TWO-DIMENSIONAL NONPARAMETRIC SPECTRAL ANALYSIS IN THE MISSING DATA CASE

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ABSTRACT

We consider two-dimensional (2-D) nonparametric complex spectral estimation (with its 1-D counterpart as a special case) of data matrices with missing samples occurring in arbitrary patterns. Previously, the MAPES-EM algorithms were developed for the general 1-D missing-data problem and shown to have excellent spectral estimation performance. In this paper, we present 2-D extensions of MAPES-EM and develop another 2-D MAPES algorithm, referred to as MAPES-CM, which solves a maximum likelihood problem iteratively via cyclic maximization (CM). Compared with MAPES-EM, MAPES-CM has similar spectral estimation performance but is computationally much more efficient.

1. INTRODUCTION

Spectral estimation is important in many fields including astronomy, communications, medical imaging, radar, and underwater acoustics. Most existing spectral estimation algorithms are devised for uniformly sampled complete-data sequences. However, the spectral estimation for data sequences with missing samples is also important in a wide range of applications [1] such as synthetic aperture radar (SAR) imaging with angular diversity.

Recently, we have proposed the one-dimensional (1-D) missing-data amplitude and phase estimation via expectation maximization (MAPES-EM) algorithms to deal with the general missing-data problem where the missing data samples occur in arbitrary patterns [1]. However, the MAPES-EM algorithms are computationally intensive. The direct application of MAPES-EM to large data sets, such as two-dimensional (2-D) data, is computationally prohibitive.

We consider herein the problem of 2-D (with its 1-D counterpart as a special case) nonparametric spectral estimation of data matrices with missing data samples occurring in arbitrary patterns. First, we present 2-D extensions of the MAPES-EM algorithms introduced in [1] in the 1-D case. Then we develop a new MAPES algorithm, referred to as MAPES-CM, by solving an ML problem iteratively via cyclic maximization (CM). MAPES-EM and MAPES-CM possess similar spectral estimation performance, but the computational complexity of the latter is much lower than that of the former.

The remainder of this paper is organized as follows. In Section 2, we review the 2-D APES algorithm. In Section 3, we present 2-D extensions of the MAPES-EM algorithms and develop the 2-D MAPES-CM algorithm. Numerical examples are provided in Section 4 to demonstrate the performance of the MAPES algorithms. Finally, Section 5 concludes the paper.

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2. 2-D APES FOR COMPLETE-DATA SPECTRAL ESTIMATION

Consider the problem of estimating the amplitude spectrum of a complex-valued uniformly sampled 2-D discrete-time signal $\{y_{n_1,n_2}\}_{n_1=0,n_2=0}^{N_1-1,N_2-1}$, where the data matrix is $N_1 \times N_2$. We require $N_1 > 1$ and $N_2 \geq 1$, where the special case of $N_2 = 1$ corresponds to the 1-D case.

For a 2-D frequency (ω_1, ω_2) of interest, the signal y_{n_1,n_2} is described as

$$y_{n_1,n_2} = \alpha(\omega_1,\omega_2)e^{j(\omega_1 n_1+\omega_2 n_2)} + e_{n_1,n_2}(\omega_1,\omega_2), \quad (1)$$

where $n_1 = 0,...,N_1-1,\ n_2 = 0,...,N_2-1,\ \omega_1,\omega_2 \in [0, 2\pi)$, $\alpha(\omega_1,\omega_2)$ denotes the complex amplitude of the 2-D sinusoidal component at frequency (ω_1,ω_2), and e_{n_1,n_2} (ω_1,ω_2) denotes the corresponding residual matrix (assumed zero-mean) which includes the unmodeled noise and interference from frequencies other than (ω_1,ω_2).

Partition the $N_1 \times N_2$ data matrix $\mathbf{Y}$ into L_1L_2 overlapping submatrices $\bar{\mathbf{Y}}_{l_1,l_2}$ of size $M_1 \times M_2$. Here $l_1 = 0,...,L_1-1$, $l_2 = 0,...,L_2-1$, $L_1 \triangleq N_1 - M_1 + 1$, and $L_2 \triangleq N_2 - M_2 + 1$. Let

$$\bar{\mathbf{y}}_{l_1,l_2} = \text{vec}[\bar{\mathbf{Y}}_{l_1,l_2}], \quad (2)$$

where vec[$\cdot$] denotes the operation of stacking the columns of a matrix on top of each other. Let

$$\mathbf{a}(\omega_1, \omega_2) = \mathbf{a}_{M_2}(\omega_2) \otimes \mathbf{a}_{M_1}(\omega_1), \quad (3)$$

where $\otimes$ denotes the Kronecker matrix product, and

$$\mathbf{a}_{M_k}(\omega_k) \triangleq [1 \quad e^{j\omega_k} \quad \cdots \quad e^{j(M_k-1)\omega_k}]^T, \quad k = 1, 2 \quad (4)$$

with $(\cdot)^T$ denoting the transpose. Then, according to (1), the snapshot vector $\bar{\mathbf{y}}_{l_1,l_2}$ can be written as

$$\bar{\mathbf{y}}_{l_1,l_2} = [\alpha(\omega_1, \omega_2)\mathbf{a}(\omega_1, \omega_2)] \cdot e^{j(\omega_1 l_1 + \omega_2 l_2)} + \bar{\mathbf{e}}_{l_1,l_2}(\omega_1, \omega_2), \quad (5)$$

where $\bar{\mathbf{e}}_{l_1,l_2}(\omega_1, \omega_2)$ is formed from $\{e_{n_1,n_2}(\omega_1, \omega_2)\}$ in the same way as $\bar{\mathbf{y}}_{l_1,l_2}$ is made from $\{y_{n_1,n_2}\}$. To estimate $\alpha(\omega_1, \omega_2)$, the APES algorithm mimics an ML estimator by assuming that $\{\bar{\mathbf{e}}_{l_1,l_2}(\omega_1, \omega_2)\}_{l_1=0,l_2=0}^{L_1-1,L_2-1}$ are zero-mean circularly symmetric complex Gaussian random vectors that are statistically independent of each other and have the same unknown covariance matrix

$$\mathbf{Q}(\omega_1, \omega_2) = E\left[\bar{\mathbf{e}}_{l_1,l_2}(\omega_1, \omega_2)\bar{\mathbf{e}}^H_{l_1,l_2}(\omega_1, \omega_2)\right], \quad (6)$$

where $(\cdot)^H$ denotes the conjugate transpose. For notational convenience, we drop the explicit frequency dependence on (ω_1, ω_2) in the following derivations.

Using the above assumptions, we get the normalized log-likelihood function of the data snapshots $\{\bar{\mathbf{y}}_{l_1,l_2}\}$ as follows:

$$\frac{1}{L_1 L_2} \ln p\left(\{\bar{\mathbf{y}}_{l_1,l_2}\} | \alpha, \mathbf{Q}\right) = -M_1 M_2 \ln \pi$$
$$- \ln|\mathbf{Q}| - \frac{1}{L_1 L_2} \sum_{l_1=0}^{L_1-1} \sum_{l_2=0}^{L_2-1} [\bar{\mathbf{y}}_{l_1,l_2} - \alpha\mathbf{a}$$
$$e^{j(\omega_1 l_1 + \omega_2 l_2)}\Big]^H \mathbf{Q}^{-1} \Big[\bar{\mathbf{y}}_{l_1,l_2} - \alpha\mathbf{a}e^{j(\omega_1 l_1 + \omega_2 l_2)}\Big] \quad (7)$$

where tr$\{\cdot\}$ and $|\cdot|$ denote the trace and the determinant of a matrix, respectively. Let $\bar{\mathbf{g}}$ denote the normalized 2-D Fourier transform of $\bar{\mathbf{y}}_{l_1,l_2}$: $\bar{\mathbf{g}} = \frac{1}{L_1 L_2} \sum_{l_1=0}^{L_1-1} \sum_{l_2=0}^{L_2-1} \bar{\mathbf{y}}_{l_1,l_2} e^{-j(\omega_1 l_1 + \omega_2 l_2)}$ and define $\hat{\mathbf{R}} = \frac{1}{L_1 L_2} \sum_{l_1=0}^{L_1-1} \sum_{l_2=0}^{L_2-1} \bar{\mathbf{y}}_{l_1,l_2} \bar{\mathbf{y}}^H_{l_1,l_2}$. Maximizing the log-likelihood function in (7) with respect to α and $\mathbf{Q}$ yields

$$\hat{\alpha}_{ML} = \frac{\mathbf{a}^H \hat{\mathbf{Q}}^{-1} \bar{\mathbf{g}}}{\mathbf{a}^H \hat{\mathbf{Q}}^{-1} \mathbf{a}} \quad (8)$$

$$\hat{\mathbf{Q}}_{ML} = \hat{\mathbf{R}} - \bar{\mathbf{g}}\bar{\mathbf{g}}^H + [\hat{\alpha}_{ML}\mathbf{a} - \bar{\mathbf{g}}][\hat{\alpha}_{ML}\mathbf{a} - \bar{\mathbf{g}}]^H. \quad (9)$$

3. 2-D MAPES FOR MISSING-DATA SPECTRAL ESTIMATION

Assume that some elements of the data matrix $\mathbf{Y}$ are missing. Due to these missing data samples, the log-likelihood function (7) cannot be maximized directly. In this section, we will show how to tackle this missing-data problem, in the ML context, through the use of the expectation maximization (EM) and cyclic maximization (CM) algorithms.

3.1. 2-D MAPES-EM (Expectation Maximization)

Letting $\boldsymbol{\theta}$ denote the vector containing the unknowns in $\{\alpha, \mathbf{Q}\}$, the EM algorithm solves the missing-data ML problem iteratively by maximizing the conditional expectation

$$\hat{\boldsymbol{\theta}}^i = \arg\max_{\boldsymbol{\theta}} \mathrm{E}\left\{\ln p(\boldsymbol{\gamma}, \boldsymbol{\mu}|\boldsymbol{\theta}) \,\Big|\, \boldsymbol{\gamma}, \hat{\boldsymbol{\theta}}^{i-1}\right\}. \quad (10)$$

where $\hat{\boldsymbol{\theta}}^i$ is the current estimate of the parameter vector, and $\boldsymbol{\gamma}$ and $\boldsymbol{\mu}$ stand for the available and missing data samples, respectively.

3.1.1. 2-D MAPES-EM1

We assume that the data snapshots $\{\bar{\mathbf{Y}}_{l_1,l_2}\}$ (or $\{\bar{\mathbf{y}}_{l_1,l_2}\}$) are independent of each other, and hence we estimate the missing data separately for different data snapshots. For each data snapshot $\bar{\mathbf{y}}_{l_1,l_2}$, let $\bar{\boldsymbol{\gamma}}_{l_1,l_2}$ and $\bar{\boldsymbol{\mu}}_{l_1,l_2}$ denote the vectors containing the available and missing elements of $\bar{\mathbf{y}}_{l_1,l_2}$, respectively. Assume that $\bar{\boldsymbol{\gamma}}_{l_1,l_2}$ has dimension $g_{l_1,l_2} \times 1$ where $1 \le g_{l_1,l_2} \le M_1 M_2$ is the number of available elements in the snapshot $\bar{\mathbf{y}}_{l_1,l_2}$. Then $\bar{\boldsymbol{\gamma}}_{l_1,l_2}$ and $\bar{\boldsymbol{\mu}}_{l_1,l_2}$ are related to $\bar{\mathbf{y}}_{l_1,l_2}$ by unitary transformations as follows:

$$\bar{\boldsymbol{\gamma}}_{l_1,l_2} = \bar{\mathbf{S}}^T_g(l_1, l_2)\bar{\mathbf{y}}_{l_1,l_2} \quad (11)$$
$$\bar{\boldsymbol{\mu}}_{l_1,l_2} = \bar{\mathbf{S}}^T_m(l_1, l_2)\bar{\mathbf{y}}_{l_1,l_2}, \quad (12)$$

where $\bar{\mathbf{S}}_g(l_1, l_2)$ and $\bar{\mathbf{S}}_m(l_1, l_2)$ are $M_1 M_2 \times g_{l_1,l_2}$ and $M_1 M_2 \times (M_1 M_2 - g_{l_1,l_2})$ unitary selection matrices such that $\bar{\mathbf{S}}^T_g(l_1, l_2)\bar{\mathbf{S}}_g(l_1, l_2) = \mathbf{I}_{g_{l_1,l_2}}$, $\bar{\mathbf{S}}^T_m(l_1, l_2)\bar{\mathbf{S}}_m(l_1, l_2) = \mathbf{I}_{M_1 M_2 - g_{l_1,l_2}}$ and $\bar{\mathbf{S}}^T_g(l_1, l_2)\bar{\mathbf{S}}_m(l_1, l_2) = \mathbf{0}_{g_{l_1,l_2} \times (M_1 M_2 - g_{l_1,l_2})}$. Because we clearly have

$$\bar{\mathbf{y}}_{l_1,l_2} = \bar{\mathbf{S}}_g(l_1, l_2)\bar{\boldsymbol{\gamma}}_{l_1,l_2} + \bar{\mathbf{S}}_m(l_1, l_2)\bar{\boldsymbol{\mu}}_{l_1,l_2}, \quad (13)$$

the joint normalized log-likelihood function of $\{\bar{\boldsymbol{\gamma}}_{l_1,l_2}, \bar{\boldsymbol{\mu}}_{l_1,l_2}\}$ is obtained by substituting (13) into (7).

From the Gaussian assumption on $\bar{\mathbf{y}}_{l_1,l_2}$, it follows that the probability density function of $\bar{\boldsymbol{\mu}}_{l_1,l_2}$ conditioned on $\bar{\boldsymbol{\gamma}}_{l_1,l_2}$ (for given $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}^{i-1}$) is complex Gaussian with mean $\bar{\mathbf{b}}_{l_1,l_2}$ and covariance matrix $\bar{\mathbf{K}}_{l_1,l_2}$ [2]:

$$\bar{\boldsymbol{\mu}}_{l_1,l_2} | \bar{\boldsymbol{\gamma}}_{l_1,l_2}, \hat{\boldsymbol{\theta}}^{i-1} \sim \mathcal{CN}(\bar{\mathbf{b}}_{l_1,l_2}, \bar{\mathbf{K}}_{l_1,l_2}), \quad (14)$$

where $\bar{\mathbf{b}}_{l_1,l_2}$ and $\bar{\mathbf{K}}_{l_1,l_2}$ are functions of $\bar{\boldsymbol{\gamma}}_{l_1,l_2}$, $\hat{\boldsymbol{\theta}}^{i-1}$, $\bar{\mathbf{S}}_g(l_1, l_2)$, and $\bar{\mathbf{S}}_m(l_1, l_2)$. Based on this observation, the optimization problem in (10) can be readily solved [1].

3.1.2. MAPES-EM2

Following the observation that the same missing data samples may enter in many snapshots, we propose a second method to implement the EM algorithm by estimating the missing data simultaneously for all data snapshots. Let

$$\mathbf{y} = \text{vec}[\mathbf{Y}] \tag{15}$$

denotes the vector of all the data samples. Similar to the MAPES-EM1 case, we let $\boldsymbol{\gamma}$ and $\boldsymbol{\mu}$ denote the vectors containing the available and missing elements of $\mathbf{y}$, respectively. Then $\boldsymbol{\gamma}$ has a size of $g \times 1$, where g is the number of available samples.

Let $\tilde{\mathbf{y}}$ denote the $L_1L_2M_1M_2 \times 1$ vector obtained by concatenating all the snapshots

$$\tilde{\mathbf{y}} \triangleq \begin{bmatrix} \bar{\mathbf{y}}_{0,0} \\ \vdots \\ \bar{\mathbf{y}}_{L_1-1,L_2-1} \end{bmatrix} = \mathbf{S}_g\boldsymbol{\gamma} + \mathbf{S}_m\boldsymbol{\mu}, \tag{16}$$

where $\mathbf{S}_g$ (which has a size of $L_1L_2M_1M_2 \times g$) and $\mathbf{S}_m$ (which has a size of $L_1L_2M_1M_2 \times (N_1N_2 - g)$) are the corresponding selection matrices for the available and missing data vectors, respectively. Due to the overlapping of the vectors $\{\bar{\mathbf{y}}_{l_1,l_2}\}$, $\mathbf{S}_g$ and $\mathbf{S}_m$ are not unitary, but they are still orthogonal to each other: $\mathbf{S}_g^T\mathbf{S}_m = \mathbf{0}_{g\times(N_1N_2-g)}$. So instead of (11) and (12), we have from (16):

$$\boldsymbol{\gamma} = (\mathbf{S}_g^T\mathbf{S}_g)^{-1}\mathbf{S}_g^T\tilde{\mathbf{y}} = \tilde{\mathbf{S}}_g^T\tilde{\mathbf{y}} \tag{17}$$

and

$$\boldsymbol{\mu} = (\mathbf{S}_m^T\mathbf{S}_m)^{-1}\mathbf{S}_m^T\tilde{\mathbf{y}} = \tilde{\mathbf{S}}_m^T\tilde{\mathbf{y}}, \tag{18}$$

where the matrices $\tilde{\mathbf{S}}_g$ and $\tilde{\mathbf{S}}_m$ introduced above are defined as $\tilde{\mathbf{S}}_g \triangleq \mathbf{S}_g(\mathbf{S}_g^T\mathbf{S}_g)^{-1}$, $\tilde{\mathbf{S}}_m \triangleq \mathbf{S}_m(\mathbf{S}_m^T\mathbf{S}_m)^{-1}$, and they are also orthogonal to each other: $\tilde{\mathbf{S}}_g^T\tilde{\mathbf{S}}_m = \mathbf{0}_{g\times(N_1N_2-g)}$.

Now the normalized log-likelihood function in (7) can be written as

$$\begin{aligned} \tfrac{1}{L_1L_2}\ln p\left(\tilde{\mathbf{y}}\,|\alpha,\mathbf{Q}\right) &= -M_1M_2\ln\pi - \tfrac{1}{L_1L_2}\ln|\mathbf{D}| \\ &\quad - \tfrac{1}{L_1L_2}\left[\tilde{\mathbf{y}} - \alpha\boldsymbol{\rho}\right]^H\mathbf{D}^{-1}\left[\tilde{\mathbf{y}} - \alpha\boldsymbol{\rho}\right], \end{aligned} \tag{19}$$

where $\boldsymbol{\rho}$ and $\mathbf{D}$ are defined as:

$$\boldsymbol{\rho} \triangleq \begin{bmatrix} e^{j(\omega_1 0+\omega_2 0)}\mathbf{a} \\ \vdots \\ e^{j[\omega_1(L_1-1)+\omega_2(L_2-1)]}\mathbf{a} \end{bmatrix}, \quad \mathbf{D} \triangleq \begin{bmatrix} \mathbf{Q} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{Q} \end{bmatrix}. \tag{20}$$

Substituting (16) into (19), we obtain the joint log-likelihood of $\boldsymbol{\gamma}$ and $\boldsymbol{\mu}$.

To derive the EM algorithm for the current set of assumptions, we note that for given $\hat{\alpha}^{i-1}$ and $\hat{\mathbf{Q}}^{i-1}$, we have (similarly to (14)):

$$\boldsymbol{\mu}|\boldsymbol{\gamma},\hat{\boldsymbol{\theta}}^{i-1} \quad \sim \quad \mathcal{CN}(\mathbf{b},\mathbf{K}), \tag{21}$$

where $\mathbf{b}$ and $\mathbf{K}$ are functions of $\boldsymbol{\gamma}$, $\hat{\boldsymbol{\theta}}^{i-1}$, $\tilde{\mathbf{S}}_g^T$, and $\tilde{\mathbf{S}}_m^T$.

Similar to MAPES-EM1, the solution of (10) in this case can be readily obtained [1].

3.2. 2-D MAPES-CM (Cyclic Maximization)

Next we consider a $K_1 \times K_2$-point DFT grid: $(\omega_{k_1},\omega_{k_2}) = (2\pi k_1/K_1, 2\pi k_2/K_2)$ for $k_1 = 0,...,K_1-1$ and $k_2 = 0,...,K_2-1$. (Usually we choose $K_1 > N_1$ and $K_2 > N_2$.) Instead of dealing with each individual frequency $(\omega_{k_1},\omega_{k_2})$ separately, we consider the following maximization problem:

$$\begin{aligned} \max_{\boldsymbol{\mu},\{\alpha,\mathbf{Q}\}} \quad & \sum_{k_1=0}^{K_1-1}\sum_{k_2=0}^{K_2-1}\Bigg\{-\ln|\mathbf{Q}| - \frac{1}{L_1L_2}\sum_{l_1=0}^{L_1-1}\sum_{l_2=0}^{L_2-1} \\ & \left[\bar{\mathbf{y}}_{l_1,l_2} - \alpha\mathbf{a}e^{j(\omega_{k_1}l_1+\omega_{k_2}l_2)}\right]^H \\ & \mathbf{Q}^{-1}\left[\bar{\mathbf{y}}_{l_1,l_2} - \alpha\mathbf{a}e^{j(\omega_{k_1}l_1+\omega_{k_2}l_2)}\right]\Bigg\}, \end{aligned} \tag{22}$$

where the objective function is the summation over the 2-D frequency grid of all the frequency-dependent complete-data likelihood functions in (7) (within an additive constant), and again, we have dropped the frequency dependence on $(\omega_{k_1},\omega_{k_2})$ for notational convenience. We solve the above optimization problem via a cyclic maximization (CM) approach.

First, assuming that the previous estimate $\hat{\boldsymbol{\theta}}^{i-1}$ formed from $\{\hat{\alpha}^{i-1},\hat{\mathbf{Q}}^{i-1}\}$ is available, we maximize (22) with respect to $\boldsymbol{\mu}$. This step can be re-formulated as

$$\min_{\boldsymbol{\mu}} \quad \sum_{k_1=0}^{K_1-1}\sum_{k_2=0}^{K_2-1}\left[\tilde{\mathbf{y}} - \hat{\alpha}^{i-1}\boldsymbol{\rho}\right]^H\left[\hat{\mathbf{D}}^{i-1}\right]^{-1}\left[\tilde{\mathbf{y}} - \hat{\alpha}^{i-1}\boldsymbol{\rho}\right], \tag{23}$$

where $\tilde{\mathbf{y}}$, $\boldsymbol{\rho}$, and $\hat{\mathbf{D}}^{i-1}$ have been defined previously. Recalling that $\tilde{\mathbf{y}} = \mathbf{S}_g\boldsymbol{\gamma} + \mathbf{S}_m\boldsymbol{\mu}$, we can easily solve the optimization problem in (23) as its objective function is quadratic in $\boldsymbol{\mu}$:

$$\hat{\boldsymbol{\mu}} = \left[\mathbf{S}_m^H\mathbf{D}_s\mathbf{S}_m\right]^{-1}\left[\mathbf{S}_m^H\mathbf{D}_v - \mathbf{S}_m^H\mathbf{D}_s\mathbf{S}_g\boldsymbol{\gamma}\right], \tag{24}$$

where $\mathbf{D}_s \triangleq \sum_{k_1=0}^{K_1-1}\sum_{k_2=0}^{K_2-1}\left[\hat{\mathbf{D}}^{i-1}\right]^{-1}$ and $\mathbf{D}_v \triangleq \sum_{k_1=0}^{K_1-1}\sum_{k_2=0}^{K_2-1}\left[\hat{\mathbf{D}}^{i-1}\right]^{-1}\hat{\alpha}^{i-1}\boldsymbol{\rho}$.

Once an estimate $\hat{\boldsymbol{\mu}}$ has become available, we re-estimate $\{\alpha\}$ and $\{\mathbf{Q}\}$ by maximizing (22) with $\boldsymbol{\mu}$ replaced by $\hat{\boldsymbol{\mu}}$. This can be done by maximizing each frequency term separately, which reduces to the 2-D APES problem introduced in Section 2.

A cyclic maximization of (22) can be implemented by the alternating maximization with respect to $\boldsymbol{\mu}$ and, respectively, α and $\mathbf{Q}$.

3.3. MAPES-EM versus MAPES-CM

Consider evaluating the spectrum for all three MAPES algorithms on the same DFT grid. Since during each iteration, all three algorithms estimate the missing samples and the spectrum, we can compare their computational complexity separately for each step. Comparing (24) with estimation of $\mathbf{b}$ and $\mathbf{K}$ (or $\bar{\mathbf{b}}_{l_1,l_2}$ and $\bar{\mathbf{K}}_{l_1,l_2}$), which have to be evaluated for each frequency $(\omega_{k_1}, \omega_{k_2})$ (and for each snapshot for $\bar{\mathbf{b}}_{l_1,l_2}$ and $\bar{\mathbf{K}}_{l_1,l_2}$), we note that the computational complexity of MAPES-CM is much lower. When they calculate the spectrum, MAPES-CM uses the standard APES algorithm which can be efficiently implemented [3]. Due to the fact that MAPES-EM uses different estimates for the missing data at different frequencies, no efficient algorithms are available to calculate the corresponding spectral estimate.

4. NUMERICAL EXAMPLES

We consider a 16×16 data matrix consisting of three 2-D sinusoids at normalized frequencies (4/16, 5/16), (6/16, 5/16), and (10/16, 9/16) and with complex amplitudes equal to 1, 0.7, and 2, respectively, embedded in zero-mean circularly symmetric complex Gaussian white noise with standard deviation 0.1. All the samples in rows 4, 8, 11, 14, and in columns 3, 6, 7, 11, 12, 14 are missing, which amounts to over 50% of the total number of data samples. The true amplitude spectrum is plotted in Figure 1(a) with the estimated amplitude values given next to each sinusoid. Each spectrum is obtained on a 64×64 grid. The data missing pattern is displayed in Figure 1(b). The WFFT spectrum for the missing data case is shown in Figure 1(c), which underestimates the sinusoids and contains strong artifacts due to the zeros assumed for the missing samples. In Figures. 1(d), 1(e), and 1(f), we show the spectra estimated by MAPES-EM1, MAPES-EM2, and MAPES-CM, respectively, with $M_1 = M_2 = 8$. The WFFT spectrum is used as the initial estimate of $\alpha(\omega_1, \omega_2)$, and the initial estimate of $\mathbf{Q}(\omega_1, \omega_2)$ is calculated from (9) with missing samples set to zero. All MAPES algorithms perform well by giving accurate spectral estimates and clearly separated spectral peaks.

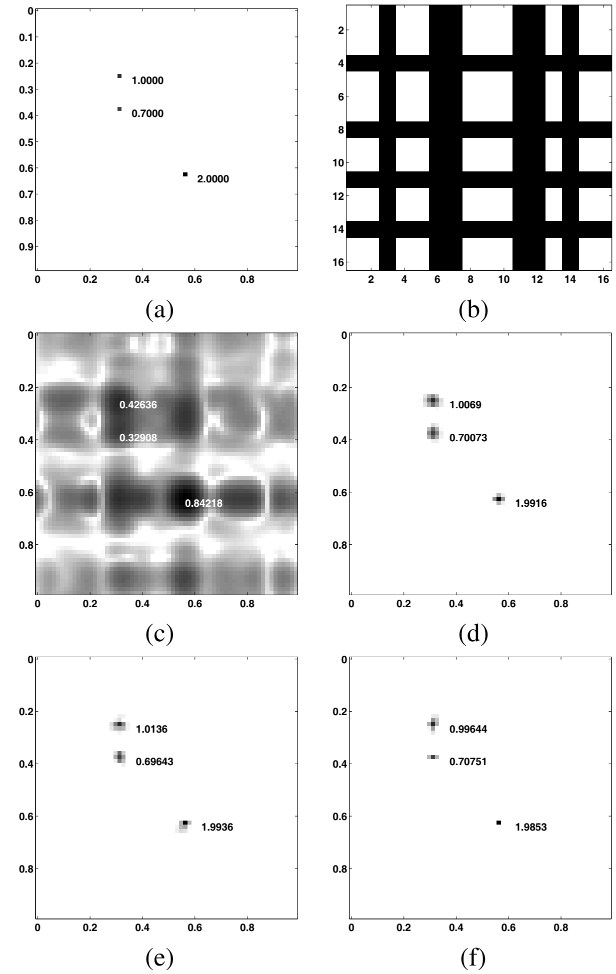

Fig. 1. Modulus of the 2-D spectra: (a) True spectrum, (b) 2-D data missing pattern, the black stripes indicate missing samples, (c) 2-D WFFT, (d) 2-D MAPES-EM1, (e) 2-D MAPES-EM2, and (f) 2-D MAPES-CM.

5. CONCLUSIONS

We have considered 2-D nonparametric complex spectral estimation (with its 1-D counterpart as a special case) for data matrices with missing samples occurring in arbitrary patterns. The previously proposed MAPES-EM algorithms have been extended to the 2-D case. We have also developed another missing-data algorithm, referred to as MAPES-CM, by maximizing an ML fitting criterion iteratively via a cyclic maximization (CM) algorithm. We have compared MAPES-EM with MAPES-CM and have shown that MAPES-CM allows significant computational savings compared with MAPES-EM, which is especially desirable for 2-D applications.

6. REFERENCES

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