the non-negativity of the modeling error autocorrelation. If the upper bound is achieved, the noise autocorrelation matrix is singular. This property is understood if one examines the noise autocorrelation matrix $\hat{\mathbf{R}}_{\mathbf{uu}}[n]$. Let $\lambda[n]$ denote the smallest eigenvalue of $\hat{\mathbf{R}}_{\mathbf{uu}}[n]$ and $\boldsymbol{\varphi}[n]$ denote the corresponding eigenvector. The noise autocorrelation matrix is nonnegative-definite and therefore

$$\lambda[n] = \boldsymbol{\varphi}^H[n]\left(\mathbf{R}_{\mathbf{yy}}[n] - \left|\hat{A}_c[n, f]\right|^2 \mathbf{e}[f]\mathbf{e}^H[f]\right)\boldsymbol{\varphi}[n] \,. \tag{9a}$$

If we denote $\mu_0[n]$ as the smallest eigenvalue of $\mathbf{R}_{\mathbf{yy}}[n]$ and notice that the matrix $\mathbf{e}[f]\mathbf{e}^H[f]$ has only one nonzero eigenvalue $\mathbf{e}^H[f]\mathbf{e}[f] = p+1$, then

$$\lambda[n] \ge \mu_0[n] - \left|\hat{A}_c[n, f]\right|^2 (p+1) \,. \tag{9b}$$

In general, $\mu_0[n]$ and $\lambda[n]$ are nonnegative for each instant n. Suppose that for some value of $\left|\hat{A}_c[n, f]\right|$, $\hat{\mathbf{R}}_{\mathbf{uu}}[n]$ is positive-definite. In this case, $\mu_0[n]$ is strictly positive and the value of $\left|\hat{A}_c[n, f]\right|$ can always be increased to the point where $\hat{\mathbf{R}}_{\mathbf{uu}}[n]$ is singular i.e. $\lambda[n]=0$. If we denote the upper bound of $\left|\hat{A}_c[n, f]\right|^2$ by $\hat{P}[n, f]$, then for nonzero eigenvector $\boldsymbol{\eta}[n]$,

$$\hat{\mathbf{R}}_{\mathbf{uu}}[n]\boldsymbol{\eta}[n] = (\hat{\mathbf{R}}_{\mathbf{yy}}[n] - \hat{P}[n, f]\mathbf{e}[f]\mathbf{e}^H[f])\boldsymbol{\eta}[n] = 0\cdot\boldsymbol{\eta}[n] \,. \tag{10}$$

By rearranging (10), we obtain

$$\hat{P}^{-1}[n, f]\boldsymbol{\eta}[n] = \hat{\mathbf{R}}_{\mathbf{yy}}^{-1}[n]\mathbf{e}[f]\mathbf{e}^H[f]\boldsymbol{\eta}[n] \,. \tag{11}$$

The eigenvector $\boldsymbol{\eta}[n]$ is also the eigenvector of the matrix $\hat{\mathbf{R}}_{\mathbf{yy}}^{-1}[n]\mathbf{e}[f]\mathbf{e}^H[f]$. Left multiplication of (11) by $\mathbf{e}^H[f]$ and considering the eigenvector $\boldsymbol{\eta}[n]$ which is not orthogonal to $\mathbf{e}^H[f]$ and solving for $\hat{P}[n, f]$ yields,

$$\hat{P}[n, f] = \frac{1}{\mathbf{e}^H[f]\hat{\mathbf{R}}_{\mathbf{yy}}^{-1}[n]\mathbf{e}[f]} \;, \tag{12}$$

which is identical to the CTFR that was previously given by Özgen in [1]. Therefore, the CTFR at a particular frequency f and instant n can be interpreted as an upper bound on the range of non-unique solutions for the problem of time-varying power estimation of complex sinusoid in an unknown noise environment. If the signal is stationary, then (12) reduces to the case of Capon's spectral estimator.

3. NAR AUTOCORRELATION

A nonstationary autocorrelation estimator is required for (12). The accuracy of this estimator directly affects the quality of the time frequency estimated spectrum. Özgen in [1] used Kayhan's autocorrelation with Fourier basis function. We propose a new NAR autocorrelation for this purpose. Based on the estimation of the NAR model parameters a recursive algorithm is given for the modeling of the NAR autocorrelation.

The NAR model characterizes nonstationary signals using a linear prediction model. The coefficients are allowed to vary with time as a linear combination of known basis functions,

$$y[n] = -\sum_{i=1}^{p}\sum_{k=0}^{q} \alpha_{ik} b_k[n-i]y[n-i] + v[n],\ 0 \le n \le N-1\,. \quad (13)$$

The nonstationary signal $y[n]$ is modeled by a p-order AR model with time-dependent coefficients. The driving noise $v[n]$ is a zero-mean white random variable with variance σ_v^2, and $\{b_k[\cdot]\}_{k=0,\dots,q}$ denotes the basis functions set. The basis functions set can be arbitrary. For example the Fourier basis functions set was defined in [7] as, $b_k[n] = \cos(\pi kn/N)$, $b_k[n] = \sin(\pi kn/N)$, for even and odd values of k, respectively. The NAR parameters are composed of the NAR coefficients $\{\alpha_{ik}\}$ and the driving noise variance σ_v^2. The notation of p was chosen here and in the CTFR to point out that in both cases we use the same number of past samples of the signal for the estimation procedure.

The autocorrelation of a zero-mean NAR process, $y[n]$, is defined as

$$\gamma[n,m] = E(y[n]y^*[n-m]) = \gamma^*[n-m,-m], \quad (14)$$

where $(\cdot)^*$ denotes the complex conjugate, n is the time index and m is the time deference. Inserting the NAR process $y[n]$ given in (13) into the autocorrelation given in (14), yields

$$\begin{aligned}\gamma[n,m] = &-\sum_{i=1}^{p}\sum_{k=0}^{q} \alpha_{ik} b_k[n-i] \underbrace{E(y[n-i]y^*[n-m])}_{\gamma[n-i,m-i]} \\ &+ E\left[v[n]y^*[n-m]\right].\end{aligned} \quad (15)$$

If $|m| > p$, then the autocorrelation function of NAR(p,q) processes is zero. Therefore, we will consider time difference indices $|m| \le p$. The second term of (15) is zero if $m > 0$ since $v[n]$ is white, and equal to the deriving noise variance for m=0. In general, the NAR parameters are unknown and therefore should be estimated. The estimated NAR coefficients $\{\hat{\alpha}_{ik}\}$ are obtained by a method presented in [6]. The estimated noise variance $\hat{\sigma}_v^2$ is obtained using Maximum Likelihood Estimation (MLE) under the assumption that $v[n]$ is white Gaussian noise. Hence for $m \ge 0$,

$$\gamma[n,m] = -\sum_{i=1}^{p}\sum_{k=0}^{q} \hat{\alpha}_{ik} b_k[n-i]\gamma[n-i,m-i] + \hat{\sigma}_v^2\delta[m]\,, \quad (16)$$

where $\delta[\cdot]$ denotes the Kronecker delta. If $y[n]$ is a stationary process, then q=0 and $b_0[n] = 1$. In this case, the autocorrelation is no longer a function of the time index n, and (16) reduces to the well-known Yule-Walker Equations (YWE). However, the difference between YWE and this method is that the autocorrelation is unknown and the NAR parameters were estimated at a previous stage.

Eq. (14) implies that for nonstationary processes one cannot produce the autocorrelation $\gamma[n-m,-m]$ previous to instant n, since it is equal to the conjugated value of $\gamma[n,m]$ which is practically the future. Therefore, we will decompose (16) into two elements and apply (14) on the second element,

$$\begin{aligned}\gamma[n,m] = &-\sum_{i=1}^{m}\sum_{k=0}^{q} \hat{\alpha}_{ik} b_k[n-i]\gamma[n-i,m-i] - \\ &\sum_{i=m+1}^{p}\sum_{k=0}^{q} \hat{\alpha}_{ik} b_k[n-i]\gamma^*[n-m,i-m] + \hat{\sigma}_v^2\delta[m],\end{aligned} \quad (17)$$

Eq. (17) is a recursive algorithm for estimating the autocorrelation from its past elements. In order to obtain the autocorrelation with negative values of m and for each time instant, one should estimate the autocorrelations for $0 \le n \le N+p-1$, $0 \le m \le p$ and apply the relation given in (14). In order to calculate (17), it is further assumed that $\gamma[n,m] = 0$ for n<0. Since the process is excited by white noise, it is reasonable to initialize the autocorrelation by the deriving noise variance,

$$\gamma[0,m] = \hat{\sigma}_v^2\delta[m]\,. \quad (18)$$

The elements of the nonstationary autocorrelation $\mathbf{R_{yy}}[n]$ required for the computation of the CTFR given in (12) could be calculated using NAR autocorrelation function $\gamma[n,m]$. The functionality of this method lies in the fact that the NAR model is spread in numerous fields such as speech processing, EEG, ECG and etc. Furthermore only a single data record of the nonstationary signal is needed to obtain the NAR autocorrelation. The values of p,q and the basis functions can be determined from a finite set of parameters using model order criterion such as the AIC, which was developed in [9]. Thus, the CTFR using the NAR autocorrelation and an appropriate model order criterion does not involve an arbitrary selection of parameters such as window length or number of basis functions.

4. SIMULATION RESULTS

For examining the influence of the time-varying autocorrelation estimation on the CTFR, a NAR(2,1) process is generated. The CTFR with Kayhan autocorrelation and with NAR autocorrelation of this process are compared to the expected NAR spectral estimator [6] that is defined as

$$P[n,f] = \sigma_v^2\left|1 + \sum_{i=1}^{p}\sum_{k=0}^{q} \alpha_{ik} b_k[n-i]\exp(-j2\pi if)\right|^{-2}. \quad (19)$$

The NAR process parameters are $\alpha_{10} = \alpha_{11} = \alpha_{20} = 0$, $\alpha_{21} = -r_{21}\exp(j2\pi f_{21})$, where $r_{21} = 0.95$, $f_{21} = 0.4$ and with $\sigma_v^2 = 0.2$. The NAR(2,1) spectral estimator with exponential basis functions $b_k[n] = \exp\left(j\pi nk/N\right)$ is given by,

$$P[n,f] = \sigma_v^2\left|1 - r_{21}\exp(j2\pi(f_{21} + (n-2)/2N + 2f))\right|^{-2} \quad (20)$$

where $n = 0,\dots,N-1$, and $N = 128$. Fig. 1 presents the CTFR with NAR autocorrelation matrix (Fig. 1a) and with Kayhan autocorrelation matrix (Fig. 1b). In both cases the autocorrelations were estimated from the signal using exponential basis functions with p=2 and q=1. The autocorrelations here and in the next example were first calculated and then Kayhan autocorrelation is normalized as follows,

$$\gamma_K[n,m] = \gamma_K[n,m]\left(\max(\gamma_{NAR}[n,m])/\max(\gamma_K[n,m])\right) \quad (21)$$

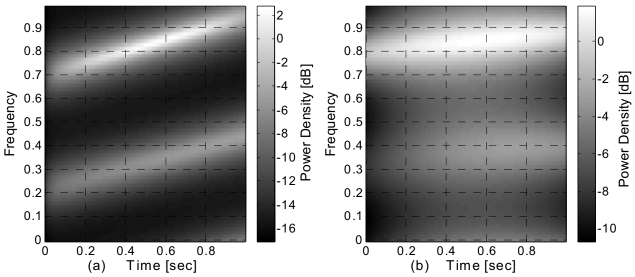

Fig.1. The CTFR of NAR(2,1) process using NAR autocorrelation (a) and Kayhan autocorrelation (b).

where $\gamma_K[n,m]$ is the Kayhan autocorrelation and $\gamma_{NAR}[n,m]$ is the NAR autocorrelation. The NAR spectrum reaches its maximum approximately when $f = f_{21}/2 + n/4N$ and $f = f_{21}/2 + n/4N + 0.5$. This is clearly seen in Fig. 1a. The CTFR with Kayhan autocorrelation (Fig. 1b) failed to represent the true nature of the signal with p=2 and q=1. The simulations shows that by significant increase of the model order (for example when p=6 and q=32), the CTFR with Kayhan autocorrelation for the above NAR(2,1) process provide satisfactory results.

In the second example, the signal is a sum of two chirp signals one with decreasing and increasing frequencies $y[n] = A[n](e^{j\omega_1 n} + e^{j\omega_2 n}) + u[n]$ where $n = 0,\ldots,N-1$, and $N = 128$. The process $u[n]$ is white Gaussian noise with variance 0.05. The amplitude is given by $A[n] = n(N-n)/N^2$ and $\omega_1 = \pi/2 - \pi n/2N$, $\omega_2 = \pi n/2N$. The CTFR of the signal with NAR autocorrelation (Fourier basis function with p=12 and q=4) and Kayhan autocorrelation (Fourier basis function with q=64 and p=12) are shown in Figs. 2a and 2b, respectively. The STFT with Kaiser window of length 32 (β=5) is shown in Fig. 2c, and the WVD is presented in Fig. 2d. The CTFR's, shown in Figs. 2a and 2b, outperform the STFT and WVD presented in Figs. 2c and 2d respectively. The CTFR explores the time behavior of each chirp with better resolution than the STFT (Fig. 2c) with the same window length. (See a comparison between the CTFR and the STFT in [10]). Furthermore, the CTFR displays no cross terms, while the WVD cross terms in Fig. 2d overwhelm the true component of the signal. The number of basis functions with Kayhan autocorrelation in comparison to the NAR autocorrelation (q=64 versus q=4) is dramatically higher. It should be noted that in the second example a poor CTFR with Kayhan autocorrelation was obtained when the number of basis functions, q, was less than 32.

5. CONCLUSIONS

In this paper, we showed that the CTFR is an upper bound for estimating the power of a sinusoid at a given frequency in an unknown noise environment. The CTFR suppresses cross terms and provide better resolution with respect to the conventional STFT. The CTFR with NAR autocorrelation does not involve arbitrary parameter selection such as window length and number

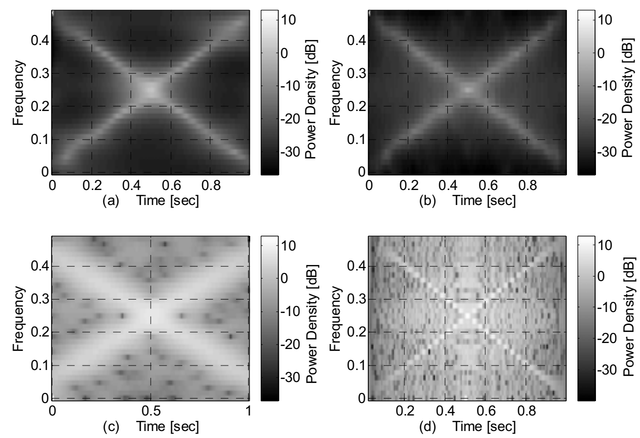

Fig. 2. The CTFR of the signal using NAR autocorrelation (a) and using Kayhan autocorrelation (b), STFT (c), and WVD (d).

of basis functions. The NAR autocorrelation is obtained by single data record. If the nonstationary signal is a NAR process then a reduced set of parameters is needed for obtaining an efficient and accurate CTFR.

6. REFERENCES

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