IS SPATIAL SUPER-RESOLUTION FEASIBLE USING OVERLAPPING PROJECTORS?

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ABSTRACT

An interesting issue of contemplation amongst researchers working on multi-projector displays is whether spatial super-resolution can be achieved by overlapping images from multiple projectors. This paper presents a thorough theoretical analysis to answer this question using signal processing and perturbation theory. Our analysis is supported by results from a simulated overlapping projector display. This analysis shows that achieving spatial super-resolution using overlapping projectors is practically infeasible.

Keywords: Fourier transform, signal processing, super resolution images, multi-projector display,

1. INTRODUCTION

Recently overlapping projectors are being used in a number of applications like parallel rendering, creation of depth of focus effects and removal of shadows cast on the screen [1, 2]. A salient issue for such displays is, can we achieve a resolution higher than that of individual projectors when multiple projectors overlap? This problem is the dual of creating super-resolution images using multiple lower resolution camera images [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17], which has been explored with reasonable amount of success. Thus, it is interesting to investigate if this duality leads to comparable results in projectors. This paper makes the first attempt to study this problem in a detailed manner using signal processing theory. This analysis shows that such a super-resolution display is only possible under certain strict conditions. Next, using results from perturbation theory, we find that the probability of the occurrence of these conditions in a practical system is zero. Thus, we infer that super-resolution images from multiple overlapping projectors is practically infeasible.

2. THEORETICAL ANALYSIS

The analysis and illustrations in this section are presented for one dimensional signals (one scan line of the display) for easy comprehension. The results are extended to two-dimensional images using a simulator that generates the image of four overlapping projector seen by a camera. Our simulator uses popular geometric models for projectors and cameras as in [18, 19]. The resolution of the camera is modeled to be sufficiently high to visualize the different effects of interleaving pixels from overlapping projectors.

2.1. Image from a Single Projector

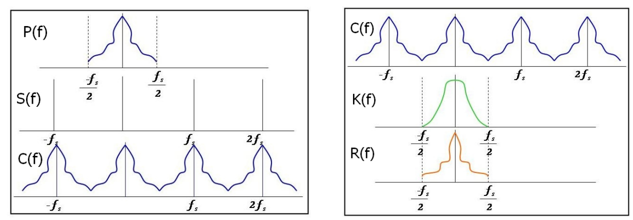

Fig. 1. *In the frequency domain, sampling (left) and reconstruction (right) of a single scan-line for a single projector. Left: The signal $P(f)$ bandlimited by $\frac{f_s}{2}$ is convolved with the sampling signal $S(f)$ to generate the sampled signal $C(f)$. Right: The sampled signal $C(f)$ is multiplied by the reconstruction kernel $K(f)$ of width $\frac{f_s}{2}$ to reconstruct the bandlimited signal $R(f)$.*

We consider a single scan-line of a projector represented by a continuous signal in the spatial domain, whose frequency response is denoted by $P(f)$. Projecting this signal involves two steps: *sampling* and *reconstruction* [20].

Sampling Function: We assume a uniform sampling represented in the spatial domain by a periodic comb function with period T, where T is the pixel width of a projector, i.e. the distance between adjacent pixels. The frequency response $S(f)$ of this function is another periodic comb with period $f_s = \frac{1}{T}$, where f_s is the *sampling frequency*. So,

$$S(f) = \begin{cases} 1 & \text{if } f = \frac{n}{T}; \\ 0 & \text{if } f \neq \frac{n}{T}; \text{where } n \text{ is an integer.} \end{cases} \quad (1)$$

Sampling: During sampling, the signal is first bandlimited by $\frac{f_s}{2}$ in the frequency domain, to avoid aliasing. This assures that $P(f) = 0$, if $|f| > \frac{f_s}{2}$. Next, in the spatial domain, this bandlimited signal is *sampled* (multiplied) by the sampling signal to get the *sampled signal*, $C(f)$. This is equivalent to convolving the two signals in the frequency domain leading to replication of the frequency spectrum of $P(f)$ at the harmonics of the sampling frequency. Thus,

$$C(f) = P(f) \star S(f) = P(f - \lfloor \frac{f + \frac{f_s}{2}}{f_s} \rfloor f_s) \quad (2)$$

Reconstruction: In the spatial domain, reconstruction is achieved by convolving the sampled signal with the point-spread function of a pixel. In the frequency domain, this is equivalent to multiplying the fourier transforms of the sampled signal $C(f)$ and the reconstruction kernel $K(f)$. For $K(f)$ of bandwidth f_p, the reconstructed signal $R(f)$ is

$$R(f) = C(f)K(f) = \begin{cases} C(f) & \text{if } f \leq f_p; \\ 0 & \text{if } f > f_p. \end{cases} \quad (3)$$

If the width of the reconstruction kernel same as the bandwidth of the signal, ($\frac{f_s}{2} = f_p$), proper reconstruction would occur since the original frequency spectrum $P(f)$ will be extracted eliminating the replicas at the harmonics. This whole process is illustrated in Figure 1. However, for a kernel with a larger width ($f_p > \frac{f_s}{2}$), for frequencies f, $\frac{f_s}{2} < f \leq f_p$, the replica at the first harmonic would contribute introducing aliasing (commonly called pixelization). On the other hand, if the kernel has a smaller width ($f_p < f < \frac{f_s}{2}$), the frequencies f, $f_p < f \leq \frac{f_s}{2}$, are lost leading to blurring. This theory is extended to two dimensional images to create these artifacts on our simulator (Figure 5).

2.2. Image from Overlapping Projectors

Next, we investigate projecting a super-resolution scan-line from two overlapping projectors, denoted in the frequency domain by $P_o(f)$. The spatial resolution of $P_o(f)$ is the sum of the resolutions of the component projectors. This doubling in the spatial resolution doubles the bandwidth of the signal to f_s. Thus, $P_o(f) = 0$, if $|f| > f_s$.

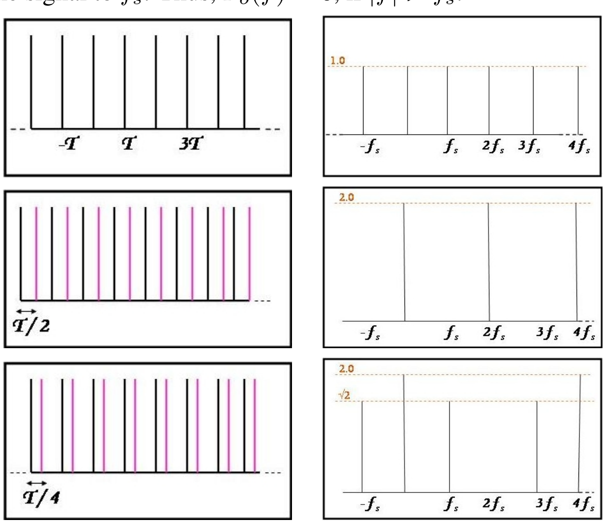

Fig. 2. *Left: In the spatial domain, the sampling function $S(f)$ from a single projector separated by pixel width T (top), the sampling function $S_o(f)$ for interleaving of pixels for a scan-line from two overlapping projectors (one in black and another in pink) shifted by $\frac{T}{2}$ (middle), $\frac{T}{4}$ (bottom). Right: The corresponding frequency response of the sampling functions on the left.*

Sampling Function: In the spatial domain, the sampling signal for two overlapping projectors, $S_o(f)$, is given by the *superposition* (addition) of two periodic comb functions (each from a different projector) shifted by a distance $l \leq T$. The frequency response of the function is

$$S_o(f) = 2S(f)cos^2\pi fl = \begin{cases} 2cos^2\pi fl & \text{if } f = \frac{n}{T}; \\ 0 & \text{if } f \neq \frac{n}{T}, \end{cases} \quad (4)$$

where n is an integer. Figure 2 shows the spatial and the corresponding frequency response of such $S_o(f)$ for different values of l ($l = \frac{T}{2}, \frac{T}{4}$). When $l = \frac{T}{2}$ (half the pixel width), $S_o(f)$ becomes zero at odd n generating a *periodic comb* with double the sampling frequency, i.e.,

$$S_o(f) = \begin{cases} 2 & \text{if } n = 0, 2, 4, 6, \ldots \\ 0 & \text{if } n = 1, 3, 5, 7, \ldots \end{cases} \quad (5)$$

This indicates a uniform but denser sampling in the spatial domain. More importantly, for any other l, $S_o(f)$ is an *aperiodic comb* and is always positive at the first harmonic, f_s.

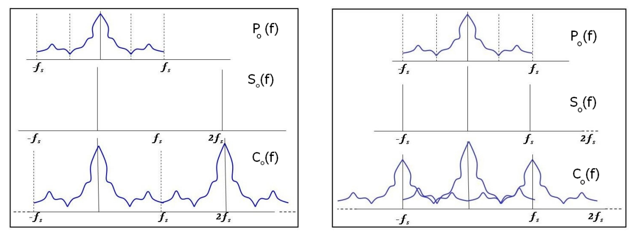

Fig. 3. *In frequency domain, sampling of a super-resolution scan-line $P_o(f)$ by $S_o(f)$ from two overlapping projectors with their pixels interleaved by $\frac{T}{2}$ (left) and $\frac{T}{4}$ (right). In other word, the super resolution signal $P_o(f)$ (top) is convolved sampling signal $S_o(f)$ (middle) to generate sampled signal $C_o(f)$(bottom).*

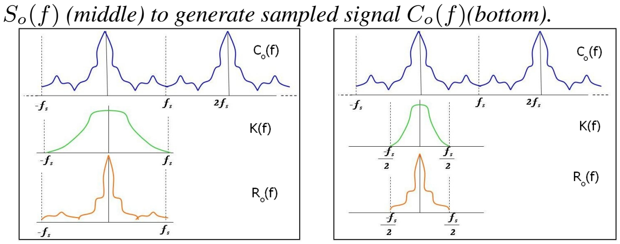

Fig. 4. *In frequency domain, reconstruction using a kernel of support f_s (left) and $\frac{f_s}{2}$ (right) of the super-resolution signal $P_o(f)$ sampled by two overlapping projectors interleaved by a distance $\frac{T}{2}$. For each, the sampled signal $C_o(f)$ (top) is multiplied by the reconstruction kernel $K(f)$ (middle) to generate the reconstructed signal $R_o(f)$ (bottom).*

Sampling of super-resolution signal: When $P_o(f)$ is convolved with $S_o(f)$ to form the sampled signal $C_o(f)$, for $l = \frac{T}{2}$, the frequency spectrum of $P_o(f)$ is amplified and replicated at even multiples of f_s with no contamination from a replica of $P_o(f)$ at the adjacent harmonic (Figure 3).

$$C_o(f) = P_o(f) \star S_o(f) = P_o(f - \lfloor \frac{f + f_s}{2f_s} \rfloor 2f_s) \quad (6)$$

But, for the more general scenario of $l \neq \frac{T}{2}$, the first harmonic f_s being positive, the spectrum of $P_o(f)$ at 0 is contaminated by the adjacent replica at f_s (Figure 3). Thus,

$$C_o(f) = 2P_o(f - \lfloor \frac{f}{f_s} \rfloor f_s)cos^2\pi \lfloor \frac{f}{f_s} \rfloor l + 2P_o(f - \lceil \frac{f}{f_s} \rceil f_s)cos^2\pi \lceil \frac{f}{f_s} \rceil l. \quad (7)$$

Reconstruction of super-resolution signal: Hence, when reconstructing signal $R_o(f)$ from $C_o(f)$ using a reconstruction kernel $K(f)$ of width $f_p = f_s$ (double the width of the pixel spread function for single projector), when $l = \frac{T}{2}$, the original *super-resolution signal* $P_o(f)$ can be reproduced with no contamination (Figure 4). This doubling of the kernel width in the frequency domain indicates sharper pixels in the spatial domain. Thus, *if we can interleave the pixels from two projectors at exactly* $\frac{T}{2}$ *and reduce the width of the point spread functions of each projector by exactly half, we can achieve super resolution.*

However, reconstruction with $K(f)$ of width $f_p = \frac{f_s}{2}$ generates just the regular signal $P(f)$ bandlimited by $\frac{f_s}{2}$. Thus, even if we start with a super-resolution signal, we can obtain lower resolution signal (like in a single projector) if we do not use sharper pixels.

For the more general case of $l \neq \frac{T}{2}$, the $C_o(f)$ generated (Figure 3) poses the well-known scenario of aliasing, irrespective of being reconstructed with $K(f)$ of width f_s or $\frac{f_s}{2}$. A pertinent question here is, can these aliasing artifacts be made imperceptible by reducing the high frequencies to noise using stochastic sampling methods [21], especially by overlapping off-axis projectors when the projector sampling functions become aperiodic? The analysis of this situation shows that due to the underlying uniform sampling of the projector imaging device, the aperiodicity is purely due to projection and hence well defined [18]. [22] shows that the frequency response of an aperiodic comb is another aperiodic comb, and nothing close to the white noise achieved by stochastic sampling. Hence, the aliasing artifacts generated by these configurations cannot be eliminated by off-axis projection. The above theory is extended to two dimensional images to generate results for a four-projector system on our simulator (Figure 5)).

Sampling and reconstruction of regular resolution signal: However, if $P_o(f)$ is not of super-resolution, i.e., $P_o(f) = P(f)$, the replicas of $P(f)$ at the harmonics of f_s will not be contaminated by the adjacent replicas in $C_o(f)$, when using overlapping projectors. Thus, there will be no artifacts when reconstructing with $K(f)$ of width $f_p = \frac{f_s}{2}$. Hence, we do not observe artifacts when we overlap projectors casually across their boundaries in multi-projector displays. The only difference of this process from the case of a single projector in the amplification of the reconstructed signal manifested as higher brightness in the overlap region.

In summary, the results can be extended to two dimensional images from n overlapping projectors. To achieve super-resolution, the *width* and *alignment* of the the pixels from the n projectors should be manipulated such that (1) the sum of the pixel widths of all overlapping projectors is equal to T, and (2) the projectors are aligned in such a fashion that the modified narrower pixels from one projector does not overlap with that of another.

2.3. Practical Feasibility

The next question is, 'What is the probability that a physical system of multiple overlapping projectors will satisfy the above conditions?' For this, we consider two interleaved pixels from projector 1 (in white) and projector 2 (in black) in Figure 6, denoted by the quadrilaterals $abcd$ and $efgh$ respectively. To achieve super-resolution, the right edge, bc, of the white pixel from projector 1 should coincide with the left edge, eh, of the black pixel from the projector 2. So, bc and eh, which are both lines, must intersect at a line.

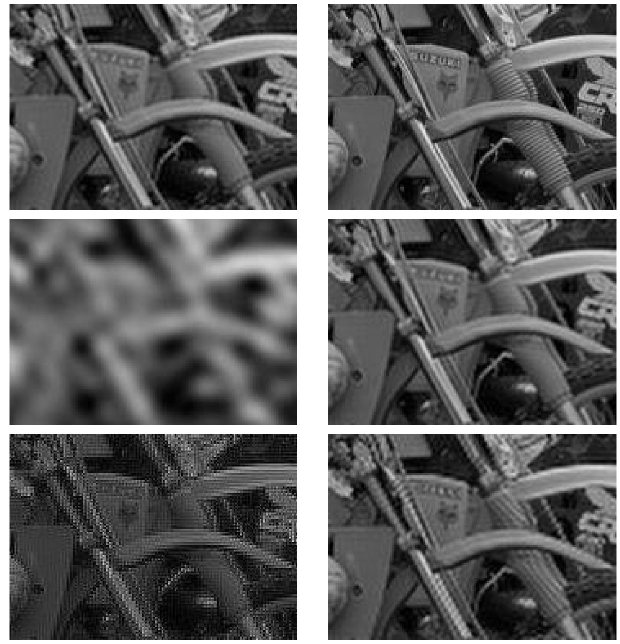

Fig. 5. *Results from the simulator when the theory is extended to images. Left: The true resolution image $P(f)$ from a single projector reconstructed using kernel of bandwidth $\frac{f_s}{2}$ (top), $0.1 \times \frac{f_s}{2}$ leading to blurring (middle), and $1.8 \times \frac{f_s}{2}$ leading to pixelization (bottom). Right: The super-resolution image $P_o(f)$ from four overlapping projectors with pixels interleaved at $\frac{T}{2}$ and reconstructed with kernel of width f_s (top). Compare this with the regular resolution image (top left) to verify that super resolution is achieved. This is sharper than the image reconstructed using a kernel of support $\frac{f_s}{2}$ (middle). When the pixels are not interleaved by $\frac{T}{2}$, aliasing artifacts are generated (bottom). Note the difference in the embedded text and the spirally engraved rod of the bike at the back in the images.*

[23] shows that two geometric entities of dimensions d_1 and d_2 respectively embedded in a space of dimension d_e can intersect in a non-degenerate manner at a geometric entity of dimension $d_1+d_2-d_e$. For a multi-projector display, $d_e = 2$. Since, bc and eh are both lines, i.e., $d_1 = d_2 = 1$, so they can meet in a non-degenerate manner at a geometric entity of dimension $d_1 + d_2 - d_e = 0$, i.e, a point. Thus, intersection of two lines in 2D resulting in another line, which is a necessary condition for super-resolution images, is a degenerate case. [23] shows that the probability of occurrence of a degenerate case is zero, and can be disturbed by arbitrarily small perturbation of system parameters, like projector position, orientation and lens parameters.

Intuitively, this result is easy to visualize. The first condition of super-resolution demands interleaving the pixels from projectors in a precise manner which is impossible practically. The second condition requires pixels of each projector to be manyfold sharper, i.e., image from the projector will look pixelated when used individually, but can generate super-resolution images when used in multi-projector configuration. No current projectors that we are aware of can achieve the required sharpness in practice.

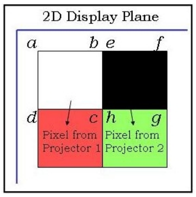

Fig. 6. *The alignment of pixels from four overlapping projectors (represented by white, black, red and green) in order to achieve super-resolution images.*

Thus, we find that though super-resolution images can be generated using multiple low resolution cameras images [8], the same is not true for projectors even though it is the dual of a camera geometrically [18]. This is because the methods for generating super-resolution with cameras use complex mathematics like least square or quadratic minimization [9, 4, 5, 12], geometric or photometric image processing and optimizations [10, 11, 13, 17], and analytical probabilistic methods [16, 14, 15]. On the contrary, achieving such complex mathematics from the projector would involve (a) sub-pixel accuracy geometric calibration of the overlapping projectors, (b) complex pre-processing of images to be projected, and (c) controlling the width of different pixels of the projector differently. Even if we assume that (a) and (b) are possible theoretically, (c) is an impossible engineering feat to achieve, especially when the only mathematical operation available from overlapping projectors is addition via superposition of light.

3. CONCLUSION

In conclusion, in this paper we have showed that generating super-resolution images using overlapping projectors is practically infeasible.

4. REFERENCES

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