SET-MEMBERSHIP ADAPTIVE FILTERING WITH PARAMETER-DEPENDENT ERROR BOUND TUNING

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ABSTRACT

This paper considers set-membership filtering (SMF) when the error-bound specification is hard to determine. Improper error-bound specification could cause overbounding or underbounding, both of which can result in degraded performance for SMF algorithms. This paper introduces a novel variable error bound and presents a different SMF criterion. It is shown that the recursive algorithm derived from this new set-membership filtering criterion has less risk of overbounding/underbounding and outperforms conventional SMF algorithms with one fixed error-bound specification, particularly when insufficient knowledge is available to determine the bound. The proposed algorithm is more suitable to time-variant environments. Frequency-domain equalization for broadband wireless communications is used as an example to illustrate the proposed criterion and recursive solution. Simulation results that show convergence performance and tracking of a time-variant channel are presented.

1. INTRODUCTION

Set-Membership Filtering (SMF) and a class of recursive algorithms have been developed and investigated extensively, see, e.g., [1, 2, 3]. With a pre-specified error bound, SMF algorithms seek a set of filter coefficients that yield bounded filter output errors [4]. Adaptive SMF algorithms have been employed for a variety of applications that includes, but not limited to, speech coding [5], adaptive equalization [6] and mitigation of multiple access interference in wireless communications [7].

The adaptive SMF algorithms generally have excellent convergence and tracking performance thanks to their feature of optimized step size for each update step and data-dependent *selective* update. Those properties are derivatives of presumed *a priori* knowledge of an error bound. The performance of SMF may depend critically on this error-bound specification. In practice, however, we are often unable to determine the error bound accurately. The reason is that there is usualy insufficient knowledge about the underlying system, or even the "true" error bound may be time-variant due to the changing environments. In such cases, choosing one error bound value arbitrarily is unreliable and has the risk of overbounding (i.e., the error bound is larger than it really is) or underbounding (i.e., the bound is too small); both of which can result in performance degradation. These situations call for alternative SMF algorithms with the ability of adjusting the error bound in accordance with the changes in the system.

This paper presents an approach to adjusting the error bound and introduces a novel *error-bound function*, that depends on the filter weight, with an intention to circumvent the aforementioned problems. Based on this *error-bound function*, a new SMF criterion is formulated to determine the filter weight vector. This SMF criterion is a general one that encompasses the conventional SMF that has one fixed error-bound specification. Recursive algorithms based on this new SMF criterion can automatically tune and track the error bound. This proposed approach to bound-tuning is different from the existing approaches (see e.g.,[8, 9]) in that it does not require the assumption that the "true" error bound is constant. Thus it is more suitable to changing environments. Adaptive frequency-domain equalization (FDE) for broadband wireless communications is presented as an application example for the proposed SMF algorithm. It will be shown that the recursive algorithm using this variable error-bound specification offers better performance than conventional adaptive SMF algorithms, e.g., SM-NLMS [4], in terms of convergence and tracking, and is less likely to overbound or underbound. Furthermore, the proposed algorithm offers better convergence performance than the conventional adaptive normalized least-mean-square (NLMS) algorithm.

2. SET-MEMBERSHIP FILTERING

Consider a general linear-in-parameter filter with input $\mathbf{x}$, weight $\mathbf{w}$ and output y. Let d be the desired output, and the filtering error is $e(\mathbf{w}){=}d{-}\mathbf{w}^T\mathbf{x}$. To determine the parameter vector $\mathbf{w}$, SMF employs a bounded error criterion. With a fixed error-bound specification γ, conventional SMF algorithms seek to find $\mathbf{w}$ such that

$$|e(\mathbf{w})|^2 \leq \gamma^2, \quad \forall(\mathbf{x}, d) \in \mathcal{S} \tag{1}$$

where $\mathcal{S}$ is the model space comprising of all input vector-desired output pairs $(\mathbf{x}, d)$. The desired region estimate, termed *feasibility set* $\mathcal{W}$, is given by

$$\mathcal{W} = \bigcap_{(\mathbf{x},d)\in\mathcal{S}} \left\{\mathbf{w} \in \mathcal{C}^N : |d - \mathbf{x}^T\mathbf{w}| \leq \gamma\right\}. \tag{2}$$

A recursive solution called SM normalized least-mean-square (SM-NLMS)[4] is given by

$$\mathbf{w}_k = \mathbf{w}_{k-1} + \mu_k e_k \mathbf{x}_k^* / ||\mathbf{x}_k||^2 \tag{3}$$

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with

$$\mu_k = \begin{cases} 1-\gamma/|e_k| & |e_k| > \gamma \\ 0 & |e_k| \leq \gamma. \end{cases}$$

In essence, at each time instant k, with input $\mathbf{x}_k$, training data d_k and the previous parameter estimate $\mathbf{w}_{k-1}$, the SM-NLMS algorithm seeks a new estimate $\mathbf{w}_k$ so as to minimize $||\mathbf{w}_k - \mathbf{w}_{k-1}||^2$ with the constraint that $|d_k - \mathbf{x}_k^T \mathbf{w}_k| \leq \gamma$.

The error-bound specification can be critical to SMF performance. Both overbounding and underbouding may impair SMF algorithms. Underbounding could result in a void feasibility set. On the other hand, overbounding could slow down the convergence of the adaptive algorithm since the constraint condition on filter weight is too loose. It could also degrade the steady-state MSE (mean-squared error) performance and lead to an inconsistent estimator.

In many practical problems, however, it is not easy to choose the error bound adequately. For example, in equalization of communication channels, it is hard to choose equalization error bound. Another problem is that error-bound specification may need to change in accordance with the changing characteristics of the channel. Setting the error bound at an arbitrarily fixed value risks overbounding or underbounding, resulting in performance degradation of adaptive equalization.

3. SMF WITH ERROR-BOUND FUNCTION SPECIFICATION

To resolve the problem of overbounding or underbounding in SMF, we propose impose a variable *error bound* that depends on filter weights. To begin with, denote the error bound as $\gamma(\mathbf{w})$ to emphasize its dependence on the filter parameter vector $\mathbf{w}$. Its value is the desired error bound if $\mathbf{w}$ is in the desired region, i.e., $\mathbf{w}$ is a feasible solution.

With the specified error-bound function $\gamma(\mathbf{w})$, the SMF criterion requires that the output error $e(\mathbf{w})$ not be greater than $\gamma(\mathbf{w})$ for all input vector-desired output pair $(\mathbf{x}, d)$ in the model space $\mathcal{S}$. Thus the objective is to find $\mathbf{w}$ that satisfies

$$|e(\mathbf{w})| = |d - \mathbf{x}^T\mathbf{w}| \leq \gamma(\mathbf{w}) \quad \forall (\mathbf{x}, d) \in \mathcal{S}. \tag{4}$$

Geometrically, for each $(\mathbf{x}, d)$, (4) represents a region between two conicoids. In contrast, (1) represents a region between two hyperplanes. Using the formulation of (4) makes the adaptive filter more adept at tracking system variations. The feasibility set $\mathcal{W}$ is the set of parameter vectors that satisfy (4) with the specified error bound $\gamma(\mathbf{w})$:

$$\mathcal{W} \triangleq \bigcap_{(\mathbf{x},d)\in\mathcal{S}} \{\mathbf{w} \in \mathcal{C}^N : \quad |d - \mathbf{x}^T\mathbf{w}| \leq \gamma(\mathbf{w})\}. \tag{5}$$

One should note that this criterion encompasses the constant error-bound specification in the conventional SMF, (1). If there is sufficient information to determine the error bound, $\gamma(\mathbf{w})$ is just γ, and criterion (4) becomes (1).

The formulations in (4) and (5) call for a different SMF solution. For those filtering problems with additive input noise as in adaptive equalization, the filter's output noise naturally depends on the filter weight, which is $v_o = \mathbf{v}_i^T\mathbf{w}$ where $\mathbf{v}_i$ is the input noise vector. $\gamma(\mathbf{w})$ can be set based on the bound of v_o. To derive a recursive solution based on this SMF criterion, assume that a sequence of data pairs $(\mathbf{x}_k, d_k) \in \mathcal{S}$ is available for "training." At time k, a constraint set $\mathcal{H}_k$ is defined as the set of all parameter vectors that satisfy the specification of (4) for the $(\mathbf{x}_k, d_k)$ pair:

$$\mathcal{H}_k = \{\mathbf{w} \in \mathcal{C}^N : \quad |d_k - \mathbf{x}_k^T\mathbf{w}| \leq \gamma(\mathbf{w})\}. \tag{6}$$

A recursive solution can be derived with a *point-wise approach* similar to that of [4]. At time k, given the parameter vector estimate $\mathbf{w}_{k-1}$, and input-desired output $(\mathbf{x}_k, d_k)$, a new estimate $\mathbf{w}_k$ is found so as to minimize the Euclidean norm of the change in the estimate given by $||\mathbf{w}_k - \mathbf{w}_{k-1}||$, subject to the constraint that $\mathbf{w}_k \in \mathcal{H}_k$.

Note that during the adaption process, for each parameter estimate, a parameter-dependent error bound specification is used to judge whether the previous parameter estimate should be retained or a new estimate is needed. In essence, error-bound specification is changed according to the current parameter estimate, which, supposedly, reflects timely the system charateristics. So the problem of determining error bound with insufficient information (or not knowing how the parameter will vary) as described in the previous section can be circumvented. Thus this parameter-dependent error-bound specification can result in a more reliable adaptive algorithm than what can be achieved with a fixed error-bound specification. Note that the recursive algorithm still enjoys the data-dependent selective update featured by conventional SMF algorithms.

4. A DESIGN EXAMPLE

Rapid growth in wireless communications results in more demand on the network to provide high-bandwidth, low-cost and reliable mobile wireless services comparable to wireline communications. A fundamental challenge is to mitigate inter-symbol interference (ISI) with low computational complexity. Single carrier frequency domain equalization (SC-FDE) has gained a good deal of interest recently because of its ability of mitigating effectively the ISI with low complexity. It has been considered as one of the options for fixed broadband wireless communications in IEEE 802.16. As stated previously, adaptive equalization is an application example where error bound is hard to determine. In this section, SC-FDE is presented as an application example of the proposed SMF criterion.

Consider a general adaptive FDE [10] with M bins for broadband wireless communications. The symbols are transmitted block-wise with a cyclic prefix in each transmitted block. At the receiver, each received block $\mathbf{r}_k$ is transformed into the frequency domain $\mathbf{R}_k$ through Fast Fourier Transform (FFT) after the cyclic prefix is removed, and $\mathbf{R}_k$ can be described as:

$$\mathbf{R}_k = \mathbf{H}\mathbf{D}_k + \mathbf{V}_k \tag{7}$$

where $\mathbf{H}$ is a diagonal matrix whose elements are the FFT of the impulse response of the frequency-selective fading channel between the transmitter and the receiver, $\mathbf{D}_k$ is the FFT of the k-th block of transmitted symbols and $\mathbf{V}_k$ is the FFT of k-th block of additive Gaussian noise. Since FFT is a linear operation, the transform domain noise $\mathbf{V}_k$ remains to be Gaussian. Assume that its variance is σ_n^2. Then equalization can be done bin-by-bin in the transform domain with an operation for each bin:

$$Y_{k,m} = W_m R_{k,m} = W_m H_m D_{k,m} + W_m V_{k,m} \tag{8}$$

where W_m is the equalizer weight for the mth bin, $m = 0, 1, ..., M-1$. Each bin is an adaptive filtering subsystem. To simplify the discussion, we focus hereafter on one bin and drop the bin index m in all variables, for the results are equivalently applicable to all bins.

At the kth block, denote the filter input by R_k and the desired output by D_k. The filter parameter estimate at time $k-1$ is W_{k-1}. The approach here is to employ the SMF criterion proposed in Section III and the point-wise approach [4] to derive an adaptive solution for W_k. First, the error bound $\gamma(W)$ needs to be determined. If the equalizer parameter is W, the noise term in equalizer output is WV_k, which is a Gaussian random variable with zero mean and variance $|W|^2\sigma_n^2$. The error bound can be approximated by $\gamma(W)=\sqrt{\alpha|W|^2\sigma_n^2}$ with $\alpha > 1$. The recursive solution is found as follows:

1. If
$$|E_k|^2 \triangleq |D_k - R_k W_{k-1}|^2 \leq \alpha |W_{k-1}|^2 \sigma_n^2,$$
then $W_k = W_{k-1}$ and no parameter update is needed.
2. Else, W_k is found by
$$\min \quad |W_k - W_{k-1}|^2 \tag{9}$$
$$\text{subject to:} \quad |D_k - R_k W_k|^2 \leq \alpha |W_k|^2 \sigma_n^2. \tag{10}$$

Define $\beta_k \triangleq |R_k|^2 - \alpha\sigma_n^2$. Depending on the value of β_k, constraint (10) may lead to different solutions for W_k. The adaptive algorithm for FDE is summarized as follows:

If
$$|E_k| \leq \sqrt{\alpha |W_{k-1}|^2 \sigma_n^2} \tag{11}$$
then
$$W_k = W_{k-1},$$
else
$$\lambda_k = \sqrt{\alpha}\sigma_n |D_k| \tag{12}$$
$$\beta_k = |R_k|^2 - \alpha\sigma_n^2 \tag{13}$$
$$\xi_k = D_k R_k^* - \beta_k W_{k-1} \tag{14}$$

$$W_k = \begin{cases} \frac{1}{2}\left(W_{k-1} - W_{k-1}^* \frac{\xi_k}{\xi_k^*} + \frac{\xi_k}{|R_k|^2}\right) & \text{if} \quad \beta_k = 0 \\ W_{k-1} + \left(1 - \frac{\lambda_k}{|\xi_k|}\right)\frac{\xi_k}{|\beta_k|} & \text{if} \quad \beta_k > 0 \\ W_{k-1} + \left(1 - \frac{|\xi_k|}{\lambda_k}\right)\frac{\lambda_k \xi_k}{|\beta_k| . |\xi_k|} & \text{if} \quad \beta_k < 0. \end{cases} \tag{15}$$

If the noise is nonGaussian, one can employ the Central Limit Theorem (CLT) [11] to approximate the noise distribution in the transform domain with a Gaussian distribution and employ the recursive algorithm (11)$\sim$(15). The parameter-dependent error bound specification and the variable step size make the adaptive algorithm less vulnerable to overbounding and underbounding, thus providing better convergence and tracking performance.

5. SIMULATIONS

Simulation results are given here to examine the proposed criterion and the performance of the associated recursive algorithm. In all simulation examples presented here, FDE with a 64-point FFT is considered. The channel model used in the simulation is a microwave radio channel obtained from actual field measurements [12]. A QPSK (Quadrature Phase-Shift Keying) signaling is assumed. The value of α in defining $\gamma(W) = \sqrt{\alpha|W|^2\sigma_n^2}$ is set to be 5. The results of the proposed algorithm are compared with those of the SM-NLMS algorithm which uses a fixed error bound [4].

Example 1: Convergence performance with Gaussian noise. In this simulation, the input noise is additive white Gaussian noise with zero mean and variance 0.1. The variance of noise in the frequency domain is $\sigma_n^2 = 6.4$. The convergence performance of the proposed algorithm is compared to SM-NLMS with different error bounds, and the results are shown in Fig.1. It is seen clearly that arbitrarily choosing the error bound with insufficient information could cause overbounding or underbounding. Simulation results also show that the proposed algorithm enjoys *sparse* data-dependent updates and the update frequency is significantly less than that of SM-NLMS. In this example, only 4% updates are needed for the proposed algorithm. For SM-NLMS, the updates needed are 46%, 32%, 13%, 3.2% and 2.5% for the bounds of $0.1\sigma_n^2$, $0.5\sigma_n^2$, σ_n^2, $3\sigma_n^2$ and $5\sigma_n^2$, respectively. We also compare the performance between the proposed algorithm and NLMS algorithm, shown in Fig.2. The results show taht the proposed algorithm exhibits a faster convergence and lower steady-state error than NLMS algorithm.

Example 2: Equalization with time-variant channel. The channel model used in previous simulation is used here as the initial state of the channel, and then the coefficients vary randomly.

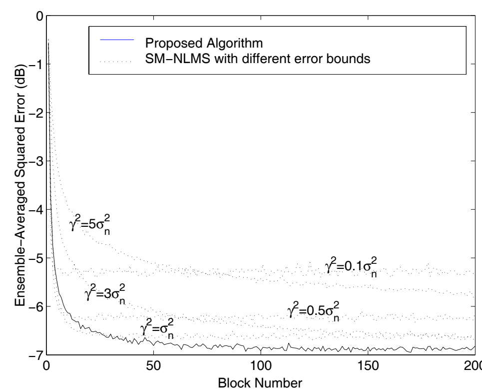

Fig. 1. Convergence performance with Gaussian noise: proposed algorithm vs SM-NLMS with different error bounds.

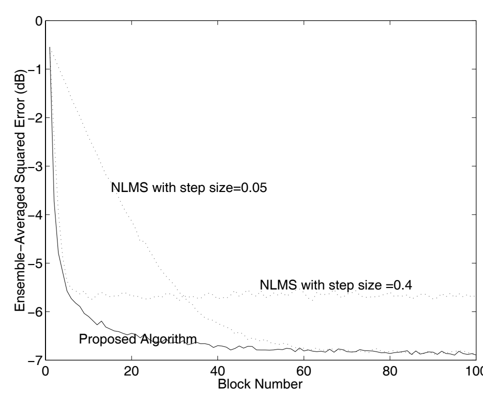

Fig. 2. Convergence performance with Gaussian noise: proposed algorithm vs NLMS algorithm

The time variations in the coefficients are introduced by having random Gaussian jumps every 200 blocks as shown in Fig.3. The curves shown in Fig.4 are learning curves of the proposed algorithm, comparing to SM-NLMS with various error bounds. It is obvious that the proposed algorithm can track the channel with good performance, while arbitrarily choosing error bound can not ensure satisfactory equalization performance. SM-NLMS algorithm with $\gamma^2 = \sigma_n^2$ can provide similar performance to the proposed algorithm in some intervals, but inferior performance in other intervals. The reason is that the "true" error bound is time-variant due to changing channel.

Other experiments have been performed to compare the performance of the proposed algorithm to SM-NLMS with nonGaussian noise. The details are omitted here due to space limitation. Once again, the proposed algorithm offers more robust and better performance than SM-NLMS algorithm does.

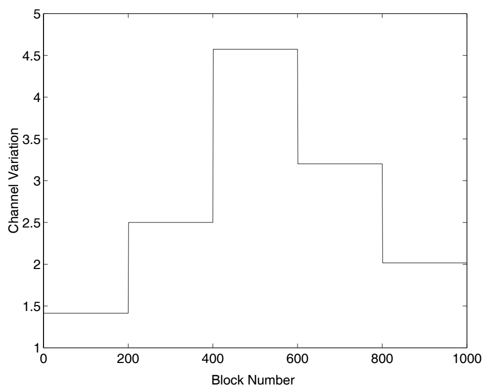

Fig. 3. Time variations of channel

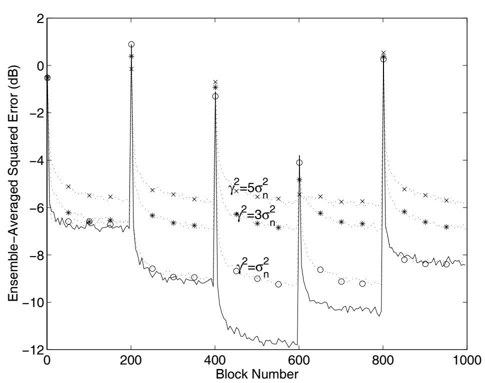

Fig. 4. Tracking of time-variant channel: the proposed algorithm (solid line) vs SM-NLMS with different error bounds (dotted lines with markers)

6. CONCLUSION

This paper has investigated the set-membership filtering when insufficient information is available to determine accurately the error bound. A variable error bound is introduced to address its dependence on the filter weight and an alternative SMF algorithm is proposed. The proposed criterion and the resulting algorithm employ a parameter-dependent error-bound specification such that the error bound is specified in accordance with the parameter estimate, thereby reducing the risk of overbounding/underbounding. The proposed algorithm is applied to SC-FDE as an example. Simulation results show that the proposed algorithm outperforms both NLMS algorithm and SM-NLMS algorithm especially when the required error bound information is not available or inaccurate.

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