Two-Dimensional Closely Spaced Frequency Estimation Using Decimation Technique

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ABSTRACT

Based on the two-dimensional harmonic model, this paper studies the problem of estimating the frequencies of closely spaced complex exponentials in the presence of colored noise, and presents a new estimation approach using the two-dimensional decimation technique. By using the capability of decimation in the time domain to increase the frequency intervals, the proposed method separates the frequencies in the frequency domain, and gives the exact frequency estimation by using the cumulant based matrix pencil method. The proposed method is easy to realize and has successfully improved the performance of the existing two-dimensional frequency estimation methods in the case of closely spaced frequencies. Simulations are provided to show its performances.

1. INTRODUCTION

Two-dimensional harmonic frequency estimation in the presence of noise is widely used in various applications, such as radar, sonar, biomedicine and physical geography. Many high-resolution frequency estimation methods [1]-[4] have been proposed according to different noise environment. Subspace-based method, matrix pencil method and linear prediction method are considered to be effective in the presence of white noise. When the additive colored noise is considered, there also exist other effective frequency estimation methods using higher order statistics based on the statistical characteristics of colored noise, e.g. the method proposed in [5] which is suitable to the MA modeled noise.

When two-dimensional frequencies are closely spaced in frequency domain, all the above-mentioned estimation methods perform poorly. In other words, when the frequency intervals are reduced to a certain extent, the traditional methods restricted by frequency resolutions can no longer give exact estimations [6], and this frequency resolution problem cannot be simply resolved by over sampling. In fact, for closely space frequencies the variance of the estimations increases with over sampling due to the decrease in the frequency spacing despite the increase in the number of samples [7]. Because decimation can increase the frequency intervals [8], in this paper we propose a new two-dimensional closely spaced frequency estimation method by using two-dimensional decimation on the basis of over sampling, which has a greatly improved performance compared with other traditional methods.

We first present an cumulant based matrix pencil method (CMP), which can accurately estimate two-dimensional frequencies in the presence of Gaussian colored noise restrained by fourth order cumulant [9]. Then we use the two-dimensional decimation technique to modify the CMP method to estimate closely spaced frequencies. Because of the fact that the fourth order cumulant of the over sampling data is of the same form with that of the two-dimensional harmonics, we decimate the fourth order cumulant instead of the noisy observed data, and get more accurate estimation results. The simulation results in the final of the paper illustrate the effectiveness of the proposed method. Because of the simpleness of the two-dimensional decimation technique proposed in this paper, it can also be used in other two-dimensional frequency estimation methods, which perform poorly in the case of closely spaced frequencies.

2. IMPROVED MATRIX PENCIL METHOD BASED ON THE FOURTH ORDER CUMULANT

The two-dimensional harmonics is assumed to be composed of K components

$$x(m,n)=\sum_{k=1}^{K} a_k \exp(j2\pi f_{k1}m + j2\pi f_{k2}n + j\varphi_k) \tag{1}$$

Where $\{x(m,n)\}$ contains K components, a_k , φ_k , f_{k1} and f_{k2} respectively represent the amplitude, phase, the first and the second normalized frequency of the k th component. φ_k is uniformly distributed on $[-\pi,\pi)$. In the presence of noise, the observed signal is

$$y(m,n)=x(m,n)+w(m,n) \tag{2}$$

Where $\{w(m,n)\}$ is the additive colored Gaussian noise with variance of σ^2.

Because the fourth order cumulant is able to restrain observed signal is equal to that of the two-dimensional

$$C_{i_1 i_2} = \begin{bmatrix} C(i_1,i_2) & C(i_1,i_2+\beta_2) & \cdots & C(i_1,i_2+\beta_2(\frac{N}{\beta_2}-1)) \\ C(i_1+\beta_1,i_2) & C(i_1+\beta_1,i_2+\beta_2) & \cdots & C(i_1+\beta_1,i_2+\beta_2(\frac{N}{\beta_2}-1)) \\ \vdots & \vdots & \ddots & \vdots \\ C(i_1+\beta_1(\frac{M}{\beta_1}-1),i_2) & C(i_1+\beta_1(\frac{M}{\beta_1}-1),i_2+\beta_2) & \cdots & C(i_1+\beta_1(\frac{M}{\beta_1}-1),i_2+\beta_2(\frac{N}{\beta_2}-1)) \end{bmatrix}$$
$$= \begin{bmatrix} C_{i_1 i_2}(0,0) & C_{i_1 i_2}(0,1) & \cdots & C_{i_1 i_2}(0,\frac{N}{\beta_2}-1)) \\ C_{i_1 i_2}(1,0) & C_{i_1 i_2}(1,1) & \cdots & C_{i_1 i_2}(1,\frac{N}{\beta_2}-1) \\ \vdots & \vdots & \ddots & \vdots \\ C_{i_1 i_2}(\frac{M}{\beta_1}-1,0) & C_{i_1 i_2}(\frac{M}{\beta_1}-1,1) & \cdots & C_{i_1 i_2}(\frac{M}{\beta_1}-1,\frac{N}{\beta_2}-1) \end{bmatrix} \quad (6)$$

$$\begin{aligned} C_{i_1 i_2}(t_1,t_2) &= \sum_{k=1}^{K} -a_k^4 \exp\{j2\pi f_{k1}(t_1\beta_1+i_1)\}\exp\{j2\pi f_{k2}(t_2\beta_2+i_2)\} \\ &= \sum_{k=1}^{K} -a_k^4 \exp\{j2\pi(f_{k1}i_1+f_{k2}i_2)\}\exp\{j2\pi\beta_1 f_{k1}t_1\}\exp\{j2\pi\beta_2 f_{k2}t_2\} \\ &= \sum_{k=1}^{K} a_k^{'} \exp\{j2\pi f_{k1}^{'}t_1\}\exp\{j2\pi f_{k2}^{'}t_2\} \end{aligned} \quad (8)$$

harmonics. We define

$$\begin{aligned} &C_{4x}(\tau_1,\tau_2;\tau_3,\tau_4;\tau_5,\tau_6) \\ &= \sum_{k=1}^{K} -a_k^4 \exp\{j2\pi(f_{k1}(\tau_3-\tau_1-\tau_5) \\ &\qquad + f_{k2}(\tau_4-\tau_2-\tau_6))\} \end{aligned} \quad (3)$$

From (3) we can see that, when $(\tau_1,\tau_2)=(\tau_5,\tau_6)=(0,0)$ and $(\tau_3,\tau_4)=(m,n)$, the fourth order cumulant of the observed signal is

$$\begin{aligned} &C(m,n) = C_{4x}(0,0;m,n;0,0) \\ &= \sum_{k=1}^{K} -a_k^4 \exp\{j2\pi(f_{k1}m+f_{k2}n)\} \end{aligned} \quad (4)$$

It is clear from (4) that the fourth order cumulant of the observed signal is of the same form with that of the two-dimensional harmonics. So we consider to use the fourth order cumulant of the observed signal $C(m,n)$ in the matrix pencil method proposed in [2].

We first use the following cumulant matrix to substitute the data matrix in [2]

$$C = \begin{bmatrix} C(0,0) & C(0,1) & \cdots & C(0,N-1) \\ C(1,0) & C(1,1) & \cdots & C(1,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ C(M-1,0) & C(M-1,1) & \cdots & C(M-1,N-1) \end{bmatrix} \quad (5)$$

Then according to the matrix pencil (MP) method we can easily estimate the two-dimensional frequencies (f_{k1},f_{k2}) in the colored noise.

3. CLOSELY SPACED FREQUENCY ESTIMATION BASED ON TWO-DIMENSIONAL DECIMATION TECHNIQUE

The above-mentioned CMP method performs poorly in the case of closely spaced frequencies. Now we use the two-dimensional decimation technique to the CMP method in order to estimate closely spaced frequencies in colored Gaussian noise.

First, we get an over sampling serial $y(m,n)$, and the two-dimensional sampling frequencies are $\{f_{sample1}, f_{sample2}\}$. Because the fourth order cumulant of the observed data not only restrains the effect of the colored Gaussian noise, but also has the same form with two-dimensional harmonics, we decimate the fourth order cumulant matrix C, which is composed as shown in (5), instead of the over sampling serial $y(m,n)$. With the two-dimensional decimation factor $\{\beta_1,\beta_2\}$, the decimated matrixes $C_{i_1 i_2}$ can be written as (6), Where $i_1=0,1,\cdots,\beta_1-1$, $i_2=0,1,\cdots,\beta_2-1$.

In order to avoid the distortion of the original signal, the two-dimensional decimation factor should satisfy

$$\beta_1 < f_{sample1}/(2f_1)\,,\quad \beta_2 < f_{sample2}/(2f_2) \quad (7)$$

where

$$f_1 = \max\{f_{k1}, k = 1, \cdots, K\}$$
$$f_2 = \max\{f_{k2}, k = 1, \cdots, K\}$$

Using the model of (4), the components of $C_{i_1 i_2}$ can be written as (8), Where $a_k^{'} = -a_k^4 \exp\{j2\pi(f_{k1}i_1 + f_{k2}i_2)\}$, $f_{k1}^{'} = \beta_1 f_{k1}$, $f_{k2}^{'} = \beta_2 f_{k2}$, $k = 1,2,\cdots,K$, $t_1 = 0,1,\cdots,M/\beta_1 - 1$, $t_2 = 0,1,\cdots,N/\beta_2 - 1$. From (8) we can see that after the two-dimensional decimation practice, the new assumed signal frequencies $\{f_{k1}^{'}, f_{k2}^{'}\}$ are β_1 and β_2 times of the original signal frequencies $\{f_{k1}, f_{k2}\}$, so the frequency separation is artificially increased. However, the matrix $C_{i_1 i_2}$ only contains a part of the available data. In order to get more accurate frequency estimations, we should make full use of the decimated matrixes $\{C_{i_1 i_2}; i_1 = 0,1,\cdots,\beta_1 - 1$, $i_2 = 0,1,\cdots,\beta_2 - 1\}$. A joint decimated matrix can be linearly combined as

$$C_T = \sum_{i_1=0}^{\beta_1-1}\sum_{i_2=0}^{\beta_2-1} C_{i_1 i_2} \tag{9}$$

Because the calculation here is linear combination, the components in the matrix C_T still have the form of two-dimensional harmonics as shown in (8). Replacing the fourth order matrix C in the CMP method by C_T, we can easily estimate the assumed two-dimensional frequencies $\{f_{k1}^{'}, f_{k2}^{'}\}$ that has been amplified by the decimation factor $\{\beta_1, \beta_2\}$.

4. SIMULATIONS

In this part we will show the high resolution and the effectiveness of the proposed method, through Monte Calro simulations. The signal model is assumed as

$$y(m,n) = \sqrt{2}\exp\{j2\pi(f_{11}m + f_{21}n)\} + \sqrt{2}\exp\{j2\pi(f_{12}m + f_{22}n)\} + w(m,n)$$

Where $w(m,n)$ is colored Gaussian noise with the variance σ^2. Signal to noise ratios (SNR) is defined as $SNR = -10\log\sigma^2$.

Experiment1. This experiment will illustrate the effectiveness of the CMP method in the colored noise. The two-dimensional frequencies are $f_{11} = 0.2$, $f_{21} = 0.36$, $f_{12} = 0.4$, $f_{22} = 0.24$, sampled at the frequency of 1, the observation matrix composed of $y(m,n)$ is of the size of 64×64. The mean squared error (MSE) curves of the CMP and MP method varying with SNR are given in figure 1, where the above two curves are of the CMP method and the bottom two curves are of the MP method. Because the assumed signal has two components, the MSE is defined as $0.5 \times (mse1 + mse2)$, where $mse1$ and $mse2$ are MSE of the first and second component frequencies. $f1$ and $f2$ represent the first and the second dimensional frequency.

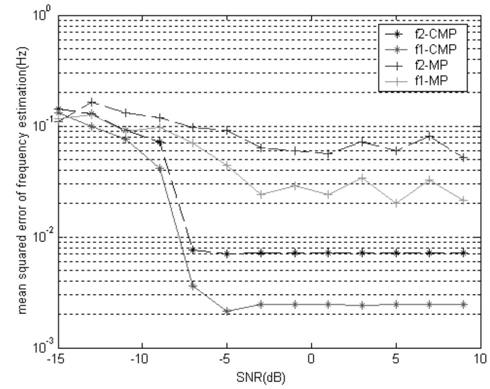

Fig. 1. Performance of the CMP and MP in different SNR

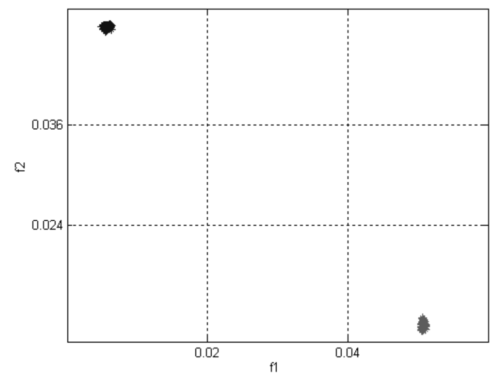

Fig. 2. Closely spaced frequencies estimation by CMP

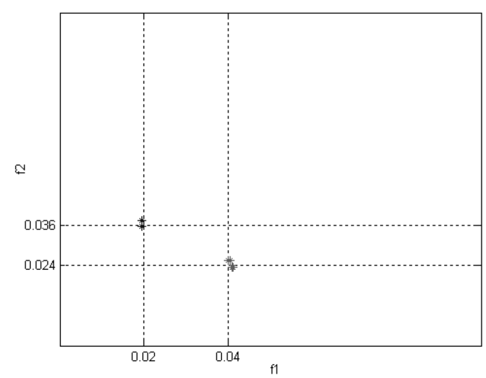

Fig. 3. Closely spaced frequencies estimation by DCMP

Experiment2. The CMP method cannot estimate closely spaced frequencies accurately. This experiment is going to show the effectiveness of the decimation based CMP (DCMP) method. We assume $f_{11}=0.02$, $f_{21}=0.036$, $f_{12}=0.04$, $f_{22}=0.024$, the observed data matrix is 128×128, and the two-dimensional decimation factor $\beta_1=\beta_2=10$. When $SNR=0$dB, the Monte Calro experiment results are shown in figure 2 and figure 3. From figure 2 and figure 3, we can see that the CMP method is ineffective in the colored noise, while the decimation based CMP method can estimate the two-dimensional closely spaced frequencies accurately. In different SNR cases, the MSE curves of the CMP method and the decimation based CMP method are given in figure 4. It is clearly illustrated that the use of the two-dimensional decimation factors can greatly improve the performance of the CMP method.

Experiment 3. In the practice of the two-dimensional decimation, the choosing of the decimation factor is of vital importance, which will influence the estimation result directly. The two-dimensional decimation factor should satisfy the formula (7), and as shown in (8), the frequency resolution of the method will improve with the decimation factors. But the decimation factors should not be chosen too large. On one hand, when the decimation factors are too large, the vector length of the decimated data will be decreased. On the other hand, confined by the resolution of the method itself, the accuracy of the frequency estimation will not increase any more when the decimation factors are too large. Figure 5 illustrates the influence of the decimation factors to the frequency estimation results. In the experiment, we choose $\beta_1=\beta_2$ for convenience, and the other parameters are the same as in experiment 2.

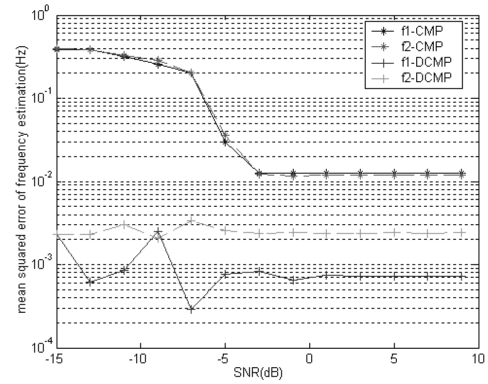

Fig. 4. Performance of the DCMP and CMP in different SNR

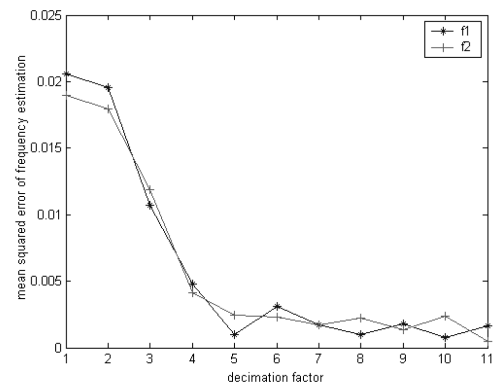

Fig. 5. The influence of the decimation factor

5. CONCLUSIONS

In this paper, we introduce the two-dimensional decimation technique to the closely spaced frequency estimation problem, and propose a new decimation based CMP method. Simulations illustrate that the use of the two-dimensional decimation technique can greatly improve the performance of the CMP method. Compared with the existed two-dimensional frequency estimation method, the method proposed in this paper is more effective in the case of the closely spaced frequencies.

ACKNOWLEDGMENT

Thanks to the Doctor Special Foundation of China (2001404) and the National Natural Science Foundation of China (60172032) who support this project.

6. REFERENCES

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