On Propagation of Self-Similar Traffic Through an Energy-Conserving Wireless Gateway

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***Abstract*—[1] It has been well established by now that high-speed wireline traffic exhibits self-similar behavior. Several studies in the past have hypothesized that wireless traffic is also self-similar but without adequate justification. In this paper we study the propagation of self-similarity as self-similar wireline traffic feeds to a gateway that interconnects a wireline to a wireless network. We model the wireline traffic as an On/Off process. We propose models for buffering and repacking performed at the gateway. Based on those models and also statistical models for the wireless channel, we study the statistics of the outgoing On/Off traffic. We show that when the On and Off state durations of the input traffic are both heavy-tail distributed, such as in the case of LAN traffic, the outgoing traffic is self-similar. On the other hand, if the On state durations are heavy-tail distributed but the Off state durations have finite variance, such as in variable-bit-rate video traffic, the self-similarity maybe disappear if the gateway has a buffer much larger than the maximum channel capacity and it operates under an energy conserving protocol.**

I. INTRODUCTION

With the increasing demands for wireless Internet access and the fast evolution of wireless techniques, various high-speed multimedia services will soon be provided via wireless networks. Statistical modeling of traffic is of great importance in network engineering, and a substantial body of literature has been devoted to it.

Over the past decade, a number of empirical studies have convincingly demonstrated that wireline network traffic generated by multimedia applications exhibits burstiness over a wide range of timescales [10] and is long-range dependent, or self-similar. Several model for wireline traffic have been proposed [10], [12], [13].

As of recently, there has been works suggesting that wireless traffic might also self-similar [16], [9] simply as an extension of the wireline traffic behavior. However, there are big differences between wired and wireless transmission, as the effectiveness of the latter is restricted by wireless channel impairments, and power limitations of the wireless transmitters/receivers.

To study the extent to which the wireline traffic statistical characteristics propagate in the wireless traffic, we here study the role of the gateway that interconnects wireline and wireless networks. In general, packet sizes are different over a heterogeneous collection of networks, and the gateway provides a means by which packets are fragmented and reassembled [8]. Since energy consumption of portable wireless terminals (e.g. PDA and laptops) is a limiting factor in the services these devices can provide, we here consider an energy conserving gateway protocol, that targets at reducing the amount of time a mobile node needs to have its receiver on. Our study also takes into account the wireless channel modeled by the two-state Markovian model of [5]. In [14] we considered the problem for the case of a gateway behaving as a small buffer system (i.e., the buffer can hold at most one packet). We here add to that work by considering a gateway that behaves like a large buffer system (i.e., the buffer size is much larger than the service rate), and also a buffer system that alternates between two different service rates, which is close to a practical system.

[1]This work has been supported by ONR under grant N00014-03-1-0123

II. BACKGROUND

The *Pareto* survival function equals:

$$\bar{F}(x;\alpha,K) = P(X \geq x) = \begin{cases} (\frac{K}{x})^{\alpha}, & x \geq K, \\ 1, & x < K, \end{cases} \quad (1)$$

where K is a positive constant and α is the tail index with $1 < \alpha < 2$.

The *On/Off process* is used to model traffic generated by a single user. The overall network traffic can be viewed as superposition of On/Off processes. The On/Off process alternates between two states: the On, during which the source generates traffic at a rate A_j, and the Off, during which the source remains silent. Let X_j and Y_j denote the duration of the j-th On and Off state, respectively. In most On/Off-type models, each of the X_j, Y_j are assumed to be independent identically distributed (i.i.d.) according to a heavy-tail distribution with infinite variance (e.g. Pareto distribution), or have finite variance [10].

The Hurst parameter of the On/Off process equals [11]: $H = \frac{1}{2}(3 - min(\alpha_0, \alpha_1)$ where α_1, α_0 are the tail indices of the On and Off durations, respectively. If the On or Off durations have finite variance then the corresponding tail index is taken as 2 when calculating H [10]. The On/Off process is self-similar if the corresponding Hurst parameter satisfies: $1/2 < H < 1$. This implies that at least one of the On or Off durations has to be heavy-tail distributed for the On/Off process to be self-similar.

III. SYSTEM MODEL

Modelling of incoming wireline traffic: The incoming traffic, $S(t)$, is modelled as a rate-limited EAFRP; this model was proposed and validated in [13]. Its On/Off durations, denoted here by X_j, Y_j are Pareto distributed according to $\bar{F}(x;\alpha_1,K_1)$ and $\bar{F}(x;\alpha_0,K_0)$, respectively. The On rates, A_j, are i.i.d. cut-off Pareto distributed with survival function: $\bar{F}_L(x;\alpha_A,K_A) = P(X \geq x) = \bar{F}(x;\alpha_A,K_A)(1-u(x-L))$ where $u(.)$ is the unit step function; L represents the rate limit imposed by competing media; and $1 < \alpha_A < 2$ [13]. The Hurst parameter of such traffic is within (0.5,1), thus the traffic is self-similar.

The wireless channel model: We follow the two-state Markovian process of [5], in which the channel strictly alternates between *good* and *bad* states, with corresponding service rates: c_g and c_b. The durations of both state are independent exponentially distributed with means $1/\beta$ and $1/\gamma$, respectively. We further assume that the channel is slowly varying, so that the alternation of channel states is relatively slow compared with that of the incoming traffic $S(n)$. In other words, the service rate of the gateway can be assumed to be constant within several On /Off periods of $S(n)$.

The gateway model: Traffic streams from one or more connections feed into a buffer. Let us assume that the service time is slotted, with time slot denoted by τ. In the sequel, we will use the notation $S(n)$, $T(n)$ instead of $S(t)$, $T(t)$, where n is the slot index. During each time slot, at most one packet can leave buffer. Since the incoming traffic is assumed to be a continuous bit flow, the server needs to repack bits into equal-sized packets and send them out via the wireless channel. For the packing operation, we here consider the following energy conserving model: if the data in the buffer are less than the packet size, the server takes no action and waits until there are enough data to form a packet. The output traffic $T(n)$, and instant buffer content $Q(n)$ equal:

$$T(n) = \begin{cases} c, & \\ & \text{if } S(n) + Q(n-1) \geq c \\ 0, & \\ & \text{if } S(n) + Q(n-1) < c \end{cases} \tag{2}$$

$$Q(n) = \begin{cases} < S(n) + Q(n-1) - c, 0 > \wedge B, & \\ \quad \text{if } S(n) + Q(n-1) \geq c \\ < S(n) + Q(n-1), 0 > \wedge B, & \\ \quad \text{if } S(n) + Q(n-1) < c \end{cases} \tag{3}$$

where $< \alpha, \beta >= max(\alpha, \beta)$ and $\alpha \wedge \beta = min(\alpha, \beta)$.

IV. The Impact of the Buffering System on The Degree of Self-similarity

Let us view the output traffic $T(n)$ (wireless traffic) as an On/Off process, in the sense that it alternates between non-zero and zero-values. Let X_j^S/Y_j^S denote the On/Off durations of $S(n)$, and X_j^T/Y_j^T, the On/Off durations of $T(n)$. In this section, we will derive the complementary distribution function (CDF) of X_j^T and Y_j^T based on the assumed statistics of $S(n)$. For mathematical tractability we assume that the durations are i.i.d., thus the index j will be dropped.

We will first consider the propagation of self-similarity through a buffering system with constant service rate. For mathematical simplicity in the sequel we only study two extreme cases: the *small* buffer system ($B = c$) and *large* buffer system ($B \gg c$). If $B > c$ but B takes moderate values, the mathematic analysis is rather intractable. However, our simulations indicate that if $B > 5c$ the corresponding buffering system acts more like a *large* buffer system, in which case the analysis shown next still applies.

Then, we will study the gateway by multiplexing the two afore-mentioned buffering systems.

The *small* buffer system: For this system the buffer can hold up to one packet. During all Off periods of $S(n)$, it holds that $Q(n) = Q(n-1) < c$ and so $T(n) = 0$, i.e., $T(n)$ is in Off state. During the On periods of $S(n)$, as new bits come into the buffer, the buffer content is updated according to (3), and thus $T(n)$ changes according to (2). The buffer system does not keep one-to-one mapping between X^S and X^T, Y^S and Y^T.

Let us make the assumptions: (A1) The minimum rate during an On period of the incoming traffic, i.e., K_A, satisfies $K_A << c$. It can be shown that this assumption can be satisfied by a light traffic load [15]. (A2) We ignore the previous buffer content, $Q(n-1)$ when calculating $T(n)$ by eq. (2). This approximation is made mainly for mathematical convenience; in the simulations section of [14], we have provided simulation results to confirm its validity.

Proposition 1: For the small buffer system with assumption (A1) (A2), the tail exponent of X^T is the same as that of X^S. The survival function of Y^T is asymptotically power-law with tail exponent $\min\{\alpha_1, \alpha_0\}$.

Proof: The proof concerning X^T was given in [14]. An expression for the CDF of Y^T was also given in [14]. Here, using properties of Pereto distributions [1], we can show that for the CDF expression it holds: $P(Y^T > y) \overset{y \to \infty}{\sim} y^{-\min\{\alpha_1, \alpha_0\}}$ (see also [15]).

The *large* buffer system: In this case the buffer can hold more than one packet. Now, since $B \gg c$, the previous buffer content $Q(n-1)$ can take very large values and thus cannot be ignored. In this case, $T(n)$ can be in the On state even when the rate of the incoming On state is $A < c$.

The CDFs of X^T and Y^T can be calculated by applying the total probability theorem as follows:

$$\begin{aligned} P(X^T > x) &= P(X^T > x | A \geq c) P(A \geq c) \\ &\quad + P(X^T > x | A < c) P(A < c) \end{aligned} \tag{4}$$

$$\begin{aligned} P(Y^T > y) &= P(Y^T > y | A \geq c) P(A \geq c) \\ &\quad + P(Y^T > y | A < c) P(A < c) \end{aligned} \tag{5}$$

Let S_j denote the value of $S(n)$ at the so-called regenerative points [2]. Those points correspond to the onset of the j-th On period, i.e., $S_j = \sum_{i=1}^{j-1} X_i^S + Y_i^S$.

In addition to assumption (A1), we will also assume that: (A3) The queue is stable, which implies that $c > E[S(n)] = \frac{\mu_A \mu_1}{\mu_1 + \mu_0}$ where μ_A, μ_1, μ_0 are the means of On state rate, On/Off durations respectively. Under (A3), the stationary distribution of $Q(S_j)$ is defined as: $Q_e \overset{d}{=} \lim_{n \to \infty} Q(S_j)$ where $\overset{d}{=}$ represents equality in distribution. (A4) For a real queue, it always holds that $L > c$, otherwise the queue would not be filled and the system would work in an inefficient way.

Proposition 2: For the large buffer system with assumption (A1)(A3)(A4), Y^T has nearly the same tail index as Y^S, while X^T is asymptotically power-low decaying with tail exponent $(\alpha_1 + 1)$, where α_1 is the tail exponent of X^S.

Proof: See Appendix A.

A. The buffering system serving the wireless channel

We here approximate the gateway's action as statistical multiplexing of two buffering systems that have the same buffer of size B, but serve at two different rates: c_g and c_b.

The CDFs of X^T and Y^T can be calculated via the total probability theorem as,

$$\begin{aligned} P(X^T > x) &= P(X^T > x \mid c = c_g) P(c = c_g) \\ &\quad + P(X^T > x \mid c = c_b) P(c = c_b) \end{aligned} \tag{6}$$

$$\begin{aligned} P(Y^T > y) &= P(Y^T > y \mid c = c_g) P(c = c_g) \\ &\quad + P(Y^T > y \mid c = c_b) P(c = c_b) \end{aligned} \tag{7}$$

If $B = c$ we have a small buffer system, while if $B \gg c$ we have a large buffer system; here c can be c_g, or c_b. So the conditional CDFs (e.g. $P(X^T > x \mid c = c_g)$) can be calculated accordingly.

For the incoming traffic we assumed that $1 < \alpha_1 < 2$ and $1 < \alpha_0 < 2$. Then, based on the previous analysis, it holds:

$$\begin{aligned} P(X^T > x) &= (\frac{K_1}{x})^{\alpha_1} P(c = c_g) + C_2 x^{-(\alpha_1 + 1)} P(c = c_b) \\ &\overset{x \to \infty}{\sim} \Big[K_1^{\alpha_1} P(c = c_g) \Big] x^{-\alpha_1} \end{aligned} \tag{8}$$

where $P(c = c_g) = \frac{\gamma}{\beta + \gamma}$, $P(c = c_b) = \frac{\beta}{\beta + \gamma}$ and $1/\beta$, $1/\gamma$ are mean durations of good and bad states. Thus, the On states will be heavy-tailed with index $\alpha_1 < 2$.

Similarly, Y^T will also be heavy-tailed, i.e.,

$$\begin{aligned} P(Y^T > y) &\sim y^{-\min\{\alpha_1, \alpha_0\}} P(c = c_g) + y^{-\alpha_0} P(c = c_b) \\ &\overset{y \to \infty}{\sim} y^{-\min\{\alpha_1, \alpha_0\}} \end{aligned} \tag{9}$$

Since the tail indices of both On and Off durations are in $(1, 2)$, the Hurst parameter of $T(n)$ is in $(0.5, 1)$, thus $T(n)$ is self-similar.

Remark: The above results were based on the assumptions that the tail indices of the incoming On and Off durations were both less than 2. This is the behavior that we saw in our experimental study in [13] that involved LAN traffic. However, for an On/Off process to be self-similar it suffices that either the On or the Off durations, are heavy-tailed [10], [3]. In [6], variable-bit-rate (VBR) video traffic was modeled as an On/Off process with heavy-tail On state durations and Off state durations with finite variance. It can be shown [15] that Proposition 2 still holds in this case. For an alternating buffering system with $c_g > c_b$ and $B >> c_g$, the gateway is always under large buffer model and thus the tail index of the On states of $T(n)$ will become greater than 2, and the Off states will maintain their finite variance. As a result, the Hurst parameter will no longer be in the $(0.5, 1)$ range, therefore the outgoing traffic will not be self-similar.

V. Simulation Results and Analysis

We here provide simulation results to validate the claim that the gateway can change the statistics of the On /Off periods of the rate-limited EAFRP.

We first generated incoming traffic $S(n)$ as follows. We took $X^S \sim \bar{F}(x; \alpha_1 = 1.6, K_1 = 1)$ and $Y^S \sim \bar{F}(x; \alpha_0 = 1.4, K_0 = 1)$, $A \sim \bar{F}_{10^{4.64}}(x; \alpha = 1.19, K = 48)$. The time unit τ was taken to be $\tau = 0.001 sec$. The wireless channel was taken to alternate between the two states c_g and c_b. The channel states durations were taken to be independently exponentially distributed with mean $1/\beta = 0.1 sec$ for the good state, and $1/\gamma = 0.0333 sec$ for the bad state. We considered packet size $P = 1,270$ (bits/ time slot) with $c = 1,200$ information bits inside, which corresponds to a overall rate or 1.27Mbps. In the sequel we only consider information bits rather than total bits. The packet is fragmented into 10 blocks each of which has 127 bits and is coded by the BCH(k, n) code, where k, n are the sizes of code word and payload respectively. By setting the BER to 10^{-6} for the good channel state, and 0.01 for the bad channel state, and following the steps of [5] we get $c_g = 1,200$ (information bits/time slot) and $c_b = 290$ (information bits/time slot).

We consider a buffer system with buffer size $B = 1,200$ bits. During good channel states the buffering system satisfies $(B = 1,200) = (c = 1,200)$, which corresponds to a *small* buffer system. During the bad channel states the system satisfies $(B = 1,200) \gg (c = 290)$, which corresponds to a *large* buffer system.

First we pass $S(n)$ through the gateway serving at a rate of 290 (large buffer model). According to Proposition 2, the tail exponent of X^T should be $\alpha_1 + 1 = 2.6$, while that of Y^T does not change. Figure 1 shows the log-log complementary distribution (LLCD) of On /Off durations of $T(n)$ (solid line) and $S(n)$ (dashed line). The tail index of each graph is also shown in the figure, confirming our expectations.

Next, we pass $S(n)$ through the buffering system that serves alternatively at rates $1,200$ and 290. The LLCDs of X^T/Y^T (solid line), and X^S/Y^S (dashed line) are plotted in Fig. 2. The tail index of X^T and Y^T were estimated to be $\alpha_{X_T} = \alpha_1 = 1.6$, and $\alpha_{Y_T} = \alpha_1 = 1.4$, respectively, which match the analysis. The Hurst parameter of $T(n)$ is $H = 0.8$, which implies self-similarity. This can also be confirmed by looking at the normalized variance-time plot [7] of $T(n)$ in Fig. 3; its slope of $2H - 2 = -0.4$ indicates that the output traffic is self-similar.

To verify the Remark of Section IV-A, we did the following simulations. We regenerate the input traffic, denoted by $S_1(n)$, as before but with $\alpha_0 = 2.2$ (i.e. X^S are heavy-tailed and Y^S have finite variance). We set $B = 6000$, $c_g = 1,200$, $c_b = 1,100$, i.e., gateway always operates under large buffer model. The normalized variance-time plots of $T_1(n), S_1(n)$ are given in Fig. 4. The slope of $T_1(n)$ is -1, or equivalently, $H = 0.5$, indicating that $T_1(n)$ is not self-similar.

VI. Conclusion

The proposed model can help us understand and study the effect of the gateway that feeds wireline traffic into the wireless network. The analysis presented here suggests that under certain conditions the self-similarity can be preserved through the gateway, while self-similarity will disappear in the case that if the On durations of input traffic are heavy-tailed while Off have finite variance and the gateway operated always under the large buffer model.

VII. Appendix

Since the event "$A < c$" is dominant over "$A \geq c$", and A can only take finite value, the event "$Q_e < c$" occurs with higher probability compared to that of "$Q_e \geq c$". During the Off periods of $S(n)$, no new bits comes into buffer, and the previous buffer content (i.e. Q_e) is less than c. Thus, no packet leaves buffer during these periods, i.e. $P(Y^T > y) \approx P(Y^S > y)$. It also holds: $P(X^T > x) \approx P(X^T > x \mid A < c, Q_e < c)$.

For a stable queue, the distribution of X^T depends on the distribution of buffer content at the regeneration points, i.e. Q_e. With the conditions that $A < c$ and $Q_e < c$, the event $X^T > x$ is equivalent to the union of the following three conditions: (B1) There might be a period of time with length l $(l = 0, 1, 2, ...)$, when $T(n) = 0$ even if $S(n) > 0$. (B2) The sum of the previous buffer content Q_e and the accumulating amount of incoming bits during the mean time is enough to form at least x packets. (B3) The corresponding On period of $S(n)$, X^S, should be larger than $x + l$.

The following observations will also be used in derivations: (C1) The buffer content Q_e can only take non-negative value. (C2) According to $P(Q_e < c) \approx 1$, we have $\min(c, L) = c$. (C3) According to $P(A < c) \approx 1$, i.e. $K_A \ll c$, we have $K_A < \frac{x-1}{x}c$, i.e. $\max(\frac{x-1}{x}c, K_A) = \frac{x-1}{x}c$. (C4) For the asymptotic complementary distribution function of Q_e it holds [13]$P[Q_e \geq b] \sim C_{Q_e} b^{(1-\alpha_1)}$, where C_{Q_e} is a constant. (C5) For the Pareto distributed On durations of $S(n)$ it holds: $P(X^S > x + l) = (\frac{K_1}{x+l})^{\alpha_1}$. (C6) We find that $0 \leq l \leq \frac{x}{x-1} < 2$, or simply $l = 0, 1$.

By combining (B1)-(B3) and (C1)-(C6), we get:

$$\begin{aligned} & P(X^T > x \mid A < c, Q_e < c) \\ \overset{x \to \infty}{\sim} & \sum_{l=0}^{1} \int_{\frac{x-1}{x}c}^{c} \Big\{ C_{Q_e}(c - la)^{(1-\alpha_1)} \\ & - C_{Q_e}[xc - (x + l)a]^{(1-\alpha_1)} \Big\} (\frac{K_1}{x + l})^{\alpha_1} f_A(a) da \end{aligned} \tag{10}$$

where $f_A(a)$ is the pdf of On state rates of $S(n)$ and it is assumed to be cut-off Pareto distribution with CDF, $\bar{F}_L(x; \alpha_A, K_A)$ $(i = 1, 2, ...)$. From (10), proposition 2 is proved.

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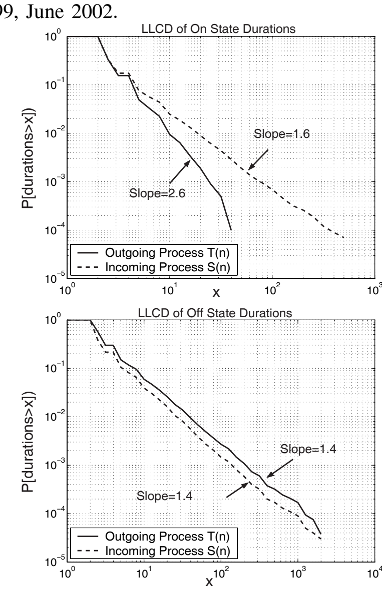

Fig. 1. The LLCD of On and Off durations of $S(n)$ (dashed line) and $T(n)$ (solid line) for the *large* buffer system ($B \gg c_b$).

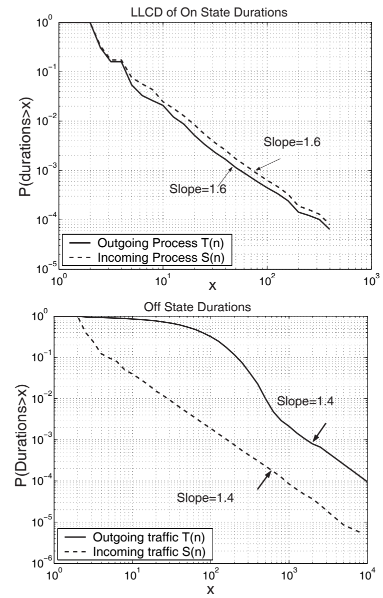

Fig. 2. The LLCDs of On periods and Off periods of $S(n)$ (dashed line) and $T(n)$ (solid line) for the *alternating* gateway.

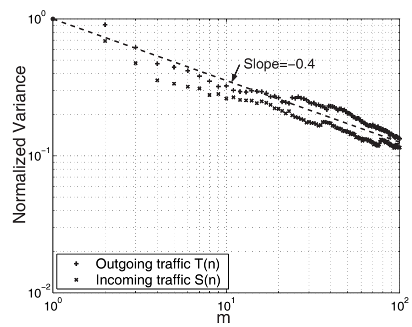

Fig. 3. Normalized variance-time plot and the gateway alternates between small and large buffer case.

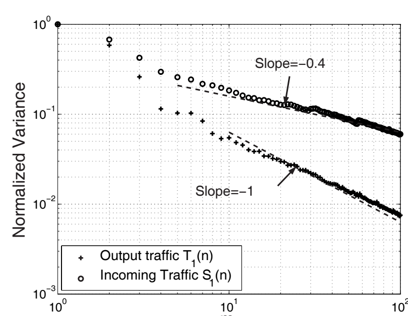

Fig. 4. Normalized variance-time plots when the gateway is always under large buffer case.