DIRECTION FINDING OF NONSTATIONARY SIGNALS USING A TIME-FREQUENCY HOUGH TRANSFORM

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ABSTRACT

We consider the problem of direction finding for nonstationary signals impinging on an array of sensors. Making use of a time-frequency representation of the data, we are able to exploit the non-stationary nature of the source signals. We employ a generalized Hough transform to estimate the time-frequency signature of each source. The proposed method also allows direction finding when there are more sources than the number of array sensors.

1. INTRODUCTION

Nonstationary signals such as frequency modulated (FM) and polynomial phase signals (PPS) arise in a number of fields including sonar, radar and telecommunications. Recently, the application of time-frequency (TF) analysis to sensor array processing for non-stationary signals has received significant attention in the literature. The use of spatial time-frequency distribution (STFD) matrices in particular has emerged as a natural means for exploiting both the spatial diversity and TF localization properties of nonstationary sources impinging on a sensor array [1].

From subspace analysis of STFD matrices [2], it was shown that performance improvement with respect to traditional approaches is significant when the sources are closely spaced and/or the SNR is very low. Further, the best performance gain is achieved when separately averaging over the TF signatures of each source. We therefore seek a means of identifying the individual TF signatures from a mixture of closely spaced sources, which does not break down at low SNR.

One may try to determine the TF signatures nonparametrically, via thresholding of TFDs [3] and subsequently perform the direction finding [4]. Such techniques are able to estimate the combined TF signature of all sources, but not identify the individual signatures of each source. Discrimination between TF-signatures based on spatial diversity has also been proposed in [5, 6]. Such methods clearly break down when the source signals are closely spaced. Suboptimal parametric methods such as the PPT [7] have a constant SNR threshold which is not improved by an increased number of observations.

The Hough transform of the Wigner-Ville distribution has been proposed for analysis of multi-component linear FM signals in [8] and shown to have good performance at low SNR, due to coherent integration in the TF plane. This idea was extended for the analysis of non-linear FM signals in [9] via a generalized Hough transform. In this paper we investigate the application of the Hough transform to STFDs for the purpose of direction finding.

In Section 2, we outline the signal model used and in Section 3 briefly review the idea behind direction finding using STFDs. Section 4 discusses the proposed direction finding algorithm based on the Hough transform and in Section 5, simulations depicting the performance of the proposed method are included. Section 6 summarizes the important conclusions drawn from this work.

2. SIGNAL MODEL

We consider an m-element sensor array observing an instantaneous linear mixture of signals emitted from d narrowband far-field sources. The vector $\boldsymbol{x}(t) \in \mathbb{C}^{m\times 1}$ represents the baseband array output waveforms at time t, which are corrupted by an additive noise process $\boldsymbol{v}(t)$. The baseband array output model is

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{v}(t), \tag{1}$$

where $\boldsymbol{A} \in \mathbb{C}^{m\times d}$ is termed the mixing matrix and $\boldsymbol{s}(t) \in \mathbb{C}^{d\times 1}$ is a (deterministic) vector of the source signals. $\boldsymbol{A}$ is assumed to be of full column rank and parameterized by a vector $\boldsymbol{\theta} \in \mathbb{R}^{d\times 1}$ corresponding to the directions of the source signals with respect to the array broadside. We also assume that the sources have different localization properties in the TF plane. The additive noise $\boldsymbol{v}(t)$ is assumed to be a stationary, spatially and temporally white, zero-mean complex random process which is independent of the source signals and has variance σ_v^2.

We consider herein a class of FM signals with constant amplitude of the form $\{s_k(t) = A_k \exp[\mathrm{j}\phi_k(t)]\}_{k=1}^d$ which have a well defined instantaneous frequency (IF) given by

$$\omega_k(t) = \frac{\mathrm{d}\phi_k(t)}{\mathrm{d}t} \; ; \; k = 1, \ldots, d$$

and thus lend themselves to a TF formulation of the direction finding problem.

3. STFD MATRICES AND DIRECTION FINDING

We make use of the idea by Amin et al [1] for direction finding based on a spatial TFD matrix, defined in terms of the auto- and cross-TFDs of the sensors as

$$[\boldsymbol{D}_{\boldsymbol{xx}}(t, \omega)]_{ij} = D_{x_i x_j}(t, \omega; \varphi) \tag{2}$$

where $D_{x_i x_j}(t, \omega; \varphi)$ is assumed to be a bilinear TFD of Cohen's class, for which the kernel function is φ.

The estimate of $\boldsymbol{\theta}$ for a subset of $d_0 \leq d$ signals is obtained via subspace decomposition of an averaged STFD matrix. Assuming snapshots of the array output at sampling instants $\{t_n\}_{n=1}^N$ are available, the averaged STFD matrix is calculated according to

$$\boldsymbol{D} = \frac{1}{N} \sum_{k=1}^{d_0} \sum_{n=1}^{N} \boldsymbol{D}_{\boldsymbol{xx}}(t_n, \omega_k(t_n)) \tag{3}$$

where a discrete-time TFD is used in the above. Compared with the sample covariance matrix of the array output, the matrix in (3) provides an effective improvement in SNR by amplification of the source eigenvalues with respect to the noise eigenvalues [2]. We note that direction finding based on (3) can be performed with the reduced constraint $d_0 < m$. The best estimator performance is achieved by using $d_0 = 1$, and successively estimating the DOAs of each source.

4. DIRECTION FINDING WITH THE HOUGH TRANSFORM

The use of (3) requires knowledge of the source signal IFs $\{\omega_k(t)\}_{k=1}^d$ for each corresponding direction parameter to be estimated [2]. In order to implement such a scheme without *a priori* knowledge of the signal IFs, we propose the use of a generalized Hough transform [9]. This approach requires us to know a general functional form that is suitable for describing the IF, but not the particular parameters for the signals being observed. By parameterizing the signal IFs we can translate the problem of estimating $\{\omega_k(t)\}$ into peak detection in a parameter space.

Denoting the parameterized IFs by $\{\omega(t; \boldsymbol{\psi}_k)\}_{k=1}^d$, where $\boldsymbol{\psi}_k$ is the vector of IF parameters for source k and choosing $d_0 = 1$ in (3) for best performance, we obtain

$$\boldsymbol{H}_{\boldsymbol{x}}(\boldsymbol{\psi}_k) = \frac{1}{N} \sum_{n=1}^{N} \boldsymbol{D}_{\boldsymbol{xx}}(t_n, \omega(t_n; \boldsymbol{\psi}_k)), \quad k = 1, \ldots, d \tag{4}$$

which we call a spatial time-frequency Hough transform (STFHT) matrix.

In order to use the Hough transform for estimating $\{\boldsymbol{\psi}_k\}_{k=1}^d$, we average the auto-sensor TFDs across the array sensor data. This is known to reduce the effect of noise and cross-source components in the TF-plane [10]. The Hough transform of the array averaged TFD is given by

$$\begin{aligned} H_x(\boldsymbol{\psi}) &= \frac{1}{mN} \sum_{n=1}^{N} \sum_{k=1}^{m} D_{x_k x_k}(t_n, \omega(t_n; \boldsymbol{\psi})) \\ &= \frac{1}{m} \sum_{k=1}^{m} H_{x_k}(\boldsymbol{\psi}), \end{aligned} \tag{5}$$

where $H_{x_k}(\boldsymbol{\psi})$ denotes the time-frequency Hough transform (TFHT) of the waveform at sensor k. The estimates $\{\hat{\boldsymbol{\psi}}_k\}_{k=1}^d$ are obtained from (5) as the values of $\boldsymbol{\psi}$ corresponding to the d largest peaks in $H_s(\boldsymbol{\psi})$. In the case of an unknown number of sources, it would be necessary to appropriately threshold $H_x(\boldsymbol{\psi})$ before applying a peak-search algorithm. However the details and performance of such a source number detection scheme will not be discussed here.

Direction of arrival estimation for the signal $s_k(t)$ is conducted by first obtaining estimates $\hat{\boldsymbol{\psi}}_k$ from (5) and forming the corresponding STFHT matrix according to (4). A high resolution second-order direction finding method such as MUSIC can be applied. We note that obtaining $\{\hat{\theta}_k\}_{k=1}^d$ requires searching for a single peak d times as opposed to searching for d peaks in, e.g. a conventional MUSIC spectrum. By appropriate choice of the IF model, we also have the possibility of only estimating $\{\hat{\theta}_k\}$ for those sources with, e.g. a strongly linear IF such as chirp signals and ignoring others with non-localized TF representations. The DOA estimation procedure is summarized in Table 1.

If the IFs of the different source signals do not significantly overlap, then each peak in the Hough transform isolates the energy of a single signal. The use of the matrices in (4) then potentially allows direction finding for an arbitrary number of source signals, using only two sensors. Of course, the peak search in the IF parameter space may become too computationally intensive if the number of parameters required to represent the IF is large. There are, however, a number of interesting cases such as low-order polynomial phase, hyperbolic FM and sinusoidal FM, where the parameter space is of low order. Such signals arise in a range of applications including radar, sonar, telecommunications and helicopter recognition [9].

1. Determine an appropriate parameterization ($\boldsymbol{\psi}$) for the signal IF.
2. Compute $H_x(\boldsymbol{\psi})$, the array averaged TFHT, according to (5).
3. Form the estimates $\{\hat{\boldsymbol{\psi}}_k\}_{k=1}^d$ from the values of $\boldsymbol{\psi}$ corresponding to the d largest peaks of $H_x(\boldsymbol{\psi})$.
4. Calculate the STFHT matrix $\boldsymbol{H_x}(\hat{\boldsymbol{\psi}}_k)$ for $k = 1, \ldots, d$.
5. Form the DOA estimates $\{\hat{\theta}_k\}_{k=1}^d$ by applying MUSIC or other sub-space based technique to the matrices formed in Step 4.

Table 1. DOA estimation algorithm using the Hough transform.

In particular, linear FM or chirp signals, which occur in a number of array processing applications, are amenable to a combined TF Hough transform. It has been shown that optimal detection of chirps can be performed using a Radon/Hough transform of the Wigner-Ville distribution [11]. Computationally there are also advantages when dealing with chirp signals. Direct implementation of the chirp TFHT is $O(N^3)$, though a more efficient implementation via time and frequency dechirping can reduce this to $O(2N \log 2N)$ [12].

5. SIMULATIONS

In the following simulations, we focus on the chirp signal, whose Hough transform corresponds to taking line integrals through the TF plane. The signal IFs are given by $\{\omega(t, \boldsymbol{\psi}_k) = 2\pi(a_k + b_k t)\}_{k=1}^d$ where the parameter vector in the TFHT is $\boldsymbol{\psi}_k = [a_k, b_k]$. The SNR for signal k is defined as A_k^2/σ^2.

To illustrate the effect of the array averaging, we show in Figure 1 the Hough transform at a reference sensor and that averaged across the array. In this example two chirp signals are impinging on an eight-sensor, uniform linear array (ULA), each with an SNR of -20 dB and respective DOAs of -10 and 10 degrees. 1024 snapshots are used and the TFD is a pseudo Wigner-Ville distribution with odd window length of 129 samples. Clearly the noise floor over the parameter space is reduced due to array averaging, and the two peaks due to the two chirp signals are enhanced. We note also that due to their oscillatory nature, the cross-terms are significantly reduced by the integration through the TF plane when calculating the TFHT. This is in addition to the reduction of cross-source components achieved by the array averaging.

In the next example, we show the overall performance in DOA estimation, for the signal mixture defined previously. Figure 2 shows the root mean-squared error (RMSE) obtained from averaging 200 Monte Carlo runs, for estimation of the first source's DOA, as obtained by the algorithm in Table 1. The estimator performance is simulated vs SNR and compared with the case of known signal IF, the conventional root-MUSIC algorithm and the Cramér-Rao Bound (CRB). We observe that the proposed algorithm performs as well as the case of known signal IF, for SNR greater than -20 dB. This shows that estimation of the signal IF is sufficiently accurate at low SNR where TF-MUSIC exhibits a significant performance gain over conventional MUSIC.

In the third simulation example, we consider the case of fewer sensors than sources. The same signals previously used are present, plus a third chirp signal, all with SNR of -5 dB. The signal DOAs are given by -12, 2 and 15 degrees respectively and the ULA has only two sensors. Due to the isolation of individual signal TF signatures by the use of the Hough transform, we are able to estimate the DOA of each source individually despite the fact that the system is under-determined ($d > m$). MUSIC spectra are plotted in Figure 3 which illustrate that the three chirp signals are able to be resolved.

6. CONCLUSIONS

A method for direction finding for a class of nonstationary signals is proposed based on STFD matrices. Prior knowledge of the source TF signatures is not assumed, but estimated according to a parametric model using the Hough transform. Two important conclusions of the work are as follows; firstly, the use of the Hough transform provides performance close to the case of exactly known TF signatures at low SNR, where there is a significant gain in performance to be achieved by using STFD direction finding, over traditional methods. Secondly, the Hough transform allows estimation of direction for more sources than sensors, in an automatic way.

7. REFERENCES

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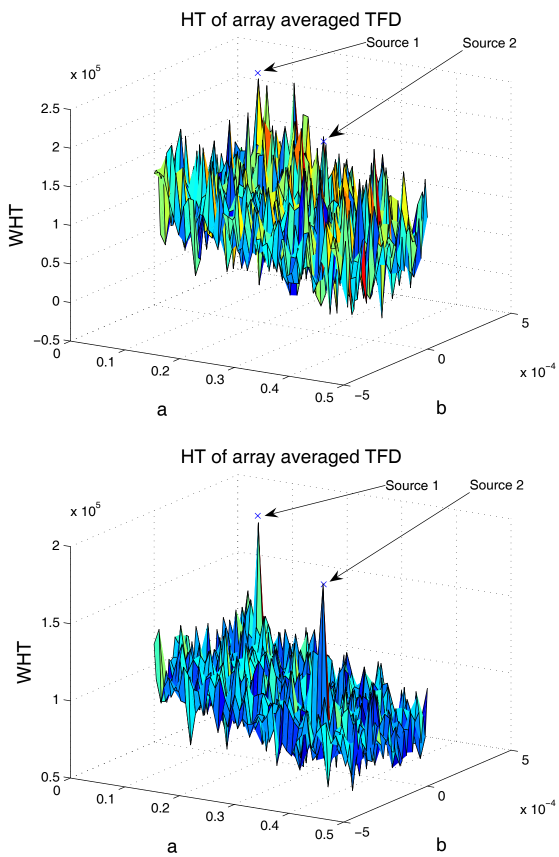

Fig. 1. Example of TFHT before (top) and after (bottom) the array averaging.

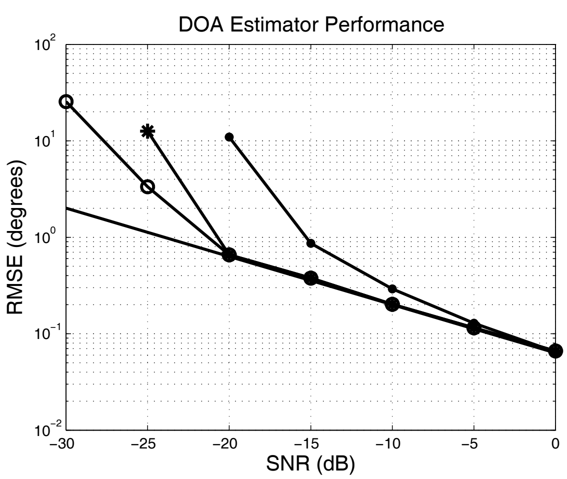

Fig. 2. RMSE of DOA estimator vs SNR. (-) CRB. (o) TF-MUSIC with known signal IF. (*) TF-MUSIC with estimated signal IF. $(\cdot)$ root-MUSIC.

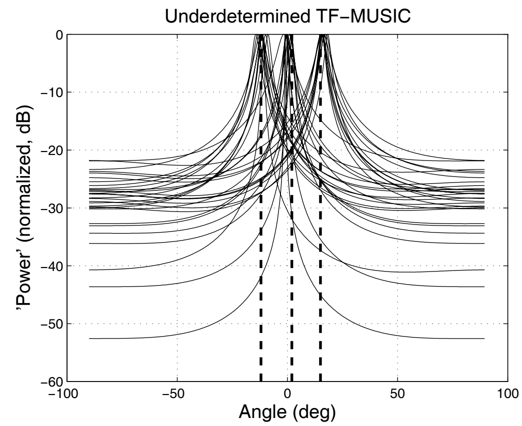

Fig. 3. MUSIC spectra in the under-determined case ($d = 3 > m = 2$) with signal DOAs $[-12, 2, 15]$ degrees.

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