L_∞-NORM BASED PARTIAL-UPDATE ADAPTIVE FILTERING ALGORITHM FOR ECHO CANCELLATION

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ABSTRACT

In this paper, we provide a framework for developing low-complexity adaptive filtering algorithm by incorporating the concept of partial-updating into the technique of finding the gradient vector in the hyperplane based on the L_∞-norm criterion. The resulting algorithm is referred to as the partial-update normalized sign LMS (PU-NSLMS) algorithm. A specific case of the PU-NSLMS algorithm, called the M-Max PU-NSLMS algorithm, based on the concept of having a minimum Euclidean length of the coefficient-update vector is considered. It is shown that this algorithm is computationally less complex compared to the partial-update normalized least-mean square (PU-NLMS) algorithm. Results concerning the mean-square analysis of the M-Max PU-NSLMS algorithm are given. The performance of this algorithm is compared with that of the PU-NLMS algorithm in the case of network echo cancellation. It is shown that the convergence rate of the proposed algorithm is comparable to that of the PU-NLMS algorithm, but with a reduced complexity, making it a good choice for applications requiring a long filter tap, especially for real-time implementations.

1. INTRODUCTION

Adaptive filtering applications such as echo cancellation require a large number of filter taps in order to model the unknown system accurately. Since the computational complexity of an adaptive filtering algorithm is proportional to its tap length, such an algorithm might become computationally prohibitive for applications requiring large number of filter taps. One approach for reducing the computational complexity of an adaptive filtering algorithm is by updating only a subset of the filter coefficients at each time step [1]-[3]. The reduction in the complexity of the algorithm is proportional to the reduction in the number of filter coefficients updated at each time step. Another efficient approach for reducing the complexity is to employ quantization in the filter coefficient updates, such as in the case of the simplified NLMS algorithm due to Nagumo and Noda [4], [5], which is based on the minimum L_∞-norm method.

The objective of this paper is to provide a framework to develop a low-complexity algorithm by incorporating the concept of partial-updating into the technique of finding the gradient vector in the hyperplane based on the L_∞-norm criterion [5]. The resulting algorithm, referred to as the PU-NSLMS algorithm, reduces the complexity not only because of the updating of only a subset of the filter coefficients at each time step, but also because of the fact that compared to the algorithm based on the L_2-norm (i.e., NLMS algorithm), algorithms based on the L_∞-norm require a smaller number of operations for updating each filter coefficient.

There are various ways of selecting the subset of the filter coefficients in partial-update algorithms. Due to space constraint, we consider only one coefficient selection technique for the PU-NSLMS algorithm in this paper. In this technique, the subset of the filter coefficients to be updated at a particular time step is chosen such that the Euclidean length of the resulting coefficient-update vector is minimum. The mean-square analysis of the PU-NSLMS algorithm using this coefficient selection technique is carried out to obtain the evolution equation for the mean-square of the filter coefficient misalignment (FCM), as well as the bounds on the step-size. The performance of this algorithm as well as that of the PU-NLMS [1] algorithm is studied in the case of network echo cancellation. It is shown that there is very little degradation in the convergence rate of the proposed algorithm as the number of filter coefficients updated at each time step is reduced. Also, for the same number of filter coefficients updated at each time step, the convergence rate of the proposed PU-NSLMS algorithm is comparable to that of the PU-NLMS [1] algorithm, but with a reduced complexity.

2. PU-NSLMS ALGORITHM

In this section, we provide the derivation of the PU-NSLMS algorithm using the error control procedure as suggested in [5]. Let $y(k)$ be the desired response of the adaptive filter, $\mathbf{W}(k) = [w_o(k), w_1(k), \ldots, w_{N-1}(k)]^T_{N\times 1}$ the filter coefficient vector, and $\mathbf{X}(k) = [x(k), x(k-1), \ldots, x(k-N+1)]^T_{N\times 1}$ the input regressor vector at time k. For an adaptive filter, the error $e(k)$ in estimating the desired response is given by

$$e(k) = y(k) - \mathbf{W}^T(k)\mathbf{X}(k) \tag{1}$$

This error is minimized by adjusting the filter coefficient-vector $\mathbf{W}(k)$ by a vector quantity $\mathbf{A}(k)\Delta(k)$ to give

$$\begin{aligned}(1-\mu)e(k) &= y(k) \\ &- \left[\mathbf{W}^T(k) + \Delta^T(k)\mathbf{A}(k)\right]\mathbf{X}(k)\end{aligned} \tag{2}$$

where μ is the step-size, $\Delta(k) \in \Re^N$ a variable vector, and $\mathbf{A}(k)$ a $N \times N$ diagonal matrix whose diagonal elements are +1 or 0 such that $trace(\mathbf{A}(k)) = M$ is a constant less than or equal to N. Substituting (1) in (2) yields

$$\mu e(k) = \Delta^T(k)\mathbf{A}(k)\mathbf{X}(k) \tag{3}$$

It should be noted that there is no unique solution for $\Delta(k)$. If, we solve for $\Delta(k)$ based on the minimum L_∞-norm method (for details see [5]), we obtain

$$\Delta(k) = \frac{\mu e(k)}{\|\mathbf{A}(k)\mathbf{X}(k)\|_1} sign\{\mathbf{A}(k)\mathbf{X}(k)\} \tag{4}$$

where $\|.\|_1$ and $sign\{.\}$ are the L_1-norm and sign operators, respectively. The resulting coefficient-update equation of the PU-NSLMS algorithm is

$$\begin{aligned}\mathbf{W}(k+1) &= \mathbf{W}(k) \\ &+ \frac{\mu e(k)}{\|\mathbf{A}(k)\mathbf{X}(k)\|_1 + \delta} sign\{\mathbf{A}(k)\mathbf{X}(k)\}\end{aligned} \tag{5}$$

where δ is a small positive number added to the denominator to prevent the division by zero.

The subset of the filter coefficients that are updated at any particular time k are determined from the matrix $\mathbf{A}(k)$, which is referred to as the coefficient selection matrix. For $trace(\mathbf{A}(k)) = N$, the PU-NSLMS algorithm reduces to the full update algorithm, namely, the simplified NLMS algorithm due to Nagumo and Noda [4], [5]. The coefficient selection matrix $\mathbf{A}(k)$ has no unique solution, if $trace(\mathbf{A}(k)) = M$ is a constant less than N. In such cases, there are various possible ways of obtaining the subset of the filter coefficients for partial-updating. In the coefficient selection technique considered in this paper, a subset of the filter coefficients are chosen such that the Euclidean length of the resulting coefficient-update vector is minimum. It can be shown that the $\mathbf{A}(k)$ for this coefficient selection technique is given by

$$\mathbf{A}(k) = diag\{a_1(k), a_2(k), \ldots, a_N(k)\} \tag{6}$$

where

$$a_i(k) = \begin{cases} 1 & if\ i = arg\, M\, maxima\ of\ |x(k-i+1)| \\ 0 & otherwise \end{cases} \qquad i \in (1, \ldots, N)$$

The coefficient selection technique of the resulting PU-NSLMS algorithm is similar to that of the M-Max NLMS algorithm [3]. Therefore, the PU-NSLMS algorithm using this technique is referred to as the M-Max PU-NSLMS algorithm. The M-Max PU-NSLMS algorithm needs to run a sorting process to obtain the subset of M filter coefficients that are updated at each time step. Fast algorithms such as the ones in [6], require $2log_2(N)+2$ comparisons to carry out this sorting. Compared to the PU-NLMS algorithm [1], which requires $N+M+2$ multiplications, one division, $N+M+2$ additions and $2log_2(N)+2$ comparisons at each time step, the proposed M-Max PU-NSLMS algorithm requires $N+1$ multiplications, one division and the same numbers of additions and comparisons as in the case of the PU-NLMS algorithm for updating the M filter coefficients at each time step.

3. MEAN-SQUARE ANALYSIS

In this section, the mean-square analysis of the proposed *M*-Max PU-NSLMS algorithm is carried out. In order to make the analysis tractable, certain assumptions are made about the statistics of the input data and desired response. The first is the *independence assumption*, which has been shown to produce theoretical results that can accurately predict the simulations (for details, refer to [7]). The second is that the input signal is generated from a stationary, zero-mean, white Gaussian process with a variance σ_x^2. The desired response of the adaptive filter $y(k)$ is defined as

$$y(k) = \mathbf{W}^T\mathbf{X}(k) + n(k) \tag{7}$$

where $\mathbf{W}$ is the optimum filter response, and $n(k)$ a stationary, zero-mean, white noise with a variance σ_n^2. The filter coefficient misalignment (FCM) vector $\widetilde{\mathbf{W}}(k)$ is defined as

$$\widetilde{\mathbf{W}}(k) = \mathbf{W} - \mathbf{W}(k) \tag{8}$$

Solving for the FCM covariance matrix $E\{\mathbf{V}(k+1)\} = E\left\{\widetilde{\mathbf{W}}(k+1)\,\widetilde{\mathbf{W}}^T(k+1)\right\}$ under the independence assumption, it can be shown that

$$\begin{aligned} E\{\mathbf{V}(k+1)\} &= E\{\mathbf{V}(k)\} \\ &-\widehat{\mu}E\left\{sign\{\mathbf{A}(k)\mathbf{X}(k)\}\mathbf{X}^T(k)\right\}E\{\mathbf{V}(k)\} \\ &-\widehat{\mu}E\{\mathbf{V}(k)\}E\left\{sign\{\mathbf{A}(k)\mathbf{X}(k)\}\mathbf{X}^T(k)\right\} \\ &+\widehat{\mu}^2E\left\{sign\{\mathbf{A}(k)\mathbf{X}(k)\}\mathbf{X}^T(k)\mathbf{V}(k)\mathbf{X}(k)\,. \right. \\ &\quad \left. .sign\left\{\mathbf{X}^T(k)\right\}\right\} \\ &+\widehat{\mu}^2E\left\{sign\{\mathbf{A}(k)\mathbf{X}(k)\}\,sign\left\{\mathbf{X}^T(k)\mathbf{A}(k)\right\}\right\}\sigma_n^2 \end{aligned} \quad (9)$$

Using (9), it can be proved that the evolution equation for the mean-square of the FCM is given by

$$\sigma_{\tilde{w}}^2(k+1) = [1-\lambda]\,\sigma_{\tilde{w}}^2(k) + \widehat{\mu}^2 M\sigma_n^2 \quad (10)$$

where

$$\sigma_{\tilde{w}}^2(k+1) = trace\left(E\{\mathbf{V}(k+1)\}\right) \quad (11)$$

$$\lambda = \frac{M}{N}\left[2\widehat{\mu}\left(\sqrt{\frac{2}{\pi}}\zeta_1\sigma_x\right) - \widehat{\mu}^2(N+\zeta_2-1)\,\sigma_x^2\right] \quad (12)$$

$$\widehat{\mu} = \frac{\mu}{M\sqrt{\frac{2}{\pi}}\sigma_x\zeta_1} \quad (13)$$

$$\zeta_1 = \frac{N\,E\{\|\mathbf{A}(k)\mathbf{X}(k)\|_1\}}{M\,E\{\|\mathbf{X}(k)\|_1\}} \quad (14)$$

$$\zeta_2 = \frac{N\,E\left\{\left\|\mathbf{A}(k)\mathbf{X}^2(k)\right\|_1\right\}}{M\,E\{\|\mathbf{X}^2(k)\|_1\}} \quad (15)$$

For the lack of space, the proofs to establish (9) and (10) are omitted.

Letting $k \to \infty$ in (10) gives, $\sigma_{\tilde{w}}^2(\infty)$, the steady-state FCM of the *M*-Max PU-NSLMS algorithm:

$$\sigma_{\tilde{w}}^2(\infty) = \left[\frac{\mu N}{2M\left(\frac{2}{\pi}\zeta_1^2\right) - \mu(N+\zeta_2-1)}\right]\frac{\sigma_n^2}{\sigma_x^2} \quad (16)$$

Stability of the evolution equation for the mean-square of the FCM of the *M*-Max-PU-NSLMS algorithm is guaranteed, if

$$\left|1 - 2\frac{\mu}{N} + \frac{\mu^2(N+\zeta_2-1)}{MN\left(\frac{2}{\pi}\zeta_1^2\right)}\right| < 1 \quad (17)$$

that is,

$$0 < \mu < \frac{2M}{(N+\zeta_2-1)}\left(\frac{2}{\pi}\zeta_1^2\right) \quad (18)$$

For large values of N, (18) can be approximated as

$$0 < \mu < \frac{2M}{N}\left(\frac{2}{\pi}\zeta_1^2\right) \quad (19)$$

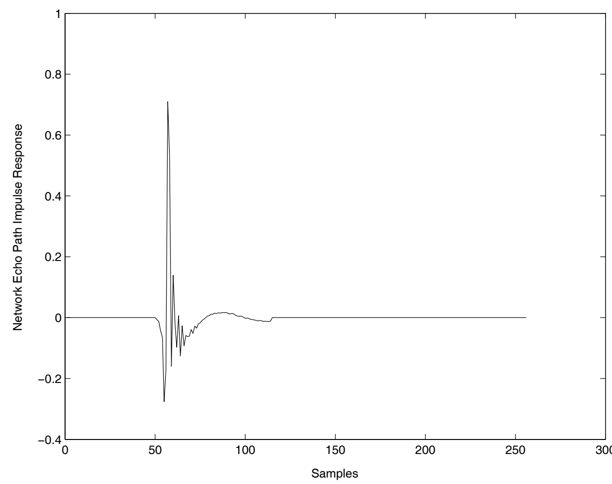

Fig. 1. Impulse response of a network echo path.

4. SIMULATIONS

In this section, we study the performance of the proposed M-Max PU-NSLMS algorithm as well as that of the PU-NLMS [1] algorithm in the case of network echo cancellation. Fig. 1 shows the impulse response $\mathbf{W}$ of the network echo path used for simulations. The input signal is generated from a stationary, zero-mean, white Gaussian process with a variance $\sigma_x^2 = 1$. The signal to noise ratio (SNR) is 40 dB. The L_2-norm of the FCM, $\|\mathbf{W}(k) - \mathbf{W}\|_2^2$, is used as the performance index. The values of the expressions ζ_1 and ζ_2 are obtained using a method similar to the one suggested in [1]. For $N = 256$ and $M = 64$, the values are $\zeta_1 = 2.0470$ and $\zeta_2 = 2.8883$.

Fig. 2 shows the FCM curves of the M-Max PU-NSLMS algorithm for the case of $N = 256$, $M = 64$, and $\mu = 0.1\mu_{\max}$, $0.2\mu_{\max}$ and $0.3\mu_{\max}$, where $\mu_{\max}$ is the upper bound given by (19). The FCM curves were obtained by averaging over 50 trials. Fig. 2 also shows the corresponding theoretical FCM curves obtained from (10). It can be seen that the theoretical results are very close to that obtained through simulations. It should also be noted that the discrepancy between the theoretical and simulation results widens with increasing step-size. This is due to the fact that the independence assumption made while deriving the theoretical expressions does not hold true for large values of the step-size.

Fig. 3 shows the FCM curves of the proposed M-Max PU-NSLMS algorithm for the case of $M = 32$, 64, 256. It also shows the FCM curves of the PU-NLMS [1] algorithm for the case of $M = 32$ and 64. These curves were obtained by averaging over 50 trials. The step-size for the two algorithms are chosen such that they achieve approx-

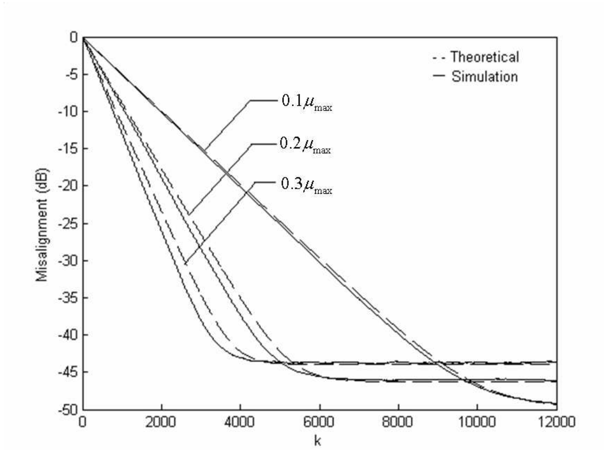

Fig. 2. Simulated and theoretical FCM curves of the M-Max PU-NSLMS algorithm for $N = 256$, $M = 64$, and $\mu = 0.1\mu_{\max}$, $0.2\mu_{\max}$, $0.3\mu_{\max}$.

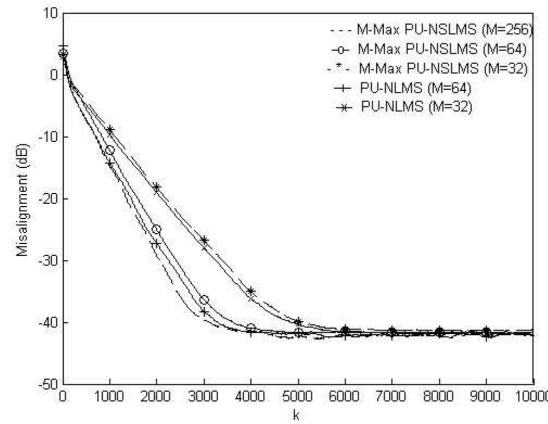

Fig. 3. FCM curves of the M-Max PU-NSLMS algorithm (M = 32, 64, 256) and PU-NLMS algorithm (M = 32, 64).

imately the same steady-state FCM. For $M = N = 256$, the M-Max PU-NSLMS reduces to the full update algorithm. There is very little degradation in the convergence rate of the proposed algorithm as the number of filter coefficients updated at each time step is reduced. Also, for the same number of filter coefficients updated at each time step, the convergence rate of the M-Max PU-NSLMS algorithm is comparable to that of the PU-NLMS algorithm.

5. CONCLUSIONS

In this paper, we have provided a framework for developing low-complexity adaptive filtering algorithm by incorporating the concept of partial-updating into the technique of finding the gradient vector in the hyperplane based on the L_∞-norm criterion. Due to space constraint, only one coefficient selection technique based on minimizing the Euclidean length of the coefficient-update vector has been considered for the proposed partial-update algorithm in this paper. Other possible choices for coefficient update will be considered in an upcoming paper. The mean-square analysis of this algorithm has been carried out to obtain the evolution equation for the mean-square of the filter coefficient misalignment as well as the bounds on the step-size. The theoretical results have been validated through simulations. The proposed algorithm has been shown to have a convergence rate comparable to that of the PU-NLMS algorithm, but with a reduced complexity, making it a good choice for applications such as echo cancellation.

6. ACKNOWLEDGEMENT

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