SUBBAND DECOMPOSITION USING MULTICHANNEL AR SPECTRAL ESTIMATION

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ABSTRACT

Subband decomposition has been shown to be a useful tool for spectral estimation, in particular when parametric methods have to be considered. Indeed, the loss of observed samples due to decimation can be compensated by the use of a suitable model, if available. This paper studies a Subband Multichannel Autoregressive Spectral Estimation (SMASE) method. The proposed method decomposes the observed signal through an appropriate filter bank and processes the decimated signals by means of a multichannel Autoregressive (AR) model. This model takes advantage of known correlations between different subband signals. This a priori knowledge allows to improve spectral estimation performance. Simulation results illustrate the interest of the proposed methodology for signals with continuous spectra and for sinusoids.

1. INTRODUCTION

Recent papers of the signal processing literature have shown the interest of subband decomposition for spectral estimation [1], [2]. Most interesting properties of subband decomposition have been demonstrated for a bank of ideal infinitely sharp bandpass filters. However, experimental results have shown that they can also apply to non-ideal filterbanks such as modified Quadrature-Mirror Filters (QMF's) or cosine modulated filterbanks [2].

The main benefits provided by subband decomposition in the case of parametric spectral estimation include model order reduction (which leads to a lower condition number for autocorrelation matrices [3]), spectral density whiteness and reduction of linear prediction error for autoregressive (AR) estimation [1]. It has also been noted that the frequency spacing and local Signal to Noise Ratio (SNR) are increased by the decimation ratio, in the case of a peaked spectrum signal [4].

Unfortunately, new problems appear when performing parametric spectral estimation on subband signals. One of these problems is spectral overlapping: the same harmonic component may appear in two contiguous subbands at two different frequencies, when using non-ideal filterbanks. Another classical problem is due to decimation: the variance of autocorrelation estimators increases because decimation reduces the number of available samples. The first disadvantage has been addressed in two recent papers [5] and [6], where (non real-time) procedures have been proposed to perform subband spectral estimation without discontinuities or aliasing (even at subband borders). These procedures are appropriate for a uniform filterbank. However, it is important to note that the methods should also be applied to any kind of filterbank. This paper studies a Subband Multichannel Autoregressive Spectral Estimation (SMASE) method in order to tackle the second drawback. In particular, the correlations between consecutive samples of a subband signal are used to build an autocorrelation estimator with reduced variance.

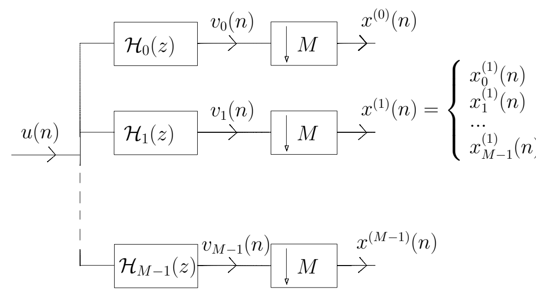

Fig. 1. Uniform Analysis filterbank with J subbands and a decimation ratio of M.

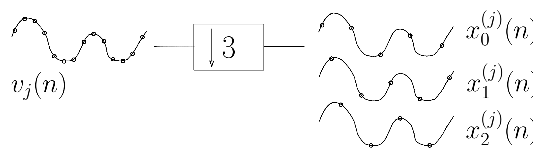

Fig. 2. Construction of $M = 3$ decimated subsequences.

Section 2 formulates the problem and introduces the SMASE model. The proposed subband spectral estimation procedure is explained in section 3. Some simulation results are presented in section 4. Conclusions and perspectives are reported in the last section.

2. SMASE MODEL

Consider the jth branch of the filterbank depicted in figure 1. Denote as $u(n), n = 0, \ldots, N-1$ the input random sequence, $\mathcal{H}_j(z)$ the transmittance of the jth subband filter and $v^{(j)}(n), n = 0, \ldots, N-1$ the output of this subband filter. The input samples are filtered by the jth subband filter and decimated M times (by M-fold decimators) yielding M subsequences $x_m^{(j)}(n)$ (each with N/M samples):

$$x_m^{(j)}(n) = v_j(Mn - m) \qquad \forall m = 0, \ldots, M-1. \tag{1}$$

The construction of the decimated sequences $x_m^{(j)}$ is illustrated on figure 2 (note that $m = 0, ..., M-1$). For brievity, the subscript j representing the subband number is omitted in what follows. Denote as $\underline{x}(n) = [x_0(n), \ldots, x_{M-1}(n)]^T$ the vector of size M constructed from the nth samples of the different decimators. The correlation matrix of this vector is defined as:

$$K_x(q) = E[\underline{x}(n)\underline{x}^H(n-q)], \tag{2}$$

where $q = -(N/M-1), \ldots, (N/M-1)$, $E[]$ denotes the mathematical expectation operator and ^{H} the hermitian transpose. The diagonal terms of the matrix $K_x(q)$ are made of autocorrelations of the M subsequences, whereas the elements of the non diagonal terms are intercorrelations between the different subsequences. Note that the intercorrelations between two decimated sequences of the same subband are defined as:

$$r_{x_{m_1}x_{m_2}}(k) = r_v(Mk + m_2 - m_1), \quad \forall m_1, m_2. \tag{3}$$

The principle of the SMASE method is the following: instead of estimating the value of $x_m(n)$ (for a given m) as a function of $x_m(n-1), \ldots, x_m(n-p)$ (where p is the model order), the whole set of observations $\underline{x}(n) = [x_0(n), ..., x_{M-1}(n)]^T$ is expressed as a function of $\underline{x}(n-1), \ldots, \underline{x}(n-p)$. This is the spirit of multichannel AR modelling, whose theory can be found in standard textbooks such as [7, p. 457]. The pth order multichannel predictor $\widehat{\underline{x}}(n)$ is recalled below:

$$\widehat{\underline{x}}(n) = -\sum_{k=1}^{p} A_k \underline{x}(n-k), \tag{4}$$

where A_k are matrices of size MxM. The corresponding prediction error $e(n)$ expresses as:

$$\begin{aligned} \underline{e}(n) &= \underline{x}(n) - \widehat{\underline{x}}(n) \\ &= \underline{x}(n) + \sum_{k=1}^{p} A_k \underline{x}(n-k). \end{aligned} \tag{5}$$

3. SUBBAND SPECTRAL ESTIMATION

3.1. Estimation of Prediction Matrices

The prediction matrices A_k of (4) are classically estimated by minimizing an appropriate mean square error (MSE). In this paper, we propose to minimize the sum of the prediction errors associated to each monodimensional component of (5). This yields the following p systems of equations [7]:

$$K_x(q) = -\sum_{k=1}^{p} A_k K_x(q-k), \quad \forall q \in \{1, ..., p\}. \tag{6}$$

The determination of matrices A_k from (6) requires the inversion of a block-matrix of Toeplitz matrices of size $(Mp) \times (Mp)$. The particular structure of this matrix allows the use of an order-recursive algorithm (closed to the classical Levinson-Durbin algorithm): the Wiggins-Robinson Algorithm. Further details can be found in [8]. Note that the main difference between the Levinson-Durbin and Wiggins-Robinson algorithms is that in the last case, forward and backward prediction coefficient matrices are no longer linked by a simple relationship. Other interesting works on block-matrices of toeplitz matrices can be found in [9].

As for classical monodimensional AR models, it is easy to show that the prediction error vector is centered ($E[\underline{e}(n)] = 0$). Moreover, the second order properties of $\underline{e}(n)$ can be summarized as follows:

$$E\left[\underline{e}(n)\underline{e}^H(n-s)\right] = \begin{cases} 0 & \text{if } s \neq 0 \\ \Sigma & \text{if } s = 0 \end{cases} \tag{7}$$

where

$$\Sigma = K_x(0) + \sum_{k=1}^{p} A_k K_x^H(k). \tag{8}$$

In other words, the prediction error vector $\underline{e}(n)$ of a multi-dimensional AR process is a vectorial white noise. Note that each component $x_m(n)$ of the vector $\underline{x}(n)$ can be modelled as an ARMA sequence [10]. An estimate $\widehat{\Sigma}$ of the matrix Σ can be easily obtained from standard autocorrelation estimators.

3.2. Spectral Analysis based on Multichannel AR Modeling

Multichannel spectral analysis is classically performed by means of a cross spectrum matrix $P_x(f)$. This matrix is defined as the discrete Fourier transform of the correlation matrix $K_x(q)$:

$$P_x(f) = DFT[K_x(q)], \tag{9}$$

where the Fourier transform is computed separately on each component of the correlation matrix. This definition guaranties that the matrix $P_x(f)$ is Hermitian symmetric. The power spectral densities (PSDs) of the monodimensional components of $\underline{x}(n)$ are located on the main diagonal of $P_x(f)$, whereas offdiagonal elements are cross spectra between the different components of $\underline{x}(n)$. Denote as

$$A = I + \sum_{k=1}^{p} A_k e^{-i2\pi fk} \tag{10}$$

where I is the $M \times M$ identity matrix (note that the matrix A has the same size $M \times M$). The AR multichannel spectral estimator can be derived as follows [7, p. 460]:

$$\widehat{P}_{AR}(f) = \left(\widehat{A}\right)^{-1} \widehat{\Sigma} \left(\widehat{A}^{-1}\right)^H. \tag{11}$$

Note that M estimated PSDs of $x_0(n)$, ..., $x_{M-1}(n)$ can be found on the main diagonal of $\widehat{P}_{AR}(f)$. The final estimates of the PSDs are obtained by averaging the appropriate M diagonal elements of $\widehat{P}_{AR}(f)$.

Remark: denote as p_{full} and p the model orders for the fullband and subband AR processes respectively. When $p = p_{full}$, the variance of autocorrelation estimates computed from subband signals are larger than the variances obtained from the fullband signal:

$$\|Var[\widehat{\underline{r}}_{sub}]\| > \|Var[\widehat{\underline{r}}_{full}]\|,$$

where $\widehat{\underline{r}}_{sub}$ and $\widehat{\underline{r}}_{full}$ are the estimated autocorrelation vectors of subband and fullband signals. Conversly, when $p = p_{full}/M$, the variances of $\widehat{\underline{r}}_{sub}$ and $\widehat{\underline{r}}_{full}$ are similar. However, it is important to note that the length of the vector $\underline{r}_{sub}$ is M times smaller than that of the vector $\underline{r}_{full}$, resulting in a loss of information. The SMASE compensates this loss of information by considering M decimated signals by branch of the filterbank.

4. SIMULATION RESULTS

SMASE modelling has been presented without any filter implementation consideration. However, it is well-known that non-ideal filters result in spectral overlapping. In order to avoid this problem, a specific procedure has been proposed in [6]. This procedure is composed of a warping device (placed before the filterbank) which is a bank of FIR modulated comb filters of order L. This results in a specific spectral estimation which differs from one frequency to another. As a consequence, the linear prediction errors also depend

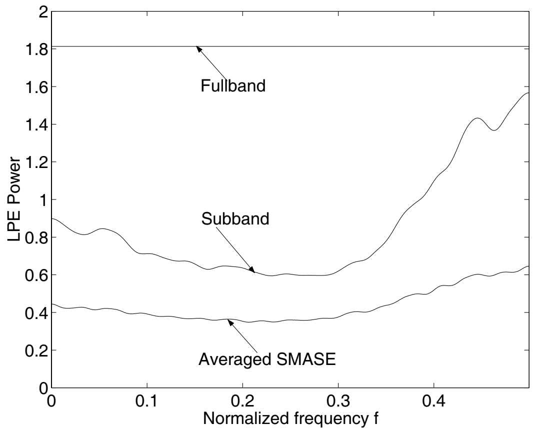

Fig. 3. Powers of Linear prediction Errors.

on the frequency, as shown for instance on figure 3. The transmittance of the jth FIR modulated comb filter has been chosen as follows:

$$|\mathcal{H}_j(e^{i2\pi f})|^2 = \begin{cases} \frac{1}{L}\frac{\sin^2 \pi(f-F_j)L}{\sin^2 \pi(f-F_j)} & \text{if } f \neq F_j, \\ L & \text{if } f = F_j, \end{cases} \tag{12}$$

where j is the subscript of the considered subband ($j \in \{0, ..., M-1\}$ and $F_j = j\frac{0.5}{M} + \frac{0.25}{M}$ is the center of the j^{th} subband. Thus, the transmittance equals periodically zero around F_j with period $1/L$. This is the alias-free condition for subband spectral estimation derived in [6]. Moreover, the filterbank parameters are $L = 16$ and $M = 4$. Two sets of simulations have been conducted for signals with continuous and line spectra. These simulations are discussed below.

4.1. Signals with Continuous Spectra

The first experiments have been carried out for 6th order moving average (MA) signals with $N = 100$ samples. These signals are obtained at the output of a FIR filter driven by a white noise $w(n)$ of variance $\sigma_w^2 = 0.8$. The MA parameter vector is

$$b = [-0.1837, 0.5373, -0.3252, 0.4351, 0.1419, 0.01174].$$

The filterbank is a uniform maximally decimated comb filterbank with $L = 16$ and $M = 4$. The model orders for fullband and subband AR spectral estimation are $p_{full} = 16$ and $p = 4$ respectively. Note that the model order in the subbands is $p = p_{full}/M$ in order to use the same autocorrelation samples from the filtered signal $v(n)$.

The Linear Prediction Errors (LPEs) obtained from fullband and subband AR modelling are depicted in figure 3. The subband LPEs are clearly lower than the fullband ones, which reflects the advantage of working in subbands. The SMASE method yields M predictors for each subband. The corresponding LPEs have been averaged and plotted in figure 3. The averaged LPEs are clearly lower than those obtained with fullband and subband signals. Considering several decimated signals per subband allows to improve prediction, as expected.

Next simulations illustrate the performance of spectral estimators based on fullband and subband signals. The biases and variances of the different estimators (computed from 50 Monte-Carlo runs) are plotted in figures 4 and 5. Note that the subband multichannel AR spectral estimator has been obtained by averaging the M spectral estimators provided by the diagonal of matrix $\widehat{P}_{AR}(f)$ (see eq.(11)). These results show that the SMASE method allows to reduce the bias and variance of spectral estimators for signals with continuous spectrum.

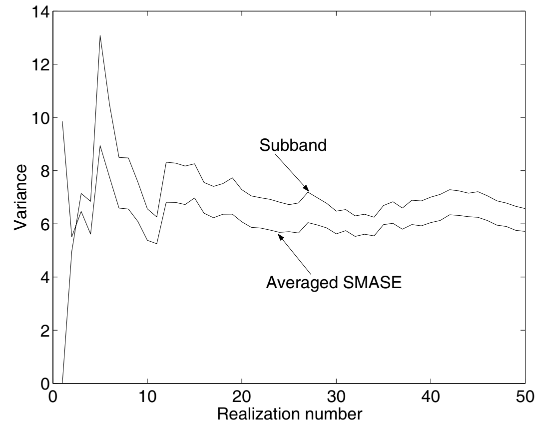

Fig. 4. Spectrum variance versus the realization number.

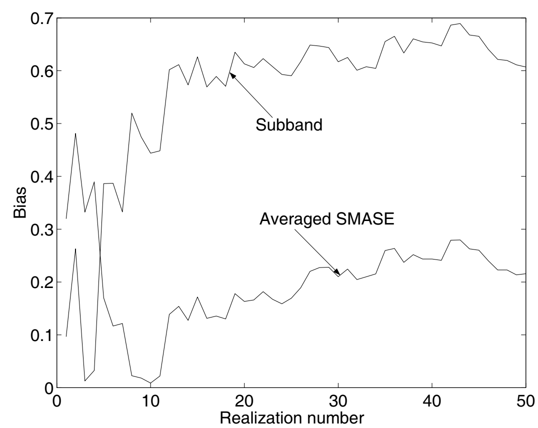

Fig. 5. Spectrum bias versus the realization number.

The next section investigates the performance of the SMASE method for noisy harmonic signals.

4.2. Noisy Harmonic Signals

This section considers a pure sinusoid with amplitude A, normalized frequency f_0 and random phase ϕ uniformly distributed over $[0, 2\pi]$ (the signal parameters are $A = 1$, $f_0 = 0.1$ and $N = 100$). This sinusoid is embedded in an additive white Gaussian noise (n) with variance $\sigma_b^2 = 0.05$ ($SNR = 10\ dB$):

$$u(n) = A \sin(2\pi f_0 n + \phi) + b(n) \tag{13}$$

Simulations are conducted with the same filterbank as in the previous section. The different results have been obtained from 250 Monte-Carlo runs.

Figure 6 shows the powers of LPEs for fullband and subband AR spectral estimators. Similarly to signals with continuous spectra, the SMASE method provides the better results in terms of pre-

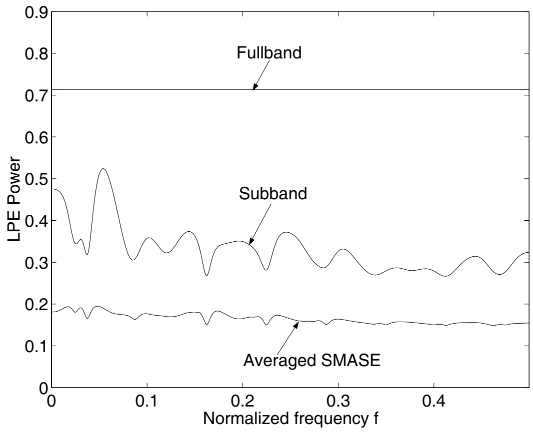

Fig. 6. Linear prediction error powers.

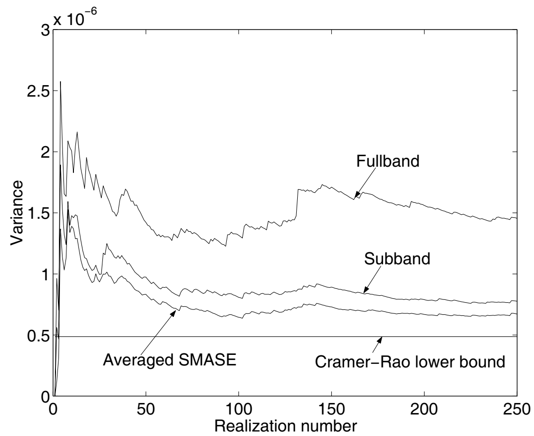

Fig. 7. Variance on f_0 estimation versus the realization number.

diction. The variances of the different frequency estimators (estimators of f_0) are compared to the corresponding Cramer-Rao lower bound in figure 7. The estimator variances are clearly reduced when using the SMASE method. This result is also highlighted on figure 8 where FFT, fullband AR, subband AR and averaged SMASE spectra are represented (zoom in the first subband). To summarize, the SMASE method provides the best spectral estimators in terms of variance and, above all, in terms of spectral resolution.

5. CONCLUSION

This paper studied a new parametric spectral estimation procedure based on multichannel AR modelling. The proposed method used the a priori information provided by the whole knowledge of the autocorrelation function of the filtered signal $v(n)$ on one branch of the filterbank. Simulations were conducted for signals with continuous and line spectra. The proposed method outperformed traditional fullband and subband AR estimators in terms of linear prediction error and bias and variance of frequency estimators.

This communication addressed the important problem of off-line spectral estimation. However, the proposed methodology based on multichannel AR modelling might also be implemented for on-line

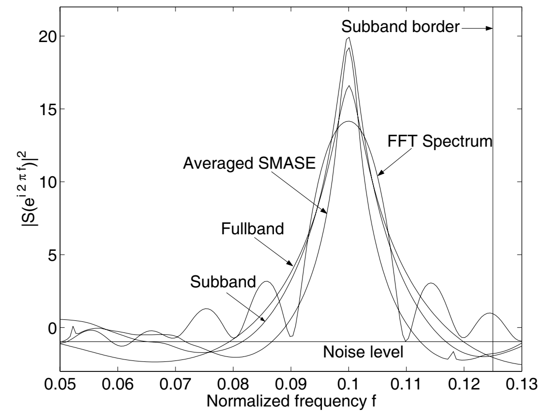

Fig. 8. Estimated spectra averaged on 250 Monte-Carlo runs.

spectral estimation. In particular, it seems interesting to design adaptive filters constructed under the alias-free condition. These filters should allow to reduce the linear prediction errors and the variance of frequency estimators. This work is currently under investigation.

6. REFERENCES

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