DETECTION OF A REFLECTIVE LAYER IN A RANDOM LAYERED MEDIUM USING TIME REVERSAL

*Oleg V. Poliannikov, Jean-Pierre Fouque**

Department of Mathematics
North Carolina State University
Raleigh, NC 27695-8205
E-mail: Oleg.Poliannikov@ncsu.edu, fouque@math.ncsu.edu

ABSTRACT

We propose a method of detecting a reflective layer in a layered random medium using the time reversal of an acoustic wave. Comparing the refocused signal with the original probing pulse enables us to detect a change it the medium's acoustic properties. The depth of the reflector is pinpointed even in the absence of a direct coherent reflection. We present experimental results validating the proposed algorithm.

1. INTRODUCTION

In this paper, we consider the problem of detecting of a reflective obstacle in a random medium. Such a task often arises in the context of searching for a land mine hidden in the earth, or a foreign object in the human body. Throughout this paper, we limit ourselves to the *layered case*, where the medium changes its properties only in one direction. The reflector is then a layer inside the medium whose acoustic properties are different from those of the background (Fig. 1). The stratifiedness of the medium is often a reasonable assumption, at least locally.

We now "probe" the medium with an acoustic pulse and listen to *reflections* generated by the medium. Our goal is to use the latter to decide if there is an object inside the medium, and further estimate the depth at which it is located. As will be elaborated later on, we are interested in the regime where a recorded signal contains no visible first arrival of the wave reflected from the object. Direct analysis of the reflected wave for our purposes therefore appears problematic.

At the heart of our approach lies the general phenomenon of time reversal. The essence of our experiment, referred to as *time reversal in reflection*, can be summarized as follows. As a source emits a pulse, and the wave propagates through the random medium, it generates reflections, which are in turn recorded at the location of the source. A part of the *recorded coda* is then resent into the medium in the reversed direction of time (last in - first out). As before, this new wave gives rise to reflections that are again recorded at the source. A striking result of the theory of time reversal (see [1]) is that *in certain regimes*, the signal recorded the second time (as opposed to the previous recording) will contain a *refocused coherent part* which is related to the original pulse (Fig. 2). The refocused signal is also a function of the medium, the structure of which is of our primary interest.

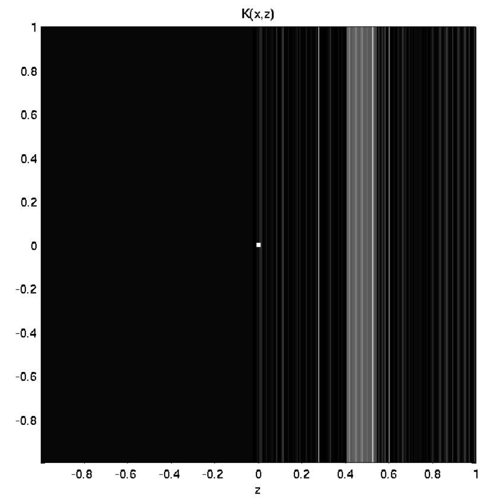
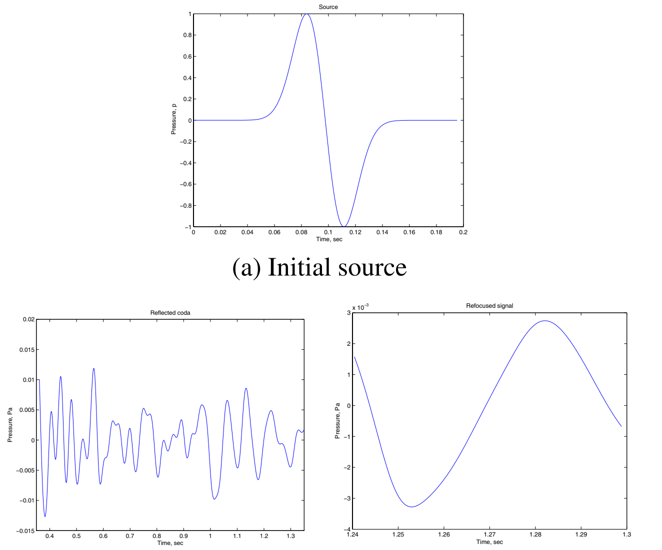


Fig. 1. The layered medium together with the embedded reflector and the location of a source (bright point).

In the spirit of [3], we show that wisely choosing parts of the coda to be resent enables us to aim at different depths in the medium. By introducing *refocusing kernels*, we then compare the properties of those regions, and ultimately design a detection tool that not only helps determine the presence of an object but also ascertains its location.

*This work was supported by ONR grant N00014-02-1-0089



(a) Initial source

(b) Reflected coda (c) Coherent part of 2nd reflection

Fig. 2. Time reversal experiment.

2. PROBLEM SETUP

2.1. Wave equations and the source term

Consider a $3D$ layered half-space with the acoustic properties $\rho(x,y,z) = \rho(z)$ and $K(x,y,z) = K(z)$ where ρ and K are the medium *density* and *bulk modulus* respectively. Assume further that a source located at the boundary of the medium described above emits a wave into the medium. For simplicity of discussion, we will consider a completely $1D$ case. All arguments below, however, can be generalized to a $2D$ or full $3D$ layered medium (see [2]). The equations for the wave propagation are written as

$$\begin{aligned} &\rho \frac{\partial u}{\partial t} + \frac{\partial p}{\partial z} = \delta(z)\, f\left(\tfrac{t}{\lambda}\right) \\ &\frac{1}{K}\frac{\partial p}{\partial t} + \frac{\partial u}{\partial z} = 0, \end{aligned} \tag{1}$$

where u, p are the velocity and the pressure of the wave, f is a fixed function (we may assume a finite support), and $\lambda \ll 1$ is a small constant. The right-hand side of the first equation corresponds to a *point source* with the wavelength of the order λ emitted at $z = 0$, which is at the boundary of the medium.

2.2. Description of the medium

The acoustic characteristics of the medium, ρ and K, are modeled as being fluctuating around their "average" values. More specifically, we set

$$\begin{aligned} &\left.\begin{aligned} &\rho(z) = \bar{\rho}\left(1 + \mu\left(\tfrac{z}{l}\right)\right) \\ &K^{-1}(z) = \bar{K}^{-1}\left(1 + \nu\left(\tfrac{z}{l}\right)\right) \end{aligned}\right., \ z \geq 0, \\ &\left.\begin{aligned} &\rho(z) = \bar{\rho} \\ &K^{-1}(z) = \bar{K}^{-1} \end{aligned}\right., \ z < 0. \end{aligned} \tag{2}$$

Here, $\bar{\rho}$ and $\bar{K}$ are fixed positive constants, μ, ν are two centered bound random processes, i.e. $|\mu|, |\nu| < C < 1$ and $l \ll 1$. In addition to being centered, the random processes μ and ν should satisfy some *mixing conditions*. For the moment, we may assume that they are simply i.i.d. random variables. A few remarks are in order regarding this setup.

- The *scale parameter* l has a physical meaning of the correlation length of the medium. We will assume that $l \ll \lambda$. Small values of l correspond to media whose acoustic properties vary on a fine scale, or equivalently, media with very thin layers (much thinner than the wavelength of the probing pulse).
- The density and the bulk modulus are chosen so that their averages have the form

$$\langle \rho \rangle = \bar{\rho}, \ \left\langle \frac{1}{K} \right\rangle = \frac{1}{\bar{K}}. \tag{3}$$

  One can use the homogenization theory (see [4]) to show that if $l \ll \lambda$ then a wave propagating through the medium (2) over a distance comparable to λ, "sees" it as being constant with the acoustic parameters $\left(\bar{\rho}, \bar{K}\right)$. The latter is called the *effective medium*.

  The average speed of wave propagation in the layered medium (and the constant speed of propagation in the effective medium) is defined by $\bar{c} = \sqrt{\bar{K}/\bar{\rho}}$.
- Finally, the medium $(z > 0)$ and the "free space" $(z < 0)$ are matched so that their effective parameters are identical. This assumption is for simplicity only, and is not crucial for the phenomena we are about to describe.

3. TIME REVERSAL EXPERIMENT

Suppose now a wave of the form $f\left(\frac{t}{\lambda}\right)$ starts at $z = 0$ and propagates into the medium. Each layer then generates small reflections that are recorded at the surface thus producing $\{A(t) = p(z = 0, t)\}_{t \geq 0}$. Part of this signal recorded from time t_1 till t_2 can then be written as $y(t) = A(t)G(t)$, where $G(t) = \mathbb{I}_{[t_1, t_2]}(t)$. One can show that when $l, \lambda \to 0$, $y(\cdot)$ is completely incoherent ("noise-like").

The time reversed signal is then defined as $f_{\mathrm{TR}}(t) = y(t_2 - t)$. This signal is then sent back to the same medium according to (1) and the new reflections, $y_{\mathrm{TR}}(t) = p(z = 0, t)$, are once again recorded.

The theory of time reversal then states that asymptotically, when $\lambda, l \to 0, \lambda \sim \sqrt{l}$, the function $y_{\mathrm{TR}}(t)$ will have a coherent part around $t = t_2$ (and only there). Furthermore, this coherent part is the convolution of the original source f with some kernel, which is a function of the statistics of the medium and the cutoff parameters t_1, t_2. More precisely,

$$s(\sigma) = \frac{1}{2\pi} \iint \Lambda_{\mathrm{TR}}(\omega, \tau) \overline{\hat{f}(\omega)} e^{-i\omega\sigma} G(\tau)\, d\omega\, d\tau, \quad (4)$$

or

$$s(\sigma) = \left(f(-\cdot) \star K_{\mathrm{TR}}^{t_1,t_2}(\cdot)\right)(\sigma), \quad (5)$$

where $s(\sigma) = y_{\mathrm{TR}}(t_2 + \lambda\sigma)$ and

$$\widehat{K_{\mathrm{TR}}^{t_1,t_2}}(\omega) = \int G(\tau)\, \Lambda_{\mathrm{TR}}(\omega, \tau)\, d\tau. \quad (6)$$

We will refer to $K_{\mathrm{TR}}^{t_1,t_2}$ as a *refocusing kernel*, and Λ_{TR} as the *kernel's density in the Fourier domain*. The equation (5) means that the time reversal experiment leads to the *refocusing* of the original signal at $t = t_2$ at the location of the source. Our goal in the next section is to take the original *and* the refocused signal to recover the refocusing kernel and use the latter to study the medium.

4. DETECTION OF A REFLECTOR USING REFOCUSING KERNELS

In this section, we show that time reversal can be used as means to detect a reflective layer embedded into the medium. We compute numerically approximations to refocusing kernels for a medium with and without a reflector, and show that not only do they contain information about its presence or absence but in the former case also enable us to reliably estimate its location.

4.1. Expanding window

Suppose we have two identical copies of the same medium. We then embed a reflective layer at the depth L in one of them, propagate the same pulse f according to (1) through both, and record the reflections $A_{1,2}(t)$ at the locations of the sources. The hyperbolicity of the equations (1) implies that the two waves will propagate in exactly the same fashion up until the time $T = 2L/\bar{c}$, which is the time it takes a wave front to get deep to the reflector and bounce back to the surface. We thus have

$$\begin{array}{l} A_1(t) = A_2(t),\ 0 \le t \le T, \\ A_1(t) \ne A_2(t),\ t > T. \end{array} \quad (7)$$

Computer simulations will invariably introduce numerical errors due to discretization. Experiments, however, clearly show the exact time when the two codas start deviating. Unfortunately, in a practical setting we will only have access to one coda and not the other. Their direct comparison therefore will be impossible.

Consider a family of expanding cut-off window functions

$$\left\{G^{t_1,\tau}(\cdot) = \mathbb{I}_{[t_1,\tau]}(\cdot)\right\}_{\tau \ge t_1},$$

where $t_1 < T$. For the same initial pulse, this family corresponds to a continuous family of time reversal experiments, which in turn gives rise to a collection of refocused signals:

$$s^{\tau}(\sigma) = \left(f(-\cdot) \star K_{\mathrm{TR}}^{t_1,\tau}(\cdot)\right)(\sigma). \quad (8)$$

It follows from (7) that the refocused signal s^{τ} will contain no reflections from the embedded layer if $\tau \le T$. If, on the other hand, $\tau > T$, then the recorded coda contains information about the reflector introduced through the refocusing kernel $K_{\mathrm{TR}}^{t_1,\tau}$.

4.2. Extracting refocusing kernels

Taking the Fourier transform of (8), we obtain

$$\widehat{K_{\mathrm{TR}}^{t_1,\tau}}(\omega) = \frac{\widehat{s^{\tau}}(\omega)}{\overline{\hat{f}(\omega)}}. \quad (9)$$

Also, since

$$\begin{aligned} K_{\mathrm{TR}}^{t_1,\tau}(\omega) &= \int G(\tau')\Lambda_{\mathrm{TR}}(\omega, \tau')\, d\tau' \\ &= \int_{t_1}^{\tau} \Lambda_{\mathrm{TR}}(\omega, \tau')\, d\tau', \end{aligned}$$

we have

$$\frac{\partial}{\partial \tau} K_{\mathrm{TR}}^{t_1,\tau}(\omega) = \Lambda_{\mathrm{TR}}(\omega, \tau).$$

An initial pulse and a refocused signal together enable one to numerically estimate the corresponding refocusing kernel and its density in the Fourier domain. In the next section, we demonstrate experimentally that given $K_{\mathrm{TR}}^{t_1,\tau}$ (or $\Lambda_{\mathrm{TR}}(\omega, \tau)$) alone, one can detect the presence of a reflective layer and reliably estimate its depth.

5. EXPERIMENTAL RESULTS

We illustrate the ideas described in the previous sections with the following numerical experiment. A $2D$ square of the size $2m \times 2m$ is divided in halves (Fig. 1). The right half contains a layered medium, and the left half is a "free space". The layered part consists 170 layers (each layer is $\sim 0.006m$ thick). The density of the medium is taken as constant, i.e. $\rho \equiv 1$, and the bulk modulus $K = \frac{1}{1+U}$, where U is uniformly distributed on $[-0.9, 0.9]$. The effective speed of propagation is therefore $\bar{c} = 1m/s$. The reflector of width $\sim 0.1m$ is buried into the medium at the depth $\sim 0.5m$ and its bulk modulus is $\sim 20Pa$ (Fig. 3).

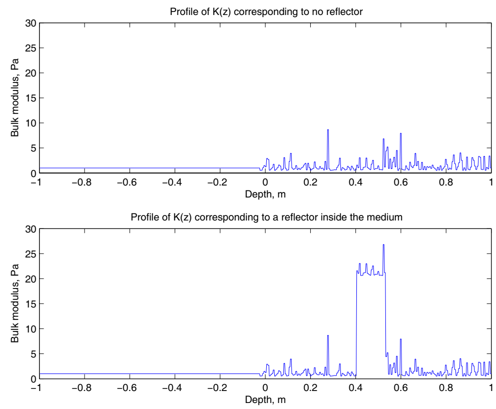


Fig. 3. Profiles of $K(z)$ without and with the embedded reflector.

We use a pulse of the wavelength $\lambda \sim 0.04m$ to penetrate the medium with and without the reflector. The two reflected codas are recorded. One observes (see Fig. 4) that the two codas are incoherent and do not contain any first arrival information. At the same time the two codas are different only when $t > T = 1$, where T is the time it takes for the pulse to reach the reflector and reflect back to the surface.

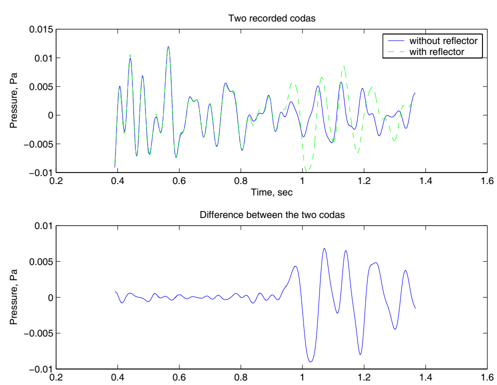


Fig. 4. Plots of $A_{1,2}(t)$ and the difference $A_1(t) - A_2(t)$.

Finally, we perform several expanding window time reversal experiments with each medium and its corresponding coda, record refocused signals, and extract the corresponding refocusing kernels and their densities as described in the previous section. The cross-sections of the resulting surfaces are presented in Fig. 5. We observe that the location of the reflective layer is clearly marked by a change in the slope of the refocusing kernel (or a jump of its density), which is absent in the case where there is no embedded object.

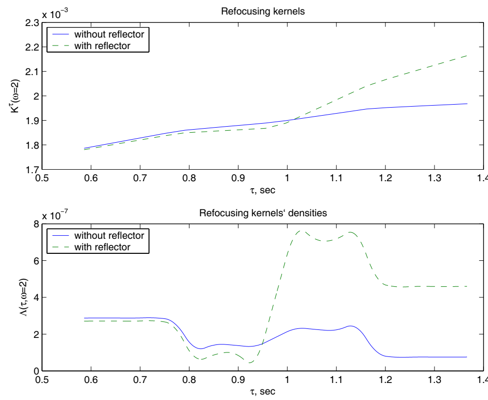


Fig. 5. Plot of $K^{t_1,\tau}(\omega = 2)$ as a function of the window parameter τ.

6. CONCLUSIONS

In this paper, we have presented an algorithm for detecting a highly reflective layer in a random finely layered medium. To do that we have relied on the phenomenon of time reversal of an acoustic wave. We have shown that while the direct reflections generated by a pulse going through the medium contain no first arrival, time reversal allows one to obtain a coherent pulse which is a convolution of the original source with the medium dependent kernel.

Furthermore, by selecting how much of a coda we want to back-propagate into the medium, we can control which depth of the medium affects the kernel. A reflective layer then translates into a sharp change in the derivative of the kernel. The time of that change is directly related to the depth of the reflector.

7. REFERENCES

[1] J.-F. Clouet and J.-P. Fouque, "A time-reversal method for an acoustical pulse propagating in randomly layered media," *Wave Motion*, vol. 25, pp. 361–368, 1997.

[2] J.-P. Fouque, J. Garnier, A. Nachbin, and K. Solna, "Time reversal refocusing for point source in randomly layered media," *submitted*, 2004.

[3] J.-P. Fouque, J. Garnier, A. Nachbin, and K. Solna, "Imaging of a dissipative layer in a random medium using a time-reversal method," *submitted*, 2004.

[4] M. Asch, W. Kohler, G. Papanicolaou, M. Postel, and B. White, "Frequency content of randomly scattered signals," *SIAM Rev.*, vol. 33, pp. 519–625, 1991.