JOINT ESTIMATION OF STATES AND TRANSITION FUNCTIONS OF DYNAMIC SYSTEMS USING COST-REFERENCE PARTICLE FILTERING

Joaquín Míguez[†], *Shanshan Xu*[‡], *Mónica F. Bugallo*[‡], *Petar M. Djurić*[‡]

[†]Departamento de Electrónica e Sistemas, Universidade da Coruña, Spain
[‡]Department of Electrical and Computer Engineering, Stony Brook University, USA
E-mail: `jmiguez@udc.es`, `{shaxu,monica,djuric}@ece.sunysb.edu`

ABSTRACT

The recently introduced cost-reference particle filter (CRPF) methodology allows for recursive estimation of unobserved states of dynamical systems *without a priori knowledge of probability distributions* of the noises in the system. In this paper, we use CRPFs in problems where we eliminate one more strong assumption about the state space model, the one of knowing the function governing the state evolution. We replace this function by a linearly combined set of basis functions where the linear combination coefficients are unknown. We show how CRPFs can be modified to cope with this scenario and demonstrate their performance for positioning a moving vehicle in a two-dimensional space.

1. INTRODUCTION

Particle filtering has lately become a powerful tool for online tracking of signals and time-varying parameters of dynamic systems [1, 2]. These methods require a mathematical representation of the dynamics of the system evolution, together with assumptions of probabilistic models.

In this paper, we continue the study of the class of particle filtering methods introduced in [3]. The main feature of these filters is that they are not based on any particular probabilistic assumptions and that the statistical reference is substituted by a user-defined cost function which measures the quality of the state signal estimates according to the available observations. Hence, methods within this class are termed cost-reference particle filters (CRPFs), as opposed to conventional statistical-reference particle filters (SRPFs).

Although the filters proposed in [3] drop all *probabilistic* assumptions regarding the system model, they still require knowledge of the model deterministic *structure*, i.e., the state transition function, and/or the observation function. We introduce the notion of unstructured CRPFs (UCRPFs), which drop both the usual probabilistic assumptions in particle filtering (as CRPFs in [3] do) and, partially or completely, the deterministic structural assumptions regarding the knowledge of the functions in the model. In this paper we investigate the adaptive estimation of approximations of the state transition function.

The fundamentals of the CRPF and UCRPF approaches are introduced in Section 2 and Section 3, respectively. In Section 4, we apply the proposed algorithms to the problem of positioning a mobile in a two-dimensional space. Finally, brief concluding remarks are made in Section 5.

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2. COST REFERENCE PARTICLE FILTERS

2.1. Problem statement

Many problems in signal processing can be stated in terms of the estimation of a hidden random signal in a dynamic system of the form

$$\mathbf{x}_t = f_x(\mathbf{x}_{t-1}) + \mathbf{u}_t \quad \text{state equation} \tag{1}$$
$$\mathbf{y}_t = f_y(\mathbf{x}_t) + \mathbf{v}_t, \quad \text{observation equation} \tag{2}$$

where $t = 1, 2, ...$ denotes discrete time; $\mathbf{x}_t$ is an $L_x \times 1$ vector that represents the system state; $f_x : \mathbb{R}^{L_x} \rightarrow I_x \subseteq \mathbb{R}^{L_x}$ is the (possibly nonlinear) state transition function; $\mathbf{u}_t \in \mathbb{R}^{L_x}$ is a state perturbation at time t; $\mathbf{y}_t$ is an $L_y \times 1$ observation vector; $f_y : \mathbb{R}^{L_x} \rightarrow I_y \subseteq \mathbb{R}^{L_y}$ is a (possibly nonlinear) transformation of the state; and $\mathbf{v}_t \in \mathbb{R}^{L_y}$ is an observation noise vector at time t.

Equation (1) describes the dynamics of the system state vector and, hence, it is usually termed *state equation* or *system equation*, whereas equation (2) is commonly referred to as *observation equation* or *measurement equation*. It is convenient to distinguish the structure of the dynamic system, which is due to the functions f_x and f_y, from the associated probabilistic model, which depends on the probability distribution of the noise signals and the *a priori* distribution of the state, i.e., the statistics of $\mathbf{x}_0$.

The ultimate aim is the online estimation of the sequence of system states, $\mathbf{x}_{0:t}$, using the available observations, $\mathbf{y}_{1:t}$.

2.2. Sequential Algorithm

In order to estimate $\mathbf{x}_{0:t}$ from $\mathbf{y}_{1:t}$ without knowledge of *any* probability density function (pdf), we substitute the statistical reference of the *a posteriori* state pdf, $p(\mathbf{x}_{0:t}|\mathbf{y}_{1:t})$, by a user-defined *cost* function that measures the quality of the state signal estimates according to the available observations. In particular, we use a real *cost* function of the form

$$\mathcal{C}(\mathbf{x}_{0:t}|\mathbf{y}_{1:t}, \lambda) = \lambda\mathcal{C}(\mathbf{x}_{0:t-1}|\mathbf{y}_{1:t-1}) + \triangle\mathcal{C}(\mathbf{x}_t|\mathbf{y}_t)$$

where the recursive structure allows to update the cost of a sequence up to time $t-1$ by looking solely at the state and observation vectors at time t, $\mathbf{x}_t$ and $\mathbf{y}_t$, respectively, which are used to compute the *incremental cost*, $\triangle\mathcal{C}(\mathbf{x}_t|\mathbf{y}_t)$. The parameter λ, where $0 < \lambda < 1$, is a forgetting factor that avoids attributing an excessive weight to *old* observations when a long series of data are collected, hence allowing for potential adaptivity. We also consider a one-step *risk* function, $\mathcal{R}(\mathbf{x}_{t-1}|\mathbf{y}_t)$, that measures the adequacy of the state at time $t-1$ given the new observation, $\mathbf{y}_t$. It is convenient to view the risk function as a prediction of the incremental cost which can be obtained before $\mathbf{x}_t$ is actually propagated. Hence, a natural choice of the risk function is

$$\mathcal{R}(\mathbf{x}_{t-1}|\mathbf{y}_t) = \triangle\mathcal{C}\left(f_x(\mathbf{x}_{t-1})|\mathbf{y}_t\right).$$

The proposed estimation technique proceeds sequentially in a manner similar to a standard particle filter [1, Chapter 1]. Particles are initialized uniformly on a set, $X_0 \subset \mathbb{R}^{L_x}$, where the initial state is known to lie, while zero initial costs are assigned. Given a set of M state trajectories and associated costs up to time t, $\left\{\mathbf{x}_{0:t}^{(i)}, \mathcal{C}_t^{(i)}\right\}_{i=1}^{M}$, where $\mathcal{C}_t^{(i)} = \mathcal{C}(\mathbf{x}_{0:t}^{(i)}|\mathbf{y}_{1:t}, \lambda)$ and $\mathbf{x}_{0:t}^{(i)}$ are the i-th cost and particle stream, respectively, the grid of state trajectories is randomly propagated when $\mathbf{y}_t$ is observed by taking the following steps:

1. **Selection of the most promising trajectories (resampling).** For $i = 1, 2, ..., M$, let
$$\begin{aligned} \mathcal{R}_{t+1}^{(i)} &= \lambda\mathcal{C}_t^{(i)} + \mathcal{R}(\mathbf{x}_t^{(i)}|\mathbf{y}_{t+1}) \\ \hat{\pi}_{t+1}^{(i)} &\propto \mu(\mathcal{R}_{t+1}^{(i)}) \end{aligned}$$
where $\mu : \mathbb{R} \to [0, +\infty)$ is a monotonically decreasing function and $\hat{\pi}_{t+1} : \{1, ..., M\} \to [0, 1)$ is a probability mass function (pmf). A new particle filter is obtained by resampling the trajectories $\{\mathbf{x}_{0:t}^{(i)}\}_{i=1}^{M}$ according to the pmf $\hat{\pi}_{t+1}^{(i)}$ and we denote it as $\left\{\hat{\mathbf{x}}_{0:t}^{(i)}, \hat{\mathcal{C}}_t^{(i)}\right\}_{i=1}^{M}$.
2. **Random propagation.** For $i = 1, ..., M$, let
$$\begin{aligned} \mathbf{x}_{t+1}^{(i)} &\sim p_{t+1}(\mathbf{x}|\hat{\mathbf{x}}_t^{(i)}) \\ \mathcal{C}_{t+1}^{(i)} &= \lambda\hat{\mathcal{C}}_t^{(i)} + \triangle\mathcal{C}_{t+1}(\mathbf{x}_{t+1}^{(i)}|\mathbf{y}_t) = \lambda\hat{\mathcal{C}}_t^{(i)} + \triangle\mathcal{C}_{t+1}^{(i)} \end{aligned}$$
where p_{t+1} is a probability density function (pdf) chosen by the designer which must satisfy $E_{p_{t+1}(\mathbf{x}|\hat{\mathbf{x}}_t^{(i)})}\mathbf{x}_{t+1}^{(i)} = f_x\left(\hat{\mathbf{x}}_t^{(i)}\right)$ (i.e., zero-mean noise is assumed).
3. **Estimation of the state.** Let $\pi_{t+1}^{(i)} \propto \mu(\mathcal{C}_{t+1}^{(i)})$ for $i = 1, \ldots, M$. The pmf π_{t+1} allows to calculate state estimates at time $t+1$ in several ways, e.g.,
$$\mathbf{x}_{t+1}^{mean} = \sum_{i=1}^{M} \mathbf{x}_{t+1}^{(i)} \pi_{t+1}^{(i)}.$$

The general procedure described above is referred to as the Cost-Reference Particle Filter (CRPF). It is apparent that many implementations are possible for a single problem. Some more detailed design guidelines are proposed in [3], as well as sufficient conditions for asymptotic convergence (as $M \to \infty$) of the method.

3. UNSTRUCTURED CRPF

The CRPFs introduced in [3], and briefly reviewed in Section 2, drop the usual statistical assumptions common to standard particle filters, namely:

- the knowledge of the *a priori* pdf of the state signal, $p(\mathbf{x}_0)$,
- the knowledge of the noise densities, $p(\mathbf{u}_t)$ and $p(\mathbf{v}_t)$, and
- the ability to evaluate the likelihood, $p(\mathbf{y}_t|\mathbf{x}_t)$, and sample from the state transition pdf, $p(\mathbf{x}_t|\mathbf{x}_{t-1})$.

However, CRPFs still require knowledge of the deterministic structure of model (1)-(2), given by the functions f_x and f_y. In order to relax these structural assumptions, it is necessary to carry out a recursive estimation of the functions f_y and/or f_x within the CRPF.

In this paper, we focus on the problem of estimating f_x, while still assuming f_y is known. Hence, we do not constrain the dynamics of the system state, $\mathbf{x}_t$, but we still assume basic physical knowledge of the signals we acquire in order to perform the estimation task, e.g., radar pulses or communication waveforms.

3.1. Estimation of f_x

A simple way to represent the state transition function, is by linear combination of a set of basis functions. Let us denote such a set as

$$B = \{\varphi_{11}, \ldots, \varphi_{1J}, \varphi_{21}, \ldots, \varphi_{2J}, \ldots \varphi_{L_x 1}, \ldots, \varphi_{L_x J}\},$$

where $\varphi_{ij} : \mathbb{R}^{L_x} \to \mathbb{R}, \quad i = 1, \ldots, L_x, \quad j = 1, \ldots, J$. Using B, we aim at building an estimate of the form

$$\begin{aligned} \hat{f}_x(\mathbf{x}_t) &= \begin{bmatrix} \sum_{j=1}^{J} \varphi_{1j}(\mathbf{x}_t) a_{j1} \\ \vdots \\ \sum_{j=1}^{J} \varphi_{L_x j}(\mathbf{x}_t) a_{jL_x} \end{bmatrix} = \begin{bmatrix} \underline{\varphi}_{1;t}^{\top} \mathbf{a}_1 \\ \vdots \\ \underline{\varphi}_{L_x;t}^{\top} \mathbf{a}_{L_x} \end{bmatrix} \\ &= \operatorname{diag}\left\{\mathbf{\Phi}_t^{\top}\mathbf{A}\right\} \end{aligned}$$

where $\underline{\varphi}_{i;t} = [\varphi_{i1}(\mathbf{x}_t), \ldots, \varphi_{iJ}(\mathbf{x}_t)]^{\top}$ and $\mathbf{a}_i = [a_{1i}, \ldots, a_{Ji}]^{\top}$ are $J \times 1$ vectors, $\mathbf{\Phi}_t = [\underline{\varphi}_{1;t}, \cdots, \underline{\varphi}_{L_x;t}]$ is a $J \times L_x$ matrix which results from the application of the basis functions on $\mathbf{x}_t$ and $\mathbf{A} = [\mathbf{a}_1, \ldots, \mathbf{a}_{L_x}]$ is a $J \times L_x$ matrix of linear combination coefficients. A straightforward criterion for the selection of the coefficients in $\mathbf{A}$ is the Least Squares (LS) method, which can be written as

$$\mathbf{a}_{l;t} = \arg\min_{\mathbf{a}} \left\{ \sum_{k=1}^{t} \left(x_{l;k} - \underline{\varphi}_{l;k-1}^{\top} \mathbf{a} \right)^2 \right\} \tag{3}$$

for $l = 1, ..., L_x$ and $\mathbf{x}_t = [x_{1;t}, \ldots, x_{L_x;t}]^{\top}$.

3.2. Outline of the UCRPF algorithm

The LS estimator (3) can be adaptively computed using the Recursive Least Squares (RLS) algorithm [4]. The RLS algorithm and the CRPF can be intertwined, in a way similar to the combination of the Kalman filter and conventional SRPFs that yields the Mixture Kalman Filter (MKF) [5], to obtain the new Unstructured CRPF (UCRPF). In the UCRPF, an RLS algorithm is run for each particle and each state dimension in order to obtain an associated estimate of the state transition function. By proceeding in this way, the particle risks and, as a consequence, the selection step of the algorithm, become dependent on the corresponding function estimates.

Unlike the former CRPF, the UCRPF consists of four elements that are recursively updated,

$$\Omega_t = \left\{ \mathbf{x}_t^{(i)}, \mathcal{C}_t^{(i)}, \left\{\mathbf{Q}_{l;t}^{(i)}\right\}_{l=1}^{L_x}, \mathbf{A}_t^{(i)}, \right\}_{i=1}^{M},$$

where $\mathbf{Q}_{l;t}^{(i)} = \left(\sum_{k=1}^{t} \underline{\varphi}_{l;k}^{(i)} \underline{\varphi}_{l;k}^{(i)\top}\right)^{-1}$, vector $\underline{\varphi}_{l;k}^{(i)}$ results from the application of the basis functions $\varphi_{l1}, \ldots, \varphi_{lJ}$ on the i-th particle at time k, $\mathbf{x}_k^{(i)}$, and $\mathbf{A}_t^{(i)} = [\mathbf{a}_{1;t}^{(i)}, \ldots, \mathbf{a}_{L_x;t}^{(i)}]$ is the linear combiner computed via the RLS algorithm using the state sequence $\mathbf{x}_{0:t}^{(i)}$ and the observations $\mathbf{y}_{1:t}$.

At time $t+1$, the estimate of $f_x(\mathbf{x}_t^{(i)})$ required to compute the i-th particle risk is obtained as

$$\hat{f}_x(\mathbf{x}_t^{(i)}) = \text{diag}\left\{\mathbf{\Phi}_t^{(i)\top} \mathbf{A}_t^{(i)}\right\},$$

where $\mathbf{\Phi}_t^{(i)} = \left[\underline{\varphi}_{1;t}^{(i)}, \ldots, \underline{\varphi}_{L_x;t}^{(i)}\right]$. After selection and propagation of the state, $\hat{\mathbf{a}}_t^{(i)}$ and $\hat{\mathbf{Q}}_{l;t}^{(i)}$ (notice the use of $\hat{}$ to indicate that particles have been resampled) can be updated to obtain $\mathbf{a}_{l;t+1}^{(i)}$ and $\mathbf{Q}_{l;t+1}^{(i)}$ as

$$\begin{aligned}
\mathbf{g}_{t+1}^{(i)} &= \frac{\hat{\mathbf{Q}}_{l;t}^{(i)} \underline{\varphi}_{l;t}^{(i)}}{1 + \underline{\varphi}_{l;t}^{(i)\top} \hat{\mathbf{Q}}_{l;t}^{(i)} \underline{\varphi}_{l;t}^{(i)}} && (4)\\
\mathbf{a}_{l;t+1}^{(i)} &= \hat{\mathbf{a}}_{l;t}^{(i)} + \mathbf{g}_{l;t+1}^{(i)} \left(x_{l;t+1}^{(i)} - \underline{\varphi}_{l;t}^{(i)\top} \hat{\mathbf{a}}_{l;t}^{(i)}\right) && (5)\\
\mathbf{Q}_{l;t+1}^{(i)} &= \hat{\mathbf{Q}}_{l;t}^{(i)} - \mathbf{g}_{l;t+1}^{(i)} \underline{\varphi}_{l;t}^{(i)\top} \hat{\mathbf{Q}}_{l;t}^{(i)}. && (6)
\end{aligned}$$

The UCRPF algorithm is summarized on the next page, in Table 1. There a Gaussian density with adaptive variance is used for propagation of the particles [3].

4. COMPUTER SIMULATIONS

In this section we present computer simulations that illustrate the validity of our approach. We considered the problem of autonomous positioning of a vehicle moving along a two-dimensional space. The vehicle has means to estimate the power of three radio signals emitted from known locations and with known attenuation coefficients. This problem can be modeled by the state-space dynamic system

$$\begin{aligned}
\mathbf{x}_t &= f_x(\mathbf{x}_{t-1}) + \mathbf{u}_t && (7)\\
y_{j;t} &= 10\log_{10}\left(\frac{P_{j;0}}{\|\mathbf{x}_t - \mathbf{r}_j\|^{\alpha_j}}\right) + v_{j;t} \quad j = 1,2,3 && (8)
\end{aligned}$$

where $\mathbf{x}_t = [x_{1;t}, x_{2;t}]^\top \in \mathbb{R}^2$ indicates the vehicle position; its dynamics are given by the state transition function

$$f_x(\mathbf{x}_t) = \left[\begin{array}{c} 1.2|x_{1,t-1}|^{\frac{3}{4}} + 1.6|x_{2,t-1}|^{\frac{2}{3}} \\ \frac{|x_{2,t}|}{x_{2,t}}\left(2|x_{2,t}|^{\frac{1}{2}} + |x_{2,t}|^{\frac{4}{7}}\right) \end{array}\right], \qquad (9)$$

and $\mathbf{u}_t$ is the state noise process modeled as a random 2×1 vector with a mixture Gaussian distribution

$$\mathbf{u}_t \sim 0.85\mathcal{N}(0, 0.35\mathbf{I}_2) + 0.14\mathcal{N}(0, 7\mathbf{I}_2) + 0.1\mathcal{N}(0, 25\mathbf{I}_2), \quad (10)$$

where $\mathbf{I}_2$ represents the identity matrix of size 2×2. The vector $\mathbf{y}_t = [y_{1,t}, y_{2,t}, y_{3,t}]^\top$ collects the received power from the three emitters located at $\mathbf{r}_j \in \mathbb{R}^2$, $j = 1,2,3$, which transmit their signals with known initial power, $P_{j,0}$, through a fading channel with attenuation coefficient α_j, and, finally, $\mathbf{w}_t$ is the observation noise modeled as a white mixture Gaussian process given by

$$\begin{aligned}
w_{j;t} \;\sim\;& 0.85\mathcal{N}(0, 0.65) + 0.14\mathcal{N}(0, 4) + 0.1\mathcal{N}(0, 25)\\
& j = 1,2,3. \qquad (11)
\end{aligned}$$

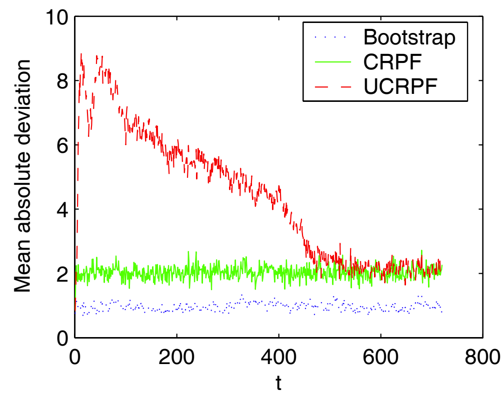

Fig. 1. Average absolute-deviation error attained by the CRPF, UCRPF and SPF algorithms.

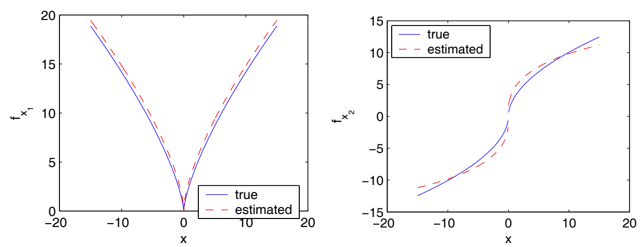

Fig. 2. **(a)** Estimate of $\hat{f}_x$ (first dimension). **(b)** Estimate of $\hat{f}_x$ (second dimension).

We applied the proposed CRPF and UCRPF algorithms in order to adaptively estimate the vehicle trajectory, $\mathbf{x}_{0:t}$, given the collected observations, $\mathbf{y}_{1:t}$. The CRPF, as described in subsection 2.2, is specified by

$$\begin{aligned}
X_0 &= (-10, +10)\\
\mathcal{C}(\mathbf{x}_0^{(i)}) &= 0 \quad \forall i \in \{1, ..., M\}\\
\triangle\mathcal{C}(\mathbf{x}_t|\mathbf{y}_t) &= \|\,\mathbf{y}_t - h(\mathbf{x}_t)\,\|^2\\
\mathcal{R}(\mathbf{x}_t|\mathbf{y}_{t+1}) &= \|\,\mathbf{y}_{t+1} - h\left(\mathbf{A}_i\mathbf{x}_t\right)\|^2\\
\mu(\mathcal{C}_t^{(i)}) &= \frac{1}{\left(\mathcal{C}_t^{(i)} - \min_k\left\{\mathcal{C}_t^{(k)}\right\} + \delta\right)^\beta}
\end{aligned}$$

where $\delta = 0.1$ and $\beta = 2$. We also chose a forgetting factor $\lambda = 0.99$. The considered form of μ was proposed and studied in [3]. As for the propagation density (p_{t+1} in Section 2.2), we considered a two-dimensional Gaussian pdf with i.i.d. components and adaptively chosen variance, as shown in Table 1, where the $\hat{}$ in $\hat{\sigma}_t^{2,(i)}$ indicates that particles have undergone resampling. The initial variance is $\sigma_0^2 = 10$. For the simulations presented below, the number of particles was fixed to $M = 200$.

The specification of the UCRPF is the same as for the CRPF, except that it needs to be completed by a set of basis functions $B = B_1 \cup B_2$, where

$$\begin{aligned}
B_1 &= \left\{\varphi_{1j}(\mathbf{x}_t) = |x_{1;t}|^{\frac{j}{J}}\right\}_{j=1}^{J}\\
B_2 &= \left\{\varphi_{2j}(\mathbf{x}_t) = \frac{|x_{2;t}|^{1+\frac{j}{J}}}{x_{2;t}}\right\}_{j=1}^{J}
\end{aligned}$$

INITIALIZATION: For $i = 1, ..., M$; $l = 1, ..., L_x$
$\mathbf{x}_0^{(i)} \sim \mathcal{U}(X_0)$; $\mathcal{C}_0^{(i)} = 0$; $\sigma_0^{2,(i)} = \sigma_0^2$; $\mathbf{Q}_{l;0}^{(i)} = 0.1\mathbf{I}_J$; $\mathbf{A}_0^{(i)} = \mathbf{0}_{J \times L_x}$

RECURSIVE UPDATE: UCRPF at time t is $\Omega_t = \left\{ \mathbf{x}_t^{(i)}, \mathcal{C}_t^{(i)}, \left\{ \mathbf{Q}_{l;t}^{(i)} \right\}_{l=1}^{L_x}, \mathbf{A}_t^{(i)}, \right\}_{i=1}^{M}$.

For $i = 1, ..., M$

Compute $\{\varphi_{lj}(\mathbf{x}_t^{(i)})\}_{l=1}^{L_x}$, then build $\left\{ \underline{\varphi}_{l;t}^{(i)} = \left[\varphi_{l1}(\mathbf{x}_t^{(i)}), \ldots, \varphi_{lJ}(\mathbf{x}_t^{(i)}) \right]^\top \right\}_{l=1}^{L_x}$ and $\mathbf{\Phi}_t^{(i)} = \left[\underline{\varphi}_{1;t}^{(i)}, \ldots, \underline{\varphi}_{L_x;t}^{(i)} \right]$.

Estimate the dynamics as $\hat{f}_x(\mathbf{x}_t^{(i)}) = \text{diag}\left\{ \mathbf{\Phi}_t^{(i)\top} \mathbf{A}_t^{(i)} \right\}$.

Compute the risk as $\mathcal{R}_{t+1}^{(i)} = \lambda \mathcal{C}_t^{(i)} + \left\| \mathbf{y}_{t+1} - f_y(\hat{f}_x(\mathbf{x}_t^{(i)})) \right\|^q$.

Build the selection pmf as $\hat{\pi}_{t+1}^{(i)} \propto \mu(\mathcal{R}_{t+1}^{(i)})$ where $\mu : \mathbb{R} \to [0, 1)$ is monotonically decreasing function.

Build the intermediate set $\hat{\Omega}_t = \left\{ \hat{\mathbf{x}}_t^{(i)}, \hat{\mathcal{C}}_t^{(i)}, \left\{ \hat{\mathbf{Q}}_{l;t}^{(i)} \right\}_{l=1}^{L_x}, \hat{\mathbf{A}}_t^{(i)}, \right\}_{i=1}^{M}$ by resampling Ω_t according to the pmf $\{\hat{\pi}_{t+1}^{(i)}\}_{i=1}^M$

For $i = 1, ..., M$

Propagate the i-th particle $\mathbf{x}_{t+1}^{(i)} \sim \mathcal{N}\left(\mathbf{x} | \hat{f}_x(\hat{\mathbf{x}}_t^{(i)}), \hat{\sigma}_t^{2,(i)} \right)$.

Compute the increment $\triangle\mathcal{C}_{t+1}^{(i)} = \triangle\mathcal{C}(\mathbf{x}_{t+1}^{(i)} | \mathbf{y}_{t+1})$ and update the cost as $\mathcal{C}_{t+1}^{(i)} = \lambda \hat{\mathcal{C}}_t^{(i)} + \triangle\mathcal{C}_{t+1}^{(i)}$.

Propagation variance update: <u>if</u> $t > 10$ <u>then</u> $\sigma_{t+1}^{2,(i)} = \frac{t-1}{t} \hat{\sigma}_t^{2,(i)} + \frac{\left\| \mathbf{x}_{t+1}^{(i)} - \hat{f}_x\left(\hat{\mathbf{x}}_t^{(i)} \right) \right\|^2}{t L_x}$, <u>else</u> $\sigma_{t+1}^{2,(i)} = \hat{\sigma}_t^{2,(i)}$.

For $l = 1, ..., L_x$: update the RLS algorithm $(\mathbf{g}_{t+1}^{(i)}, \mathbf{a}_{l;t+1}^{(i)}, \mathbf{Q}_{l;t+1}^{(i)})$ according to equations (4)-(6)

STATE ESTIMATION: $\pi_{t+1}^{(i)} \propto \mu(\mathcal{C}_{t+1}^{(i)})$, $\mathbf{x}_{t+1}^{mean} = \sum_{i=1}^M \mathbf{x}_{t+1}^{(i)} \pi_{t+1}^{(i)}$.

Table 1. Unstructured CRPF algorithm

and $J = 8$.

We simulated the evolution of the dynamic system (7)-(8) during one hour with a time step of 5 s (i.e., 720 discrete-time units) and compared the performance of the CRPF, UCRPF and a standard particle filter (SPF) (see, e.g., [1, Chapter 1]) with the same number of particles.

Figure 1 shows the average absolute-deviation error attained by the CRPF, UCRPF and BF when estimating the vehicle trajectory. The expression of the error, which was estimated from 50 independent simulation trials, is

$$e_t = \frac{1}{2 \times 50} \sum_{j=1}^{50} |x_{1;t;j} - \hat{x}_{1;t;j}| + |x_{2;t;j} - \hat{x}_{2;t;j}|,$$

where $x_{i;t;j}$ is the vehicle position on the i-th dimension, at discrete time t, in the j-th simulation run, and $\hat{x}_{i;t;j}$ is the corresponding estimate obtained from one of the three particle filtering algorithms. This plot shows that the performance of the proposed CRPF and UCRPF is worse than, but close to, that of the SPF. Note that the SPF is perfectly matched to the noise statistics given by (10) and (11) and requires perfect knowledge of the function f_x. Both the CRPF and the UCRPF are implemented without any *a priori* known probabilistic model for the noise processes, and the latter needs to estimate f_x online, together with the state trajectory.

Finally, Figures 2(a) and 2(b) show the function estimates, $\hat{f}_x$, computed by the UCRPF in a single simulation trial.

5. CONCLUSIONS

We have introduced a class of particle filters, called unstructured cost reference particle filters, which can track evolving unknowns of a dynamic system without the explicit knowledge of the noise probability distribution functions in the system *and* the function that models the evolution of the system state. The unknown function modeling the state trajectory is replaced by a set of linearly combined basis functions. The additional unknowns are the combining linear coefficients which become elements of the filter particles. The new filters have been successfully applied to the problem of positioning a moving vehicle along a two-dimensional plane.

6. REFERENCES

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