

MODELLING OF SIGNAL'S TIME-FREQUENCY CONTENT USING WARPED COMPLEX-TIME DISTRIBUTIONS

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ABSTRACT

Analytical form of the instantaneous frequency law (IFL) is very important when the physical parameters have to be evaluated from the time-frequency content of a signal. Generally, the analyzed signals are composed of several time-frequency components characterized by various non-linear IFLs. To deal with such kind of signals, we propose a new method based on the warping operators (WO) and on the complex time distribution (CTD). Using parallel structures composed of several WOs, some time-frequency components of the analyzed signal are linearized. This linearization is highlighted by using the CTD which considerably reduces the artefacts due to the complexity of the analyzed signal. This leads to an accurate estimation process, illustrated and justified by numerical and real examples.

1. INTRODUCTION

Analysis of the signals characterized by complex time-frequency behaviour is a very challenging topic, due to the richness of the information that describes the analyzed phenomena. In a large number of applications the analysis of the time-frequency (T-F) content provides an efficient solution of the problems arising in these fields [1]. The signals associated to these applications are generally characterized by many non-linear time-frequency structures. A correct characterization of each time-frequency structure could be achieved by using an adaptive, concentrated and cross terms free time-frequency representation (TFR), which will match the IFL of each signal component [2]. The choice of these TFRs implies the extraction of each time-frequency component of the signal. This could be done by approximating these components with the elementary functions from a large dictionary. This approximation is typically done by a projection algorithm such as, for example, the Matching Pursuit method. The result consists of a set of elementary functions corresponding to the time-frequency structures of the signal. Each function is completely described by its

parameters : the time and frequency centres, duration, chirp rate and the non-linearity type. According to this last parameter, the choice of the adapted TFR is automatically done. The current technique for analysis of non-linear T-F structures is based on warping operators [3].

In this paper we propose an alternative to this class of methods, based on the combination of the warping operator principle and the complex argument concept. The key point of this method is to use the warping principle not only for the design of the adapted TFR to each elementary function, but also *to estimate* their IFL. The WOs of interest are organized in a parallel structure. In order to accurately estimate the linearized T-F structures, we apply the complex time distribution, with a remarkable property related to the drastic reduction of the artefacts generated by the auto or cross terms [4], [6].

The paper is organized as follows. In Section 2 a brief presentation of the warping operator concept is done. The CTD and its connection to the warping principle is presented in the Section 3. In section 4 the Warped Complex-Time Distribution is proposed. Presented theory is justified by numerical and real examples in Section 5.

2. WARPING OPERATOR PRINCIPLE

Matching the signals with non-linear IFLs requires a joint distribution with different instantaneous frequency and group delay localization properties. One of the most known techniques is the unitary similarity transformation [3]. A frequently used unitary transform is the *axis transformation* [3]. For the signal $s(t)$, it is defined as an operator U on $l^2(\mathcal{R})$, i.e.:

$$(Us)(x) = |w'(x)|^{1/2} s[w(x)] \quad (1)$$

where w is a smooth, one-to-one function [3], called warping function. Generally, this function is chosen to ensure the “linearization” of the signal’s time-frequency behavior. Therefore, for a signal expressed as :

$$s(t) = e^{j2\pi\psi(t)} \quad (2)$$

The associated warping function, w , is defined as the inverse of $\psi(t)$, where $\psi(t)$ is the frequency modulation law and c is the modulation rate, [3].

Thus the linearization should enable that the TFR of warped signal produces a constant instantaneous frequency. Distributions from the Cohen's class (CTFR) are usually used for these TFRs. The example illustrated in the figure 1 shows the main property of the warping operator. Here, we consider the signal $s(t) = e^{j2\pi 0.08 t^{1.3}}$ and the linearization of its time-frequency content, provided by the warping operator $w(t) = t^{1/1.3}$.

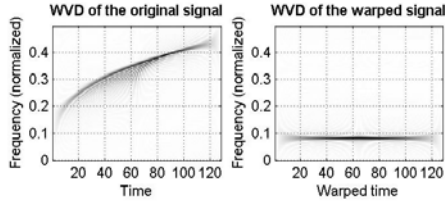


Figure 1. Linearization of T-F content using warping operator

Note that the linearization transforms the original signal into a tone with the frequency close to 0.08 – the real modulation rate.

In the case of many complex T-F structures, applicability of the warping operator concept becomes more complicated (for a simple T-F structure solution, see [2]). This problem may be modeled as

$$s(t) = \exp \left\{ j2\pi \sum_{i=1}^N c_i \psi_i(t) \right\} \quad (3)$$

where $\{\psi_i\}_{i=1, \overline{N}}$ is a set of elementary functions which can be associated to a warping operator \mathbf{W}_{ψ_i} such that :

$$(\mathbf{W}_{\psi_i} \psi_i)(t) = t \quad (4)$$

Application of one of these WOs to the signal (3) will introduce the artifacts related to the complexity of the signal. Namely, application of a WO, \mathbf{W}_{ψ_i} , to its corresponding function generates a tone, while the effect to the other functions $\{\psi_i\}_{i=1, \overline{N}, i \neq j}$ is the appearance of non-linear energetic terms superposed on the tone. This phenomenon is illustrated on figure 2, for the warped version of the signal expressed as :

$$s(t) = e^{j2\pi(c_1 t^{0.2} + c_2 t^{0.7})} = e^{j2\pi(t^{0.2} + 0.75 t^{0.7})} \quad (5)$$

To warp this signal, we use the WO associated to the elementary function $t^{0.7}$, i.e.

$$\mathbf{W}_{t^k} : t \longrightarrow t^{1/k} \quad (6)$$

where we take $k=0.7$.

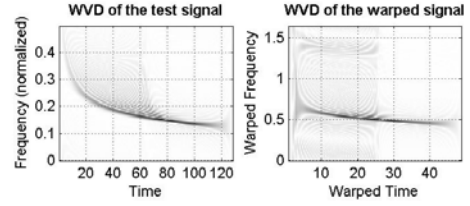


Figure 2. Artifacts due to the signal complexity

Note that the application of the WO (6) to the signal (5) induces an imperfect linearization and some artifacts, due to the existence of the term $t^{0.2}$ (figure 2). Consequently, estimation of the term with $t^{0.7}$ (the modulation rate c_2 , according to the notations introduced in (3)), using a CTFR, could be inaccurate. In the next section, we introduce the Complex Time Distribution as an efficient tool for estimation of the modulation rates in the warped domains.

3. COMPLEX TIME DISTRIBUTION

The complex time argument has been introduced in [4] as an efficient way to produce almost completely concentrated representations along the IFLs. Mathematically, the complex time distribution (CTD) is defined as :

$$CTD_s(t, \omega) = \int_{-\infty}^{\infty} s\left(t + \frac{\tau}{4}\right) s^*\left(t - \frac{\tau}{4}\right) \times s^{-j}\left(t + j\frac{\tau}{4}\right) s^j\left(t - j\frac{\tau}{4}\right) e^{-j\omega\tau} d\tau \quad (7)$$

where the continuous form of the “complex-time signal” is

$$s(t + j\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(j\omega) e^{-\omega\tau} e^{-j\omega t} d\omega \quad (8)$$

and $S(j\omega)$ is the Fourier transform (FT) of signal s .

The main property of the CTD consists in the capability to attenuate the high-order terms of the polynomial decomposition of the IFL. Hence, for the warped version of the signal given in (3) (using \mathbf{W}_{ψ_i})

$$(\mathbf{W}_{\psi_i} s)(t) = \exp(j2\pi c_i t) \exp \left\{ j2\pi \sum_{i=1, i \neq j}^N c_i \psi_i(\psi_i^{-1}(t)) \right\} \quad (9)$$

the CTD has the following expression [4] :

$$CTD_{\mathbf{W}_{\psi_i} s}(t, \omega) = 2\pi \delta(\omega - 2\pi c_i) *_{\omega} \text{FT} \left\{ e^{j2\pi Q(t, \tau)} \right\} \quad (10)$$

where τ is the lag used in (7).

The $\phi(t)$ corresponds to the artifacts generated by the combination between given WO and other elementary functions, and $Q(t, \tau)$ is the spread function which affects

the visibility of the tone peaking in c_i . As proved in [4], in the case of the CTD, this function has a fifth order dominant term (for comparison, the spectrogram and the WVD have a second and a third order dominant term, respectively), which corresponds to a drastic reduction of the higher terms of $\phi(t)$. This attenuation allows us to estimate the modulation rates as the frequency locations of the maxima of the frequency-marginal defined as :

$$\Lambda_{\psi_i}(\omega) = \int_{-\infty}^{\infty} CTD_{\mathbf{w}_{\psi_i}}(t, \omega) dt \quad (11)$$

$$c_i = \arg \max_{\omega} [\Lambda_{\psi_i}(\omega)]$$

The following example illustrates the benefit of the use of the CTD. As a test signal we use the one defined in (5).

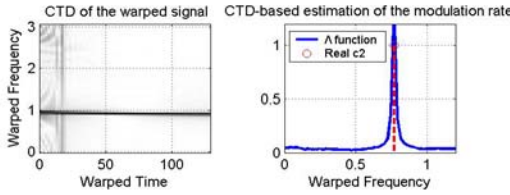


Figure 3. Artifact reduction in the CTD domain

Comparing the figures 2 and 3, one may observe that CTD makes the tone corresponding to modulation rate $c_2 = .75$ much more energetic than the artifacts, leading to a correct localization of the peak associated to c_2 . This property of the CTD will be exploited in the next section, where this method will be applied, in a parallel manner [5], for the estimation of modulation rates associated to each function which composes this structure.

4. WARPED COMPLEX TIME DISTRIBUTIONS

In the previous section we introduced the CTD as a tool for attenuation of the artifacts which appear in the T-F plane when a particular WO is applied on a signal with a complex T-F behavior (3). The problem is more complicated in the case of multi-component signals expressed as

$$x(t) = \sum_{n=1}^P A_n e^{j2\pi \sum_{i=1}^N c_{ni} \psi_i(t)} + b(t) \quad (12)$$

where P is the number of components and b is the noise. We suppose that the functions $\{\psi_i\}_{i=1, \dots, N}$ are invertible.

Therefore, one can define a set of warping operators associated, via (4), to one of the elementary functions. Estimation of the modulation rates, $\{c_{ni}\}$, could be done by reducing the corresponding function, ψ_i , to a sinusoid, thanks to the associated WO. Furthermore, the CTD is used in order to reduce the artifacts (see section 3) and to provide an accurate estimation. This can be done in a parallel manner, each branch being composed of a WO

associated to an elementary function of interest, the CTD and an estimator of the maxima positions in the CTD frequency marginal domain (11) (see the figure 4).

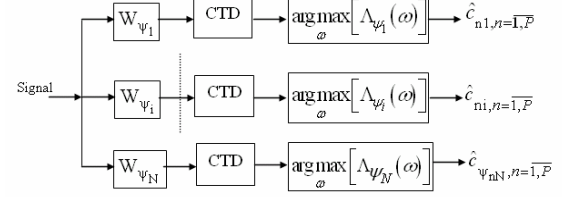


Figure 4. Diagram of the IFL modeling using the warped complex time distributions (WCTD)

Since no initial information about the analyzed signal was assumed, the set of function of interest (and the WOs) is chosen to cover the number of possibility as larger as possible. The experimental results proved that using the monomial functions, $\psi(t) = t^k$, $k = 0, 1$, it is possible to describe a large class of signals. Nevertheless, the accurateness of the modeling depends on the number of WOs involved in the parallel structure.

5. RESULTS

To illustrate the presented method, let us consider the following signal :

$$s(t) = e^{j2\pi(t^{0.2} + 2t^{0.3} + t^{0.7})} + e^{j2\pi(1.5t^{0.2} + 1.2t^{0.3} + 0.1t^{0.7})} \quad (13)$$

The estimation, via WCTDs, of the modulation rates associated to $t^{0.7}$ is illustrated in the next figure.

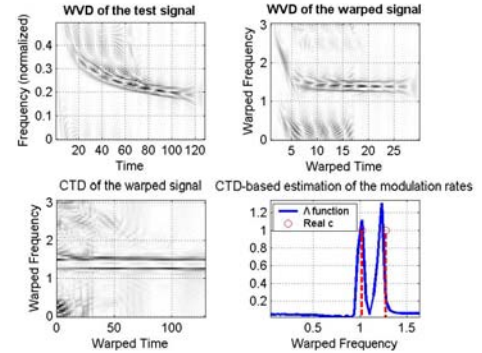


Figure 5. Estimation of modulation rates using WCTD

Due to the good concentration provided by the CTD, the frequency locations corresponding to the modulation rates are very close to the real positions (dashed lines). In spite of the fact that the two components are close (see the WVD, figure 5), the WCTD provides an accurate result.

For this example we have used the monomial functions $\psi(t) = t^k$ with k between 0.1 and 1 with an increment about 0.1. Taking into account the estimation results obtained for all branches, we obtained the IFLs plotted in the next figure.

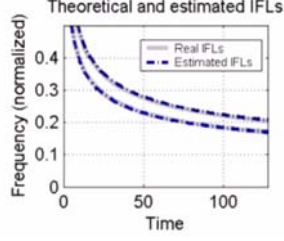


Figure 6. Theoretical and Estimated IFLs

Note that the estimated IFLs are almost superposed on the real one, indicating also the accurateness of the proposed modelling procedure.

Practically, the existence of closed-type WOs (e.g., $W_{t^{0.6}}$, $W_{t^{0.7}}$, $W_{t^{0.8}}$) induces some ambiguities in evaluation of the coefficient c . Namely, some elementary functions could not exist in the phase expression of the signal (e.g. $t^{0.6}$ and $t^{0.8}$ in the case of signal (13)). Nevertheless, the effect of their associated WOs will be a *near* linearization of the structures corresponding to the close elementary functions (in the case of signal (13) - $t^{0.7}$). This phenomenon is illustrated in the figure 7.

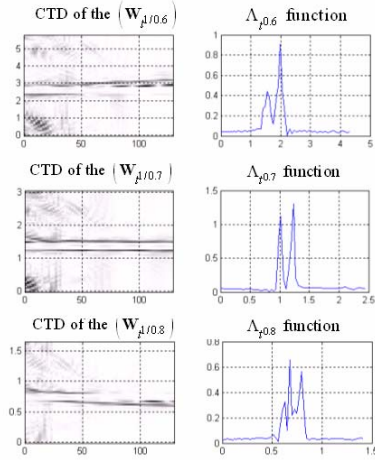


Figure 7. Ambiguity in the estimation of the modulations rates in closed branches

To overcome this problem, we compare the results provided by three close branches and we choose the coefficients which correspond to the maxima of " Λ " function with the largest value. Mathematically, this procedure, iterated for all P coefficient, is written as :

$$\begin{aligned} & \text{If } \max_{\omega} [\Lambda_{\psi_{i-1}}(\omega)] < \max_{\omega} [\Lambda_{\psi_i}(\omega)] \text{ and} \\ & \max_{\omega} [\Lambda_{\psi_{i+1}}(\omega)] < \max_{\omega} [\Lambda_{\psi_i}(\omega)] \\ & \text{Then } \hat{c}_{ij} - \text{the correct coefficient} \\ & \text{Else } i = i + 1 \end{aligned} \quad (14)$$

In the next figure, we illustrate the modeling results for a signal corresponding to a whistler-mode propagation through the Earth's magnetized plasma envelope [7].

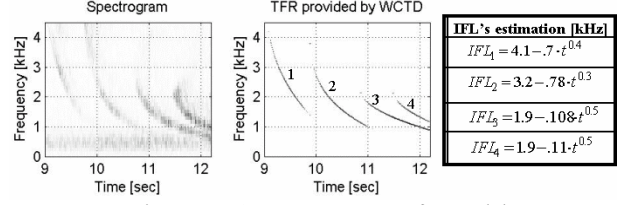


Figure 8. WCTDs vs spectrogram for a real data

We remark that the TFR corresponding to the signal modeled via WCTDs is qualitatively better than the TFR provided by spectrogram. This image, collaborated with the analytical description of the IFLs (figure 8), provide interesting information about the analyzed process.

6. CONCLUSION

In this paper, we have presented a new method for the characterization of a multi-component signal with non-linear IFLs. This method is based on the signal *warping* operation and the *estimation*, for each deformation, of the parameters of the linear time-frequency structures. The results illustrate the performances of the combination between WOs and the CTD. Otherwise, the results obtained for real data illustrate the capability of the proposed method to provide an interesting description, appropriate to physical analysis. For this reason, we intend to apply this method in the case of signals propagating through heterogeneous channels.

The proposed method could be used as a tool for the phase modeling of a multi-component signal using an arbitrary function set. It could be an alternative to the polynomial phase modeling : its main advantage is the flexibility in the choice of elementary functions.

7. REFERENCES

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