Energy Efficient Channel Estimation in MIMO Systems

Sarod Yatawatta, Athina P. Petropulu[1]
Electrical and Computer Engineering Department
Drexel University, Philadelphia PA 19104
{sarod, athina}@ece.drexel.edu

Charles J. Graff
US Army RDECOM CERDEC STCD,
Fort Monmouth, NJ 07703
Charles.Graff@usarmy.mil

***Abstract*— We consider the problem of MIMO channel estimation subject to a given error and delay constraints. Our objective is to minimize the energy spent during the channel estimation phase, which includes transmission of training symbols, storage of those symbols at the receiver, and also channel estimation at the receiver. We develop a model that is independent of the hardware or software used for channel estimation, and use a divide and conquer strategy to minimize the overall energy consumption.**

I. INTRODUCTION

The use of multiple input multiple output (MIMO) channels formed using multiple transmit/receive antennas has been demonstrated to have great potential for achieving high data rates [7]. Of concern, however, is the increased complexity associated with multiple transmit/receive antenna systems. First, increased hardware cost is required to implement multiple RF chains. Second, increased complexity and energy is required to estimate large size MIMO channels.

Energy conservation in MIMO systems has been considered in different perspectives. In [2] for instance, hardware level optimization is done to minimize energy. On the other hand, in [4],[5], energy consumption is minimized at the receiver by using low rank equalization. In [3] reducing the order of MIMO systems by selection of antennae is given as a viable option to minimize energy consumption both at the receiver and transmitter, without degrading the system performance. In [6] the transmission and circuit energy consumption per bit of information transmitted is analyzed. The authors claim in [6] that single input single output (SISO) (1×1) systems gives best performance over MIMO (2×2) systems for short range transmission.

In this paper we focus on MIMO channel estimation subject to delay and error constraints. We propose an antenna selection scheme for channel estimation that can minimize energy consumed both at the transmitter and the receiver.

We can summarize the novelty of the proposed scheme as follows. (i) we concentrate exclusively on the channel estimation phase unlike in [6] where the authors have considered the data transmission phase; (ii) we propose an antenna selection scheme to minimize energy during channel estimation unlike [3] where information theoretic performance during data transmission is considered for antenna selection; (iii) the proposed method can be applied independent of the hardware or software used for channel estimation. In fact, the hardware and software can be optimized independently of the proposed method as in [2].

The rest of the paper is organized as follows. In the next section we describe the generalized energy reduction scheme. After this we focus on minimizing energy at the transmitter and the receiver separately. Next we consider joint transmitter and receiver energy minimization. To illustrate our method we consider a scalar MIMO system of arbitrary size and give comparisons of energy and error variation for different channel estimation schemes obtained by varying the number of active transmit/receive antennas under a fixed delay and error constraint.

[1]This work has been supported by NSF under grant CNS-0435052

II. GENERAL METHODOLOGY

In this section we describe the proposed method in a general sense.

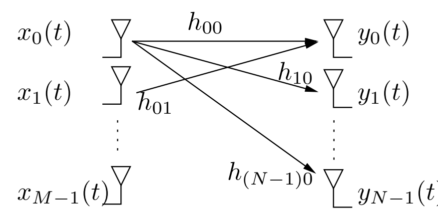

Fig. 1. MIMO channel

The fundamental property that we assume in our scheme is the modularity of hardware. For instance, when a complex hardware system is built, it is done in a modular way by assembling less complex blocks. Hence, a MIMO system can be considered as a collection of SISO systems, with respect to hardware. For instance, we assume a 4 by 4 MIMO system can operate as a 2 by 2 system by turning off some modules.

Let us consider a MIMO system with M transmitters and N receivers as given in Fig. 1. We call the set of transmitters $\mathbf{T}$ and the set of receivers $\mathbf{R}$. Their cardinalities, $|\mathbf{T}|$ and $|\mathbf{R}|$, are M and N respectively. The objective is to estimate the channels h_{ij}, $0 \leq i \leq N-1$, $0 \leq j \leq M-1$ in an energy efficient manner. The channel estimation requires the consumption of energy and time.

We make the following assumptions:

- A1 We can ignore electromagnetic interaction between antenna elements. Thus, if we estimate h_{ij} by having active only a subset of transmitters/receivers, the estimate will be the same as the estimate we would get for the same channel if all transmitters/receivers were active.
- A2 The channels are frequency flat fading and during the training phase, the channels remain time invariant.

We propose the following divide and conquer strategy. Instead of estimating all M by N channels at once, we estimate subsets of channels step by step. This seemingly gives an obvious reduction in complexity at the receiver. For instance, if we estimate all the channels at once, the complexity is $O(NM^2)$, (inversion of an N by M matrix is approximately $O(NM^2)$) assuming a matrix inversion is required. However, if we use only half the transmitters $M/2$ and all receivers N with two steps, the complexity is $2O(NM^2/4)$ which cuts the complexity by half. However, such reduction does not consider the energy required for transmission and data acquisition and so we need more detailed models.

Instead of estimating the full channel matrix at once (which we call the *naive* method), we propose to estimate the full channel matrix in K steps. On the k-th step, ($k \in [1, K]$) we select the transmitters given by the set $\mathbf{T}_k$ ($\subseteq \mathbf{T}$) and the receivers given by the set $\mathbf{R}_k$ ($\subseteq \mathbf{R}$) and estimate the channels between those transmitters and receivers. Let P_k be the power level of each transmitter at the k-th step, and l_k denote the length of training data to be used in channel estimation. Moreover, let the noise power level at the receiver be σ^2. Hence, at the k-th step, the average SNR at the receiver will be proportional to P_k/σ^2. We assume all transmitters have the same fading level, i.e., each transmitter is approximately at the same distance from the receiver and the channel is flat fading.

We will focus on minimizing the total energy consumption, both at the receiver and transmitter. We define the following functions. Let g_T be the energy spent by all the transmitters. At the receivers, the energy consumption can be broken down into two components: the energy required to perform data acquisition and storage, which we denote by g_I, and the energy needed to perform channel estimation or computations, which we denote by g_C. In our formulation, g_T, g_I, and g_C are functions of the variables K, $\mathbf{T}_k$, $\mathbf{R}_k$, l_k, P_k $k = 1, \ldots, K$. For notational convenience, this dependence in not shown in the sequel.

The total energy consumed can be given as

$$g = g_T + g_I + g_C. \tag{1}$$

Our objective is to minimize g. Next we consider the constraints involved.

- *Avoiding trivial solutions*: In order to estimate all the channels we need

$$\bigcup_{k=1,\ldots,K} \mathbf{T}_k \otimes \mathbf{R}_k = \mathbf{T} \otimes \mathbf{R} \tag{2}$$

  where $\otimes$ is the Cartesian product. In order to avoid trivial solutions we need

$$\mathbf{T}_k \neq \phi, \ \mathbf{R}_k \neq \phi, \ k \in [1, K] \tag{3}$$

  where ϕ is the null set.
- *Satisfying a channel MSE constraint*: For acceptable performance, the mean channel estimation error (MSE) at each step ϵ_k should be below a minimum threshold,

$$\epsilon_k = \epsilon_k(\mathbf{T}_k, \mathbf{R}_k, P_k, l_k) \leq \epsilon, k \in [1, K]. \tag{4}$$

  The exact expression for ϵ_k is dependent on the channel estimation method. If we consider the power level at each step, it should be higher than some threshold $\underline{P_k}$ for the channel estimation to work, and it should be lower than the maximum allowed by the transmitter P.

$$\underline{P_k} \leq P_k \leq P, \ k \in [1, K] \tag{5}$$

- *Satisfying a transmission delay constraint*: The training length at step k should be above a certain threshold $\underline{l_k}$ for the channel estimation to work and the total data length would be below the maximum delay allowed L.

$$\underline{l_k} \leq l_k, \ k \in [1, K], \ \sum_{k=1}^{K} l_k \leq L \tag{6}$$

Our objective is to find $\mathbf{T}_k$,$\mathbf{R}_k$,P_k and l_k for $k = 1, \ldots, K$ subject to the above constraints (2), (3), (4), (5), (6), that minimizes g given in (1). This is an NP hard problem. However, we pursue simplified solutions in the following sections.

Before we proceed, let us consider the feasibility of the problem. We see that all the parameters are bounded. Hence the feasibility region is bounded and in order to find feasible solutions, we should choose the limits ϵ and L in a suitable manner. For instance if we choose $\epsilon = 0$ or $L = 0$ it is obvious that no solutions exist. Hence by increasing either or both of these values, we can increase the feasibility region. In other words, we can trade off energy with channel estimation error and delay.

III. Minimizing energy at the transmitter

We make the following assumptions:

- B1 We assume the receiver has no constraints on energy because we only minimize energy at the transmitter. This allows us to always make $\mathbf{R}_k = \mathbf{R}$. In other words, we use all receivers at all steps.
- B2 We assume the antennas to be uncorrelated, so that the channel estimate will not change with the selection of $\mathbf{T}_k$ and $\mathbf{R}_k$. Moreover, we assume the only variable affecting the channel estimation error to be the sizes of $\mathbf{T}_k$ and $\mathbf{R}_k$ and not the individual elements in them.
- B3 We assume retransmissions to be costly and hence select disjoint sets of transmitters, i.e. $\mathbf{T}_k$ are disjoint. In other words each transmitter only transmit during only one step k.

Proposition 1: The channel estimation error at the k-th step

$$\epsilon_k = c_1 \frac{\sigma^2}{P_k l_k} |\mathbf{T}_k| \tag{7}$$

where σ^2 is the noise variance, c_1 is a constant.
The proof is given in Appendix I.

The total energy spent by all the transmitters can be given as

$$g_T = \sum_{k=1}^{K} c_2 P_k l_k |\mathbf{T}_k| \tag{8}$$

where c_2 is a constant. Due to $\mathbf{R}_k = \mathbf{R}$, and $\mathbf{T}_k$ being disjoint, we can simplify (2) as

$$\sum_{k=0}^{K} |\mathbf{T}_k| = |\mathbf{T}| = M. \tag{9}$$

This is a standard integer partition problem. For instance if $M = 4$ the ways we can select the number of transmitters during the K steps are $\{4\}(K = 1)$,$\{3, 1\}(K = 2)$,$\{2, 2\}(K = 2)$,$\{2, 1, 1\}(K = 3)$ and $\{1, 1, 1, 1\}(K = 4)$. Thus there are 5 possible ways in this case. If the number of possible ways of selecting $|\mathbf{T}_k|$ is $p(M)$ for $|\mathbf{T}| = M$, we have [1]

$$p(M) \approx \frac{1}{4\sqrt{3}} \left(\frac{e^{\pi \sqrt{(2/3)M}}}{M} \right). \tag{10}$$

For small values of M, i.e. $M \leq 10$, we can try all possible partitions to find the best one. Once we have enumerated $\mathbf{T}_k$ the problem reduces to

$$\min_{P_i, l_i, i \in [1,K]} \sum_{k=1}^{K} c_2 P_k l_k |\mathbf{T}_k| \tag{11}$$

subject to (4), (5), (6), where $\mathbf{T}_k$, $\mathbf{R}_k$ and K are constants.

Proposition 2: Under assumptions A1-A2 and B1-B3, the channel estimation scheme that minimizes transmitter energy is to reduce the MIMO channel into a set of single input multiple output (SIMO) channels and transmit using one transmitter only at a time. Thus each time we estimate a SIMO channel. The minimum energy is

$$\underline{g} = c_1 c_2 \frac{\sigma^2}{\epsilon} M \tag{12}$$

as opposed to the energy of the naive method

$$\overline{g} = c_1 c_2 \frac{\sigma^2}{\epsilon} M^2 \tag{13}$$

The proof is given in appendix II.

This result agrees with intuition since in this case there is reduced interference from other transmitters. However under different assumptions and different channel estimation schemes, we might get different results.

IV. Minimizing energy at the receiver

In contrast to the transmitter, the energy consumption at the receiver is due to data acquisition and computation. From Appendix I we see that the computational energy required will be

$$g_{C,k} = c_3 |\mathbf{T}_k||\mathbf{R}_k| l_k^2 \tag{14}$$

where c_3 is a constant. The energy required for data acquisition and storage will be proportional to the data length. Hence

$$g_{I,k} = c_4 |\mathbf{R}_k| l_k \tag{15}$$

where c_4 is a constant and the total energy will be

$$g_T = \sum_{k=1}^{K} c_3 |\mathbf{T}_k||\mathbf{R}_k| l_k^2 + c_4 |\mathbf{R}_k| l_k \tag{16}$$

Our objective is to minimize g_T subject to the (4), (5), (6) constraints.

Proposition 3: Under assumptions A1-A2 and B2, the channel estimation scheme that minimizes the energy consumption at the receiver is to estimate each SIMO channel individually by using one transmitter and all receivers at each step. The minimum energy is

$$\underline{g} = NM\left(c_3 c_1^2 \frac{\sigma^4}{P^2\epsilon^2} + c_4 c_1 \frac{\sigma^2}{P\epsilon}\right) \tag{17}$$

as opposed the the energy of the naive method

$$\overline{g} = NM\left(c_3 c_1^2 \frac{\sigma^4}{P^2\epsilon^2} M^2 + c_4 c_1 \frac{\sigma^2}{P\epsilon}\right) \tag{18}$$

The proof is given in Appendix III.

V. Minimizing energy both at the transmitter and receiver

From Propositions 2 and 3 we can conclude that the optimal scheme of channel estimation for a MIMO system that minimize both transmitter and receiver energy consumption is to reduce the system into a set of SIMO channels and estimate each SIMO channel individually. In other words, instead of transmitting the training symbols from all transmitters simultaneously, we have to transmit them in a sequential manner by activating only one transmitter at a time. In order to satisfy the delay requirement, each transmitter will be active only for a fraction of the time it would have been active if all transmitters were transmitting simultaneously.

VI. Numerical Example

We considered an 8×8 system with SNR 20 dB. In Fig. 3 we show results of 4 possible schemes for channel estimation. Our constraints are, maximum error $\epsilon = 10^{-3}$ and delay $L = 56$. In *scheme 1*, we used 56 symbols per each transmitter (total 448) and employed the naive method to estimate the 8×8 system. In *scheme 2* we used Proposition 2 and used 7 symbols per each transmitter (total 56) to estimate the 8×1 SIMO systems (8 times). In *scheme 3* we transmitted 7 symbols from each transmitter (total 56) and again used the naive method. Finally in *scheme 4* we used 14 symbols per each transmitter but reduced the system into 4, 8×2 systems (total 112) to estimate the channel in 4 steps. We see that scheme 1 has the lowest error but highest energy consumption. Scheme 3 has the lowest energy consumption and lowest delay, but the channel cannot be estimated because the training matrix does not have full row rank. Schemes 2 and 4 have intermediate performance in terms of error and energy, and we see that a trade off can be accomplished between error and energy. Although in this example schemes 2 and 4 have higher channel estimation error, the final conclusion can be drawn only after numerical evaluation of the performance in terms of the bit error rate. The constants P, c_1,c_2,c_3,c_4 can be calculated given the hardware or can be experimentally measured.

VII. Conclusions

Using a generic model for channel estimation error and energy consumption of a MIMO system, we have shown that the optimal channel estimation scheme in terms of minimizing energy consumption is to convert the MIMO system into a set of SIMO channels by activating each transmitter individually and performing channel estimation on each SIMO system. However, the energy reduction comes at an increase in estimation error. In our formulation, we have assumed a homogeneous, isotropic, uncorrelated set of transmitters and receivers. There is room in this area for future work on adapting this method to a MIMO channel formed by a disparate set of transmitters and receivers with different power, computation and storage capabilities and different radiation patterns.

Appendix I
Channel estimation error and energy

In this section we consider a MIMO system with frequency flat fading channels. We consider the least squares channel estimation using training symbols with all transmitters and receivers active (we call this the naive method). The basic equation can be given as $\mathbf{y} = \mathbf{H}\mathbf{x}+\mathbf{v}$ where $\mathbf{y}$ is a N by 1 vector, $\mathbf{H}$ is the N by M channel matrix, $\mathbf{x}$ is the M by 1 training vector and $\mathbf{v}$ is the N by 1 noise vector. Let us use J training blocks to estimate the channel. Grouping J blocks we have $\mathbf{Y} = \mathbf{H}\mathbf{X}+\mathbf{V}$ where $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_J]$, $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_J]$ and $\mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_J]$. For full rank condition of $\mathbf{X}$ we need $J \geq N$. The channel estimation error is

$$\boldsymbol{\xi} = \hat{\mathbf{H}} - \mathbf{H} = \mathbf{V}\mathbf{X}^{\dagger} \tag{19}$$

and the MSE

$$MSE = \frac{1}{NM}\text{trace}(\boldsymbol{\xi}\boldsymbol{\xi}^H) = \frac{1}{NM}\text{trace}((\mathbf{X}^H\mathbf{X})^{\dagger}\mathbf{V}^H\mathbf{V}) \tag{20}$$

We consider $\mathbf{X}$ having orthogonal rows and full row rank. Then $\mathbf{X}\mathbf{X}^H = \mathbf{I}$. However, since $J > N$ there is no way to choose all columns orthogonally. Hence $\mathbf{X}^H\mathbf{X}$ will not be diagonal. Similarly, $\mathbf{V}^H\mathbf{V}$ will not be diagonal if $J > M$. If we consider any generic channel estimation scheme, we know that the channel estimation error is inversely proportional to the SNR and the data length while it is directly proportional to the interference i.e. the number of transmitters. Hence we formulate the error as

$$MSE = c\sigma^2 \frac{M}{JP} \tag{21}$$

where σ^2 is the noise power, P is the signal power and c is a constant. In order to verify above formulation, we have simulated random channels and have given the result in Fig. 2. By substituting $J = l_k$, $P = P_k$, $M = |\mathbf{T}_k|$ we get (7).

Next we calculate the total number of computations required. If the rows of $\mathbf{X}$ are not orthogonal, the computation of the psuedoinverse $\mathbf{X}^{\dagger}$ requires approximately $O(MJ^2)$ operations. The multiplication $\mathbf{Y}\mathbf{X}^{\dagger}$ requires $O(MNJ^2)$ operations. Altogether we have an $O(MNJ^2)$ computation assuming $\mathbf{X}^{\dagger} = \mathbf{X}^H$.

APPENDIX II
PROOF OF PROPOSITION 2

The Lagrangian (ignoring the lower bounds for P_k and l_k) is

$$\begin{aligned}\mathcal{L} &= \sum_{k=1}^{K} c_2 P_k l_k |\mathbf{T}_k| + \sum_{k=1}^{K} \lambda_{1,k}\left(c_1 \frac{\sigma^2}{P_k l_k}|\mathbf{T}_k| - \epsilon\right) \\ &+ \sum_{k=1}^{K} \lambda_{2,k}(P_k - P) + \lambda_3\left(\sum_{k=1}^{K} l_k - L\right)\end{aligned} \quad (22)$$

where $\lambda_{1,k}$,$\lambda_{2,k}$,λ_3 are the multipliers. For optimality we need

$$\frac{\partial \mathcal{L}}{\partial l_k} = c_2 P_k |\mathbf{T}_k| - \lambda_{1,k} c_1 \frac{\sigma^2}{P_k l_k^2}|\mathbf{T}_k| + \lambda_3 = 0 \quad (23)$$

and

$$\frac{\partial \mathcal{L}}{\partial P_k} = c_2 l_k |\mathbf{T}_k| - \lambda_{1,k} c_1 \frac{\sigma^2}{P_k^2 l_k}|\mathbf{T}_k| + \lambda_{2,k} = 0 \quad (24)$$

We select a solution as follows. From (5), we select $P_k = P$. From (4) and (7) we select

$$l_k = c_1 \frac{\sigma^2}{P\epsilon}|\mathbf{T}_k| \quad (25)$$

If $\mathbf{T}_k = \mathbf{T}$, $|\mathbf{T}_k| = M$ and from (6) we need

$$l_1 = L \geq c_1 \frac{\sigma^2}{P\epsilon} M \quad (26)$$

Hence

$$\sum_{k=1}^{K} l_k = c_1 \frac{\sigma^2}{P\epsilon} \sum_{k=1}^{K} |\mathbf{T}_k| = c_1 \frac{\sigma^2}{P\epsilon} M \leq L \quad (27)$$

and we see that by selecting l_k according to (25), (6) is automatically satisfied. Hence we have a feasible solution. Next we check its optimality. Since (6) is satisfied and active, we have $\lambda_3 > 0$. From (23) we have

$$\lambda_{1,k} = (\lambda_3 + c_2 P |\mathbf{T}_k|)\frac{P l_k^2}{c_1 \sigma^2 |\mathbf{T}_k|} \quad (28)$$

which is positive. Next from (24) we have $\lambda_{2,k} = \lambda_3 \frac{l_k}{P}$ which is again positive. Hence the solution is optimal. By substitution of $P = P_k$ and (25) in (8), we get (29). The minimum transmitter energy given the partition of $\mathbf{T}$ is

$$\underline{g_T} = c_1 c_2 \frac{\sigma^2}{\epsilon} \sum_{k=1}^{K} |\mathbf{T}_k|^2 \quad (29)$$

Thus we see that the partition that minimizes (29) consists of all ones, i.e. $\{1, 1, \ldots, 1\}$. In other words, in order to minimize transmission energy, we should estimate channels selecting each transmitter individually. Substituting $|\mathbf{T}_k| = 1$ into (29) we get (12) and substituting $K = 1$,$|\mathbf{T}_k| = M$,and (26) we get (13). ■

APPENDIX III
PROOF OF PROPOSITION 3

Note that there is no transmitter power term P_k in (16) and we can select $P_k = P$. Next we select the data length as in (25). By substitution into (16) we have

$$g = \sum_{k=1}^{K} c_3 c_1^2 \frac{\sigma^4}{P^2\epsilon^2}|\mathbf{T}_k|^3 |\mathbf{R}_k| + c_4 c_1 \frac{\sigma^2}{P\epsilon}|\mathbf{T}_k||\mathbf{R}_k| \quad (30)$$

and the only constraint (6) reduces (using (26)) to

$$\sum_{k=1}^{K} c_1 \frac{\sigma^2}{P\epsilon}|\mathbf{T}_k| \leq L, \quad \text{or} \quad \sum_{k=1}^{K} |\mathbf{T}_k| \leq M. \quad (31)$$

Minimizing (30) subject to (31) is a standard discrete programming problem. It is easy to see that in order to satisfy (31) we need to partition the transmitters disjointly. In that case, the partition that minimizes (30) is $|\mathbf{T}_k| = 1$ for all k. In this case, we need to use all the receivers and the only possible partition for $\mathbf{R}_k$ is $\mathbf{R}$. Hence we can conclude that the channel estimation scheme that minimizes receiver energy consumption is to estimate each SIMO channel individually. Substituting $|\mathbf{T}_k| = 1$ into (30) we get (17) and substituting $K = 1$,$|\mathbf{T}_k| = M$,and (26) we get (18). ■

REFERENCES

[1] G.H. Hardy and S. Ramanujan, "Asymptotic formulae in combinatory analysis," in *Proc. of the London Mathematical Soc.,* no.2, vol. 17, pp. 75-115, 1918.

[2] S. Rajgopal, S. Bhashyam, J.R. Cavallaro and B. Aazhang, "Efficient VLSI architectures for Multiuser Channel Estimation in Wireless Base Station Receivers," Journal of VLSI Signal Processing, no. 31, pp. 143-156, 2002.

[3] A.F. Molisch and M.Z. Win, "MIMO systems with antenna selection," IEEE Microwave Magazine, vol. 5, no. 1, pp. 46-56, March 2004.

[4] G. Dietl and W. Utschick, "On reduced-rank approaches to matrix Wiener filters in MIMO systems," Proc. ISSPIT, Daarmstadt, Germany 2003.

[5] Y. Sun, M.L. Honig, V. Tripathi, "Adaptive iterative reduced-rank equalization for MIMO channels," in Proc. MILCOM, no. 2, pp. 1029-1033 Oct. 2002.

[6] S. Cui, A.J. Goldsmith, and A. Bahai, "Energy efficiency in MIMO and cooperative MIMO techniques in sensor networks," *IEEE Jnl. on Sel. Areas in Commun.*, vol. 22, no.6, pp. 1089-1098, Aug. 2004.

[7] G.J. Foschini and M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," Wireless Pers. Commun. vol. 6, pp. 311-335, 1998.

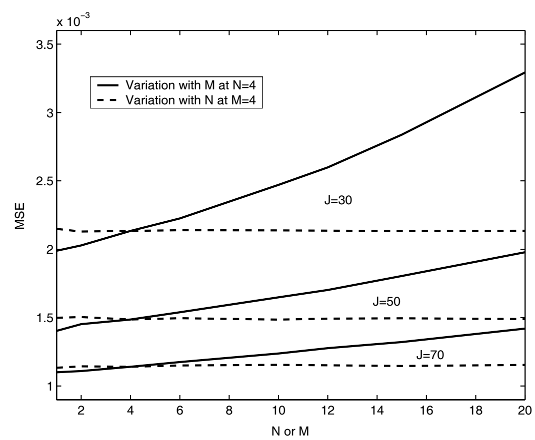

Fig. 2. MSE variation with N, M and J . We see that the MSE is independent of N, has a linear variation with M and is inversely proportional to J.

Scheme	$\text{MSE}/10^{-3}$	Energy
1	0.2	$c_2 P448 + c_3(448)^2 + c_4 448$
2	1.0	$c_2 P56 + c_3(56)^2 + c_4 448$
3	∞	$c_2 P56 + c_3(56)^2 + c_4 56$
4	0.8	$c_2 P112 + c_3(112)^2 + c_4 448$

Fig. 3. MSE and Energy for different channel estimation schemes for 50, random 8 by 8 channels .