A ROBUST MUSIC ESTIMATOR FOR POLYNOMIAL PHASE SIGNALS IN α-STABLE NOISE

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ABSTRACT

In this paper, we address the problem of estimation of the parameters of mono and multicomponent Polynomial Phase Signals (PPS) affected by alpha-stable noise using subspace methods. We propose two new estimators : a modified multiple signal classification MUSIC for PPS affected by Gaussian noise, and a modified robust MUSIC algorithm for PPS based on fractional lower-order statistics (FLOS). Simulation results show that the robust MUSIC estimator is able to estimate the values of the phase parameters in impulsive environment hence outperforming the standard estimators.

1. INTRODUCTION

Constant amplitude polynomial phase signals (PPS) are commonly used in many fields of engineering such as radar, sonar and telecommunications [1]. Many algorithms have been proposed in literature for the analysis of PPS. Non-parametric methods relying on time-frequency analysis tools such Polynomial Phase Wigner-Ville Distribution(PWVD) has been extensively used for the instantaneous-frequency (IF) estimation, as well as parametric methods such as the polynomial phase transform for the estimation of the parameters of the phase. In many practical situations, the signal under consideration may be subjected to additive noise which is assumed to be Gaussian. Several papers have considered this case as in [2]. However, the assumption of Gaussianity is not valid in some cases when noise is generated from atmospheric or underwater acoustic phenomena which displays impulsive characteristics with heavy-tailed distributions that degrade significantly the signal. Impulsive noise can be modeled by α-stable random process. The fact that α-stable random variables with $\alpha < 2$ have infinite second moment means that many techniques based on second order statistics (SOS) do not apply, and therefore, we must consider other alternatives to mitigate the effect of the non-Gaussian impulsive noise. In [3] we proposed a robust FLOS based PWVD for IF representation of PPS in α-stable noise. Recently some subspace methods have been proposed to analyse PPS. In [4], the authors derived a Capon form of the wigner distribution and polynomial periodogram. In [5], the author extended the Capon estimator to the analysis of PPS by considering a time-dependent autocorrelation sample estimates of the nonstationary signal. In [6] the MUSIC algorithm has been applied for parameter estimation of PPS in Gaussian noise by transforming the phase to a linear phase using the polynomial phase transform.
In this paper, we propose a robust MUSIC estimator for PPS of order higher than 2 corrupted by additive α-stable impulsive noise using Fractional Lower-Order Statistics (FLOS) introduced in [7] which handle robustly the presence of heavy-tailed noise in the data .
This paper is organized as follows. In section 2, we briefly review the model of PPS. Then in sections 3, we present the model of complex α-stable noise. In section 4,we introduce our FLOS based subspace method for the estimation of PPS in impulsive noise. Some simulation examples are presented in section 5. Concluding remarks are given in Section 6.

2. THE POLYNOMIAL PHASE SIGNAL MODEL

The constant amplitude polynomial phase signal of order M is given by

$$s(n) = A \exp\{j\phi(n)\} = A \exp\left\{ j \sum_{i=0}^{M} a_i n^i \right\} \tag{1}$$

where A is the amplitude of the signal, the a_i's $(i = 0, \ldots, M)$ are the phase coefficients; assumed real and unknown.
The instantaneous frequency (IF) is defined as

$$f_i(n) = \frac{1}{2\pi} \frac{d\phi(n)}{dt} = \frac{1}{2\pi} \sum_{i=1}^{M} i \; a_i \; n^{i-1}. \tag{2}$$

3. COMPLEX α STABLE NOISE

There exists many physical processes generating interference containing noise components that are impulsive in nature (e.g., atmospheric noise in radio links; and radar reflections from ocean waves, and reflections from large, flat surfaces including buildings and vehicles). The amplitude distributions of such returns are not Gaussian, and tend to produce large-amplitude excursions and occasional bursts of outlying observations. Impulsive noise profoundly degrades the performance of standard algorithms and produce poor results.
In our case, the impulsive noise is modeled by complex α stable signal $X = X_1 + jX_2$ which is better defined by its characteristic function [8]

$$\varphi(t) = E\{\exp[j\Re(tX^*)]\} = E\{\exp[j(t_1X_1 + t_2X_2)]\} \tag{3}$$

where $t = t_1 + jt_2$. X is called isotropic SαS if (X_1, X_2) has a uniform spectral measure [8]. In this case, the characteristic function reduces to

$$\varphi(t) = E\{\exp[j\Re(tX^*)]\} = \exp(-\gamma\, |t|^{\alpha}) \tag{4}$$

The stable distribution is completely characterized by the parameters α $(0 < \alpha \leq 2)$ named the characteristic exponent, γ is the dispersion ($\gamma > 0$). The characteristic exponent determines the shape of the distribution. The smaller α is, the heavier the tails of the alpha stable density. We should also note that for $\alpha = 2$ the distribution coincides with the Gaussian density. The dispersion γ determines the spread of the distribution in the same way that the variance of a Gaussian distribution determines the spread around the mean [7]. For α-stable processes only the moments of order $r < \alpha$ exist. So estimation methods based on second order statistics of the data cannot be applied. Through out this paper the value of α is assumed known.

4. TIME-COEFFICIENT REPRESENTATIONS FOR THE ANALYSIS OF PPS

The multicomponent signal is modeled by the sum of K PPS

$$x(n) = y(n) + w(n) \tag{5}$$
$$= \sum_{k=1}^{K} s_k(n) + w(n) \quad 0 \le n \le N-1 \tag{6}$$

with

$$s_k(n) = A_k e^{j\phi_k(n)} \tag{7}$$

where the A_k are constant amplitudes, ϕ_k are modeled as in (1) and the $w(n)$ is the additive noise.

4.1. Capon's estimator for PPS analysis

In [5], a modified Capon method is proposed for noiseless multicomponent PPS analysis, where the signal is passed through a time-varying filter of order p, so that only one particular PPS is selected and the others are suppressed. The output signal is given by

$$z(n) = \sum_{m=n-p}^{n} h(n,m)x(m) = \mathbf{h}^T(n)\mathbf{x}(n) \tag{8}$$

where $\mathbf{h}(n)$ is the impulse response of the filter

$$\mathbf{h}(n) = [h(n,n), h(n,n-1), \ldots, h(n,n-p)]^T \tag{9}$$

and $\mathbf{x}(n)$ is the short-time signal vector

$$\mathbf{x}(n) = [x(n), x(n-1), \ldots, x(n-p)] \tag{10}$$

Then for an input signal with phase term $e^{j\phi(n)}$, the transfer function of the filter which can be viewed as the extension of the Zadeh's generalized transfer function to PPS is given by

$$H(n,\beta) = \sum_{m=n-p}^{n} h(n,m)e^{-j[\phi(n)-\phi(m)]} = \mathbf{h}^T(n)\mathbf{b}_p(n,\beta) \tag{11}$$

The vector $\mathbf{b}_p(n,\beta)$ is given by

$$\mathbf{b}_p(n,\beta) = [1, e^{-j[\phi(n)-\phi(n-1)]}, \ldots, e^{-j[\phi(n)-\phi(n-p)]}]^T \tag{12}$$

where $\phi(n) = \sum_{r=0}^{R} \beta_r n^r$ is polynomial phase kernel functions and β is the coefficient vector $\beta = (\beta_1, \beta_2, \ldots, \beta_R)$. Minimizing the power at the filter output subject to the constraint that the signal of interest is passed undistorted, i.e. $H(n,\beta) = 1$, one obtains the time-coefficient representation (TCR) [5]

$$P_{Cap}(n,\beta) = \frac{1}{\mathbf{b}_p^H(n,\beta)R_{x,p}^{-1}(n)\mathbf{b}_p(n,\beta)} \tag{13}$$

where $R_{x,p}(n) = E\{\mathbf{x}^H(n)\mathbf{x}(n)\}$ is the time-dependent autocorrelation of $\mathbf{x}(n)$.

In [9], the authors showed that the spectrogram and the Capon estimator have the same performance in terms of resolution and estimation of the instantaneous frequency of mono and multicomponent signals. However, the Capon estimator can have a better concentration in time-frequency plane. This property is still valid in the case of PPS parameter estimation.

4.2. MUSIC estimator for PPS in Gaussian noise

If we consider that $w(n)$ is Gaussian noise, and by following the same procedure as in [5] with kernel function vector given in (12) we propose the MUSIC estimator. Using the vector $\mathbf{x}(n)$ in (10), we can write

$$\mathbf{x}(n) = \mathbf{B}\mathbf{s}(n) + \mathbf{w}(n) \tag{14}$$

where

$$\mathbf{s}(\mathbf{n}) = \quad s_1(n), \quad \ldots, \quad s_K(n) \quad ^{\mathbf{T}} \tag{15}$$

$$B = \quad \mathbf{b}_p(n,\beta_1), \quad \ldots, \quad \mathbf{b}_p(n,\beta_K) \tag{16}$$

and the noise vector

$$\mathbf{w}(n) = \quad w(n), \quad \ldots, \quad w(n-p) \tag{17}$$

It can be shown that the covariance matrix can be decomposed into two subspaces : signal and noise subspaces [10]. The MUSIC estimator can be written as

$$P_{MUSIC}(n,\beta) = \frac{1}{\mathbf{b}_p^H(n,\beta)E_{x,p}E_{x,p}^H\mathbf{b}_p(n,\beta)} \tag{18}$$

where $E_{x,p} = [e_{K+1}e_{K+2}\ldots e_p]$ is obtained by performing the eigendecomposition on the covariance matrix $R_{x,p}(n)$ and retaining eigenvectors vectors associated to the smallest $p-K$ eigenvalues of the covariance matrix.

The autocorrelation matrix in equation (13) is singular, the problem of inversion can be solved by using diagonal loading ([5], eq. 12) which leads to an additional parameter to be determined. The use of matrix decomposition allows to solve this problem. On the other hand, it is possible to reduce the computational complexity of the MUSIC estimator by using algorithms to estimate the noise subspace without eigendecomposition such as the propagator method.

4.3. Proposed FLOM-MUSIC as an estimator of PPS in impulsive noise

Many papers have treated the problem of direction of arrival estimation (DOA) in the presence of impulsive noise. algorithms like ROC-MUSIC and FLOM-MUSIC have been introduced in [8, 11]. We propose to modify the above MUSIC estimator in (18) to estimate the parameter of PPS in impulsive α-stable. In following we consider only FLOM-MUSIC [11]. Assuming that the noise $w(n)$ in (6) is impulsive with α stable distribution, the second order statistics (SOS) can not be applied. In this case the covariation matrix for α-stable processes is equivalent to the covariance matrix in the case of gaussian noise. In this paper we consider $1 < \alpha \le 2$. For $\alpha < 1$, one can use the zero-memory nonlinearity to clip the impulsive noise [12].

The (i,j)th element of the covariation matrix $\widehat{C}$ are obtained using the vector in (10) as defined in [11]

$$\widehat{C}_{x,p}(i,j) = E\{x(i)|x(j)|^{r-2}x^*(j)\} \tag{19}$$

where the value of the fractional moment r must satisfy the following inequality $1 < r < \alpha \leq 2$, so the matrix $\widehat{C}$ is bounded and can be written using (14) in the form [11]

$$\widehat{C} = B\Lambda B^H + \delta I \tag{20}$$

where B is defined in (16). Λ, and δ can be derived from ([11], theorem 2). The robust time-coefficient representation is given as follows

$$P_{FLOM-MUSIC}(n,\beta) = \frac{1}{\mathbf{b}_p^H(n,\beta)\overline{E}_{x,p}\overline{E}_{x,p}^H\mathbf{b}_p(n,\beta)} \tag{21}$$

where $\overline{E}_{x,p} = [\overline{e}_{K+1}\overline{e}_{K+2}\ldots\overline{e}_p]$ is obtained by performing the SVD on the matrix $\widehat{C}$ and retaining the left singular vectors associated to the smallest $p-K$ singular values of $\widehat{C}$.

5. SIMULATION RESULTS

In this section, we will demonstrate the performance gains when using fractional lower order statistics. In our simulations, we used signals of order $M = 4$ and kernel function phases $\phi(n) = (\pi/255)kn^4$, with $0 \leq k \leq 100$. The filter order $p = 50$. First, we consider a monocomponent PPS as given below

$$x(n) = \begin{cases} e^{j(10\pi/255)n^4}, & 0 \leq n \leq 63 \\ e^{j(50\pi/255)n^4},. & 64 \leq n \leq 254 \end{cases} \tag{22}$$

Figure 1 shows the Capon estimator for the fourth order noiseless PPS. Now, we consider a complex SαS noise with the following

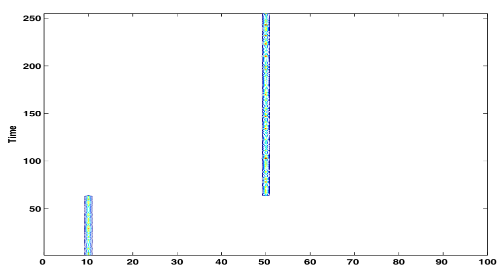


Fig. 1. Capon estimator for the fourth order PPS

parameters $\alpha = 1.2$, $\gamma = 1$. Figure 2(a) shows the effect of the implusive noise on the MUSIC estimator. Using the proposed FLOM-MUSIC with fractional moment $r = 1.1$, we can distinguish the two rays corresponding to the value of the phase parameters as shown in figure 2(b).

Now, we consider a two-component fourth order polynomial phase signal whose Capon time-coefficient representation is shown in figure 3.

$$x(n)) = e^{j(10\pi/255)n^4} + e^{j(50\pi/255)n^4}, 0 \leq n \leq 254$$

Fig. 2(b) shows again the outperforming results of FLOM-MUSIC w.r.t. standard algorithms illustrated in figures 4(a) and 4(b).

From simulations, the choice of the order of the filter p is important, an example in figure (6) shows the TCR for $p = 10$ and $p = 30$. We observe that increasing the value of p gives better

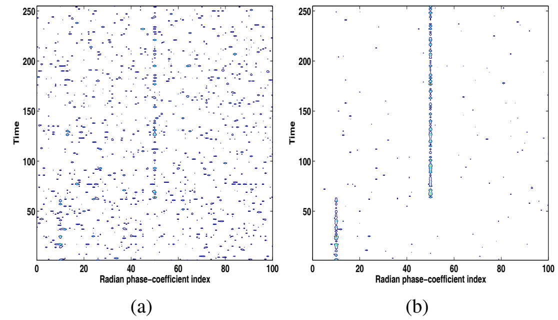


Fig. 2. (a): MUSIC and (b): FLOM-MUSIC estimators of fourth order PPS in α-stable noise with $\alpha = 1.2$ and $\gamma = 1$

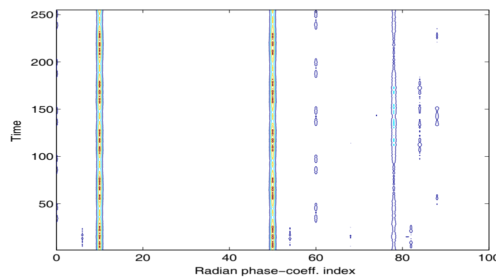


Fig. 3. Capon estimator for two fourth order PPS

representation. On the other hand, increasing the value of the dispersion γ beyond value 3 (GSNR=-5dB) with worst case $\alpha = 1.01$ leads to a degraded TCR. The GSNR is defined according to [8]

$$GSNR = 10\log \left. \frac{1}{\gamma N}\sum_{n=0}^{N-1}|s(n)|^2\right) \tag{23}$$

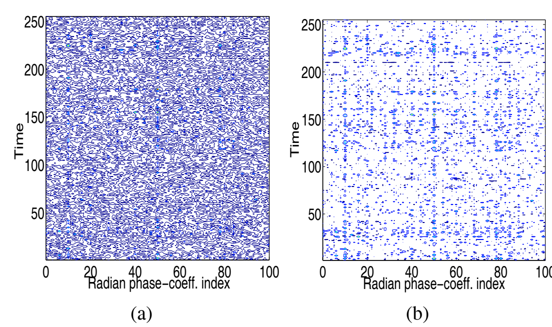


Fig. 4. (a): Capon and (b): MUSIC estimator of two fourth order PPS in α-stable noise with $\alpha = 1.2$ and $\gamma = 1$

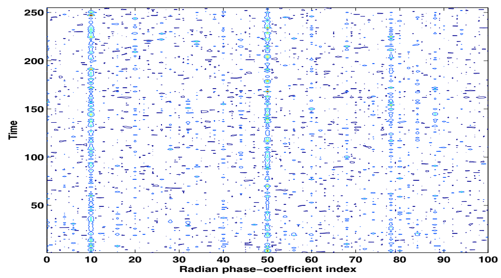
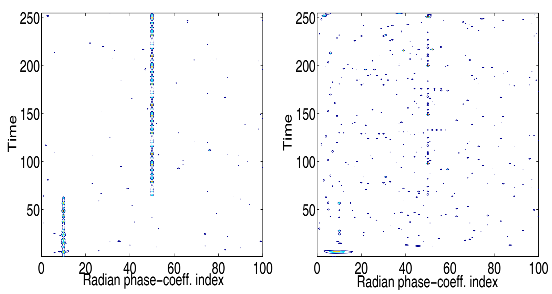


Fig. 5. FLOM-MUSIC estimator for two fourth order PPS in α-stable noise with $\alpha = 1.2$ and $\gamma = 1$



(a) (b)

Fig. 6. FLOM-MUSIC for two different values of the filter order (a): $p = 10$, (b): $p = 30$

6. CONCLUSION

In this paper, we reviewed a new time-coefficient representation (TCR) for polynomial phase signals. We proposed to use the MUSIC algorithm to estimate the values of the parameters of the phase of PPS affected by Gaussian noise. From simulation we showed that impulsive noise degrades considerably the TCR as it is the case for time-frequency representation (TFR). In order to attenuate the effect of impulsive noise, we proposed a Fractional Lower Order Moment based MUSIC to estimate these parameters for PPS affected by impulsive α-stable noise. From simulations, we observed that the approaches considered in this paper performed significantly better than the standard algorithms. Future work, we can reduce the computational complexity of the MUSIC algorithm by using recently developed techniques in array processing which compute the noise subspace without SVD or eigendecomposition.

7. REFERENCES

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