

# DYNAMIC SELECTED MAPPING FOR OFDM

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## ABSTRACT

Orthogonal frequency division multiplexing (OFDM) transmission systems generally have low power efficiency, due to the large peak-to-average power ratio (PAR) of the OFDM signal. Selected mapping (SLM) is a promising technique to reduce the PAR for OFDM. A drawback of SLM is its high computational requirement, which hinders its practical implementation. In this paper, we propose a dynamic, two-buffer structure to reduce the computational requirement without sacrificing the PAR reduction capability. Performance analysis and simulations of the proposed technique are also carried out.

## 1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a promising technique for high speed data transmission and has been adopted by many standards, such as IEEE 802.11a/g in the US, and DAB, DVB, HiperLAN/2 in Europe. One serious drawback of OFDM, however, is its large peak-to-average power ratio (PAR), which causes problems when nonlinear components such as power amplifiers and mixers are encountered during transmission. A large PAR also demands a large dynamic range of the analog-to-digital converter (ADC). For the power amplifier or the mixer, a large back-off is needed when the PAR is high, resulting in poor power efficiency. PAR reduction is often necessary to reduce the cost and improve the power efficiency of the transmission system.

There has been a great deal of research on PAR reduction for OFDM. One can pursue PAR reduction algorithms with distortion or without distortion. Deliberate clipping is the simplest PAR reduction method with distortion. However, it causes increase in the symbol-error-rate (SER) and/or spectral regrowth. Among all distortionless PAR reduction algorithms, selected mapping (SLM) is one of the most promising [1]. SLM chooses one signal with the lowest PAR from a set of “equivalent” signals which are related in the frequency domain by a series of phase rotations. However, SLM requires a large amount of additional computations, which may hinder its practical use in high speed data transmissions. Tone reservation and tone injection [2] are also distortionless PAR reduction methods, but the continuous parameter optimization involved can be computationally very intensive.

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To reduce the computational requirement of SLM, a simple approximation of the inverse discrete Fourier transform (IDFT) is proposed in [3], but the price paid is degradation in PAR reduction performance. In this paper, we propose a dynamic SLM method that can greatly reduce the computational requirement of the SLM method without sacrificing the PAR reduction capability.

This paper is organized as follows: In Section 2, we describe the SLM method and point out its computational requirement as a challenge. In Section 3, we propose a dynamic SLM method which can greatly reduce the number of mappings required and thus the computational requirement as well. Section 4 concludes this paper.

## 2. PAR REDUCTION AND SLM

In OFDM,  $N$  frequency-domain sub-symbols  $\{X_k\}_{k=0}^{N-1}$  are transformed into the time-domain by the IDFT to yield  $\{x_n\}_{n=0}^{N-1}$  via

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j(2\pi k/N)n}. \quad (1)$$

Denote by  $\text{PAR}_1$  the PAR of the original OFDM signal,

$$\text{PAR}_1 = \text{PAR}\{x_n\} = \frac{\max_{0 \leq n \leq N-1} |x_n|^2}{E[|x_n|^2]}, \quad (2)$$

where  $E[\cdot]$  denotes statistical expectation.

Assume that  $\{X_k\}_{k=0}^{N-1}$  is stationary and that  $X_k$  and  $X_l$  are uncorrelated for  $k \neq l$ . Based on the Central Limit Theorem,  $\{x_n\}_{n=0}^{N-1}$  is approximately independently and identically distributed (i.i.d.) complex Gaussian when  $N$  is large [2]. The complementary cumulative distribution function (CCDF) of  $\text{PAR}_1$ ; i.e., the probability that  $\text{PAR}_1$  exceeds a certain threshold  $\gamma$ , can be calculated as [1]:

$$\Pr\{\text{PAR}_1 > \gamma\} = 1 - (1 - e^{-\gamma})^N. \quad (3)$$

SLM was first proposed in [1] to reduce the PAR of a given OFDM block. The assumption is that the same phase table  $\{\phi_k^{(d)}\}$ ,  $0 \leq k \leq N-1$ ,  $1 \leq d \leq D$ , where  $\phi_k^{(1)} = 0$ ,  $\forall k$ , is available to both the transmitter and the receiver. In SLM, we first rotate the phases of  $X_k$  as in

$$X_k^{(d)} = X_k e^{j\phi_k^{(d)}}, \quad (4)$$

and then take the IDFT to obtain  $x_n^{(d)}$ . Although  $X_k^{(d)}$  and  $X_k$  contain the same information,  $x_n^{(d)}$  and  $x_n$  can have very different PAR values. In SLM,  $x_n^{(d)}$  which has the lowest

PAR among the  $D$  equivalent sequences, is transmitted; i.e.,

$$\tilde{d} = \arg \min_{1 \leq d \leq D} \text{PAR}\{x_n^{(d)}\}. \quad (5)$$

We denote the associated lowest PAR value by

$$\text{PAR}_D = \min_{1 \leq d \leq D} \text{PAR}\{x_n^{(d)}\}. \quad (6)$$

The CCDF of  $\text{PAR}_D$  is given by [1]

$$\Pr\{\text{PAR}_D > \gamma\} = (1 - (1 - e^{-\gamma})^N)^D. \quad (7)$$

SLM is simple and effective. However, there are a few points to consider:

(i) There is power cost associated with the  $D - 1$  extra sets of computations involved in implementing SLM: phase rotations, IDFTs and PAR calculations. It was shown in [4] that the amount of power that can be saved by SLM well exceeds the amount of power required for its implementation.

(ii) The receiver needs to know the optimal phase sequence index  $\tilde{d}$  in order to decode. Blind SLM methods that avoid the transmission of such side information have been investigated; see [5], [6].

(iii) In SLM, a fixed number of  $D$  mappings are performed at the transmitter, which consume approximately  $D$  times the computational resources as compared to the original OFDM. For simplicity, let us approximate the computational overhead of SLM by the amount of computation needed for the IDFTs. For example, an  $N$ -point IDFT can be implemented very efficiently on the Texas Instruments 'C6x<sup>®</sup> DSP which requires a total of  $(2N + 16) \log_2 N + 25$  clock cycles. When  $D = 16$  and  $N = 128$ , the required number of DSP clock cycles is 30864, which implies that the sampling rate cannot exceed  $128 \times 150 \times 10^6 / 30864 = 622$  thousand-samples-per-second at a 150 MHz clock rate, and thus the signal bandwidth cannot exceed 311 kHz, which is very limiting for modern communications applications. A larger bandwidth can be accommodated by lowering  $D$ , at the expense of less PAR reduction in SLM.

The objective of this paper is to build upon the SLM framework, devise a PAR reduction method that has similar performance as SLM, but that requires much less computational resources than SLM.

### 3. DYNAMIC SLM SCHEME FOR PAR REDUCTION

The idea of SLM is to find among  $D$  equivalent signal representations, the one that has the lowest PAR, to transmit. Since physical devices (such as ADCs, PAs) have a finite dynamic range, there is a practical limit  $\gamma$  for the PAR. From (7), for finite  $N$  and  $D$  values, there is always a non-zero probability that even after  $D$  mappings, SLM will not be able to reach a given PAR reduction threshold, and thus the peak of the OFDM block will be clipped. Often times in practice, the goal is to meet a certain threshold on the PAR, and minimizing the PAR for each OFDM block is not necessary. In that spirit, our problem can be stated as: Given a PAR threshold  $\gamma$  and a small positive number  $\epsilon$ , devise an efficient PAR reduction scheme to make  $\Pr\{\text{PAR} > \gamma\} \leq \epsilon$ .

For a given sequence  $\{X_k\}$ , each mapping can be regarded as a Bernoulli trial that reduces the PAR of the mapped sequence to below the threshold  $\gamma$  with probability  $(1 - e^{-\gamma})^N$  (c.f. (3)). Denote by  $Z$  the random variable corresponding

to the first successful trial that realizes the goal  $\text{PAR} < \gamma$ ;  $Z$  has a Geometric distribution; i.e.,

$$\Pr\{Z = d\} = a(1 - a)^{d-1}, \quad (8)$$

where

$$a = (1 - e^{-\gamma})^N, \quad (9)$$

and  $d = 1, 2, \dots$

Let us consider an example, where  $N = 128$  and  $\gamma = 7$  dB (i.e.,  $\gamma = 5.012$ ). We find from (7) that  $\Pr\{\text{PAR}_1 \leq \gamma\} = 0.4252$ ,  $\Pr\{\text{PAR}_2 \leq \gamma\} = 0.6696$ ,  $\dots$ ,  $\Pr\{\text{PAR}_{16} \leq \gamma\} = 0.9999$ . This means that there is a 43% chance that a given OFDM block already has a low enough PAR and thus PAR reduction is not necessary. There is a 67% chance that SLM with  $D = 2$  is sufficient in meeting the PAR goal. If we want  $\Pr\{\text{PAR}_D > \gamma\} \leq 10^{-4}$ , in the above example, the required  $D$  is 16. However, as we have seen,  $D = 16$  mappings are not necessary for some of the OFDM blocks. In fact, the expected number of mappings  $Z$  needed to achieve a target PAR threshold  $\gamma$  can be calculated as follows [4]

$$E[Z] = \sum_{d=1}^{\infty} d \times a \times (1 - a)^{d-1} = \frac{1}{a} = \frac{1}{(1 - e^{-\gamma})^N}. \quad (10)$$

For  $N = 128$ , we infer from (10) that  $E[Z] = 2.35$  mappings are required on average to satisfy  $\text{PAR} \leq 7$  dB.

A simple, modified SLM algorithm can be considered: If a given OFDM block has  $\text{PAR}_1 \leq \gamma$ , transmit it as is; otherwise, try an increasing number of mappings until  $\text{PAR}_d \leq \gamma$  is realized. If after the maximum allowed  $D$  number of mappings,  $\text{PAR}_D$  is still  $> \gamma$ , stop trying and transmit the signal representation with the lowest PAR. This method reduces the computational load, but causes a delay jitter (variable latency) problem that is undesirable.

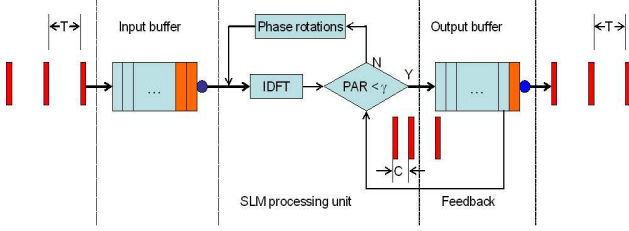
Our goal in this paper is to improve upon SLM, to reduce the computational demand without introducing delay jitter and without sacrificing the PAR reduction performance. We shall achieve this by employing input and output buffers and a dynamic SLM mechanism.

#### 3.1. Queuing Model of Dynamic SLM

A dynamic SLM scheme with two buffers is shown in Fig. 1. The input buffer contains OFDM blocks to be processed (PAR reduced). The output buffer contains OFDM blocks that have been processed and are ready to be transmitted. In the SLM processing unit, the task is to reduce the PAR of a given OFDM block by different mappings until one mapping has a PAR that is less than  $\gamma$ . This processed OFDM block will then be transferred to the output buffer. If the input buffer is not empty, the next available OFDM block in the input buffer will be retrieved by the SLM processing unit. Let us denote by  $T$  the OFDM block arrival interval at the input buffer. Denote by  $C$  the processing time needed for one mapping (phase rotations, IDFT, and PAR calculation). Assume that  $L = T/C$  is an integer. Assume that  $M \times T$  is the total delay between an OFDM block's arrival at the input buffer and its departure from the output buffer. The total number of OFDM blocks in the two buffers and in the SLM processing unit is  $M$  which is a constant.

To describe the queuing behavior of the input and output buffers, we introduce the following notations:  $t_h = (hT)$ ,  $t_h^- = (hT) - \delta$ ,  $t_h^+ = (hT) + \delta$ , where  $\delta$  is a positive but infinitesimally small number.  $t_h^-$  and  $t_h^+$  stand for the time instants immediately before and after  $t_h$ , respectively. We

assume that at  $t_h^+$ , one OFDM block arrives at the input buffer and one OFDM block departs from the output buffer. If one OFDM block is retrieved by the SLM processing unit from the input buffer at  $t_h^+$ , and  $d$  mappings are carried out for PAR reduction, then this block will arrive at the output buffer at time instant  $(t_h + dC)^-$ . The queue lengths of both buffers are measured at time instants  $t_h$ ,  $h = 1, 2, 3, \dots$



**Figure 1. Queuing model for the proposed dynamic SLM scheme.**

The arrival process of the input buffer is a deterministic process with a fixed interval  $T$ . The service time of the SLM processing unit is the time used to map the sequences and search for a qualified phase rotation sequence to meet the PAR threshold. The departure process of the input buffer and the arrival process of the output buffer have the same statistical behavior; their rates are determined by the number of mappings needed to meet the PAR threshold. To avoid any delay jitter, we require that the departure process of the output buffer be a deterministic process with the same fixed interval  $T$  as well. This fixed departure rate requires that the output buffer cannot be empty at any time instant  $t_h$ ,  $h = 1, 2, 3, \dots$

During one sample interval  $T$ ,  $L$  mappings can be performed. Denote by random variable  $Y$  the number of OFDM blocks that can be processed by the SLM processing unit in the  $h$ th sample interval between time instants  $t_h$  and  $t_{h+1}$ , and treat one mapping as a Bernoulli trial for PAR reduction. Then  $Y$  is the number of successes in  $L$  trials, which is Binomial distributed if there is no underflow in either the output buffer or the input buffer:

$$Pr\{Y = y\} = B_a(y|L) = \binom{L}{y} (1-a)^{L-y} a^y, \quad (11)$$

for  $y = 0, 1, 2, \dots, L$ .

Denote by random variable  $S$  the queue length of the output buffer at time instant  $t_h$ . Therefore, the number of OFDM blocks in the input buffer and the SLM processing unit is  $M - S$  at time instant  $t_h$ . Because one new OFDM block arrives at the input buffer at time instant  $t_h^+$ ,  $M - S + 1$  is the upper bound for  $Y$ . If  $M - S + 1 \geq L$ , the probability mass function of  $Y$  is also given by (11). On the other hand, if  $M - S + 1 < L$ , the probability mass function of  $Y$  is:

$$Pr\{Y = y\} = \begin{cases} B_a(y|L), & \text{if } y = 0, 1, \dots, M - S, \\ \sum_{i=y}^{M-S+1} B_a(i|L), & \text{if } y = M - S + 1. \end{cases} \quad (12)$$

However, with this arrival rate to the output buffer, the output buffer will underflow when  $Y = 0$  happens  $S$  times consecutively, thus causing delay jitter in the departure process of the output buffer. For that reason, a feedback path from the output buffer to the SLM processing unit is added. If at time instant  $t_h$ , there is only one OFDM block left in the output buffer ( $S = 1$ ), and the SLM algorithm cannot find a suitable mapping to reduce the PAR to  $\gamma$  for the given data between time instants  $t_h$  and  $t_{h+1}$ , the SLM

processing unit will be forced to produce an output to be pushed to the output buffer at  $t_{h+1}^-$  even though its PAR is higher than  $\gamma$ . For the case with  $S = 1$  and  $M \geq L$ , the probability mass function of  $Y$  including the feedback is:

$$Pr\{Y = y\} = \begin{cases} B_a(0|L) + B_a(1|L), & \text{if } y = 1, \\ B_a(y|L), & \text{if } 2 \leq y \leq L. \end{cases} \quad (13)$$

For the case with  $S = 1$  and  $M < L$ , the probability mass function of  $Y$  including the feedback is:

$$Pr\{Y = y\} = \begin{cases} B_a(0|L) + B_a(1|L), & \text{if } y = 1, \\ B_a(y|L), & \text{if } 2 \leq y \leq M - 1, \\ \sum_{i=y}^L B_a(i|L), & \text{if } y = M. \end{cases} \quad (14)$$

For the case with  $S > 1$ , (11) holds for  $M - S + 1 \geq L$ , and (12) holds for  $M - S + 1 < L$ .

### 3.2. Performance analysis

As in SLM, dynamic SLM can only guarantee that the resulting PAR is smaller than  $\gamma$  with a certain probability. Denote by  $R(\gamma)$  the probability that the dynamic SLM fails to reduce PAR to  $\gamma$ . The failure only happens when  $S = 1$  at  $t_h$  and the Bernoulli trial fails  $L$  times between time instants  $t_h$  and  $t_{h+1}$ . Therefore, we have

$$R(\gamma) = Pr\{S = 1\} B_a(0|L). \quad (15)$$

For given  $L$  and  $M$  values,  $R(\gamma)$  is proportional to  $Pr\{S = 1\}$ , the probability that there is only one OFDM block in the output buffer at time instant  $t_h$ . The queue length of the output buffer is analyzed in this section using a Markov model.

Assume that the OFDM blocks are mutually independent and that the phase rotation sequences are mutually independent. The queue length of the output buffer at time instant  $t_h$ ,  $h = 0, 1, 2, \dots$ , represents the status of the dynamic SLM scheme, which can be modeled by a Markov chain with  $M$  states. The probability transition matrix can be written as  $\mathbf{P} = [P_{il}]_{M \times M}$ , where  $P_{il}$  is the probability that  $S = i$  at time instant  $t_{h+1}$  conditioned on  $S = l$  at time instant  $t_h$ . That means  $Y = i - l + 1$  OFDM blocks arrive at the output buffer and one OFDM block departs from the output buffer between time instants  $t_h$  and  $t_{h+1}$ .

Due to the feedback from the output buffer to the input buffer,  $P_{11} = B_a(0|L) + B_a(1|L)$ , which is  $Pr\{Y = 1\}$  in (13). If  $M - L + 1 \leq l \leq M$ , the input buffer has a certain probability to underflow, so that  $P_{Ml} = \sum_{y=M-l+1}^L B_a(y|L)$ , which is  $Pr\{Y = M - S + 1\}$  in (12). For the case without feedback and underflow,  $P_{il} = B_a(i - l + 1|L)$ , which is  $Pr\{Y = y\}$  in (11). In summary, the value of  $P_{il}$  can be expressed as:

$$P_{il} = \begin{cases} \sum_{y=0}^{2-l} B_a(y|L), & i = 1, \text{ and } 1 \leq l \leq 2, \\ B_a(i - l + 1|L), & 2 \leq i \leq M - 1, \text{ and } \max(1, i + 1 - L) \leq l \leq i + 1, \\ \sum_{y=M-l+1}^L B_a(y|L), & i = M, \text{ and } \max(1, M - L + 1) \leq l \leq M, \\ 0, & \text{otherwise.} \end{cases}$$

Denote  $\boldsymbol{\pi} = [\pi_1, \pi_2, \dots, \pi_M]^T$  as the steady state vector of the Markov chain, and  $\sum_{k=1}^M \pi_k = 1$ . The element  $\pi_i$  represents the probability that  $i$  blocks are in the output buffer at time  $t_h$ .  $\boldsymbol{\pi}$  is the eigen vector of the probability transition matrix  $\mathbf{P}$  corresponding to the eigen value 1. Then the probability that  $PAR > \gamma$  is given by

$$R(\gamma) = \pi_1 B_a(0|L), \quad (16)$$

which happens when the SLM algorithm fails to reduce the PAR to below  $\gamma$  between time instants  $t_h$  and  $t_{h+1}$  for  $S=1$ .

SLM can be viewed as a special case of the dynamic SLM with  $L = D$  and  $M = 1$ , for which  $\pi_1 = 1$ . Therefore, (16) reduces to  $R(\gamma) = 1 \times B_a(0|L) = (1-a)^D$ , which agrees with (7).

### 3.3. Example

In this example, we assume that the OFDM signal has  $N = 128$  sub-carriers, the information symbol is randomly picked from a QPSK signal constellation. Assume that the dynamic SLM uses  $L = 3$ , and  $M = 4$ . The corresponding Markov model has 4 states. The state diagram is shown in Fig. 2 and the transition probability matrix  $\mathbf{P}$  is:

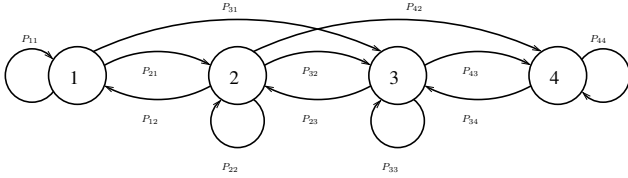
$$\begin{bmatrix} \sum_{i=0}^1 B_a(i|L) & B_a(0|L) & 0 & 0 \\ B_a(2|L) & B_a(1|L) & B_a(0|L) & 0 \\ B_a(3|L) & B_a(2|L) & B_a(1|L) & B_a(0|L) \\ 0 & B_a(3|L) & \sum_{i=2}^3 B_a(i|L) & \sum_{i=1}^3 B_a(i|L) \end{bmatrix}.$$

By solving  $\pi = \mathbf{P}\pi$ , we obtain

$$\pi = \frac{1}{C} \begin{bmatrix} (1-a)^9, (1-a)^6 a^2 (3-2a), \\ (1-a)^3 a^3 (1+6a-9a^2+3a^3), \\ a^5 (3-2a)(2+3a-6a^2+2a^3) \end{bmatrix}^T,$$

where  $C = 1 - 9a + 39a^2 - 103a^3 + 186a^4 - 234a^5 + 221a^6 - 150a^7 + 60a^8 - 10a^9$ , and

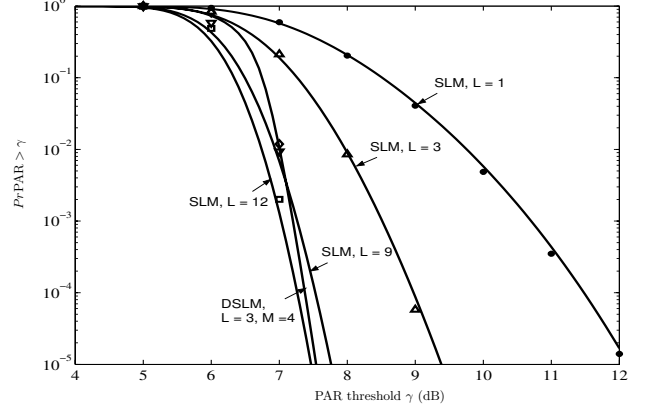
$$R(\gamma) = \pi_1 B_a(0|3) = \frac{(1-a)^{12}}{C}. \quad (17)$$



**Figure 2. State diagram for the Markov Chain.**

50,000 OFDM blocks were used for the verification of the CCDF expressions. Fig. 3 shows the theoretical  $R(\gamma)$  (c.f. (17)) as a solid line (indicated by DSLM) for  $L = 3$ ,  $M = 4$ , as well as the sample  $R(\gamma)$  values as marked points. For comparison, the CCDF curves of SLM (c.f. (7)) with  $L = 1, 3, 9, 12$ , are also included. In Fig. 3, we observe that the dynamic SLM scheme with  $L = 3$ ,  $M = 4$  outperformed the SLM method with  $L = 3$ , although they have the same computational requirement. On the other hand, the dynamic SLM scheme with  $L = 3$ ,  $M = 4$  could not outperform the SLM method with  $L = 12$ ; the computational requirement of the latter is 4 times that of the former. Note that in the dynamic SLM scheme, the number of mappings conducted for each OFDM block is a random number between  $L = 3$  and  $L \times M = 12$ . At the  $10^{-4}$  CCDF level, the dynamic SLM algorithm with  $L = 3$ ,  $M = 4$  reduced the PAR by 1.5 more dBs as compared to the SLM algorithm with  $L = 3$ . The PAR reduction performance of the dynamic SLM approach ( $L = 3$ ,  $M = 4$ ) was comparable to that of the SLM algorithm with  $L = 12$  (within 0.2 dB). The results of this example demonstrate that the proposed dynamic SLM technique reduces the computational load for each OFDM block by approximately 4 times, and thus the

same DSP hardware can handle a sampling rate that is approximately 4 times of what is possible with the original SLM.



**Figure 3.  $N = 128$  sub-carriers. CCDF curves of the conventional SLM with  $L = 1, 3, 9, 12$ , and the CCDF of the dynamic SLM scheme with  $L = 3$ ,  $M = 4$ .**

## 4. CONCLUSIONS

PAR reduction is often necessary in order to improve the power efficiency of an OFDM system. SLM is one of the most promising PAR reduction methods. However, its high computation resource requirement can hinder its use in high-speed data transmissions. In this paper, we proposed a two-buffer, dynamic SLM scheme to reduce the computational requirement of SLM. Once the prescribed PAR threshold is met, the algorithm stops striving for lower PAR values. The number of mappings to try by the SLM processing unit is dynamically assigned. The proposed algorithm reduces the computational requirement without sacrificing the PAR reduction capability and without creating any throughput jitter.

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