METHOD OF IMAGE ENHANCEMENT BY SPLITTING-SIGNALS

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ABSTRACT

In this paper the method of tensor representation of an image with respect to the Fourier transform and its application for image enhancement is described. The method is based on the fact that the two-dimensional (2-D) image can be represented by a set of 1-D signals that split the 2-D Fourier transform of the image into different groups of frequencies. Each splitting-signal carries information of the spectrum in a specific group. The processing of the image is reduced to processing splitting-signals. The effectiveness of such approach is illustrated through the processing image by α-rooting method of enhancement. We propose to enhance image by processing only one or a few splitting-signals, to achieve image enhancement which in many cases can exceed the enhancement by α-rooting method. The selection of such splitting-signals is described.

1. INTRODUCTION

The traditional transform-based methods of image processing [1, 2] are based on calculation of an 2-D unitary transform $\mathcal{F}$, for instance the discrete Fourier transform (DFT), of the image, transformation of all, or part of spectral components of the transform, and then calculation of the inverse transform $\mathcal{F}^{-1}$. The 2-D $N \times N$-point DFT can be split by different subsets of frequencies by separate 1-D DFTs, and the problem of 2-D image processing in the frequency domain can thus be reduced to processing separately spectral components at these subsets. We here consider the splitting developed by Grigoryan, which is called the tensor representation, when the 2-D DFT is calculated by minimum number of 1-D N-point DFTs [3, 4]. A modification of the tensor representation, which is called the paired representation and reduces the 2-D DFT to a minimal number of short 1-D DFTs [3], can be considered in a similar way.

In this paper a representation of an image in the form of the certain totality of 1-D "independent" splitting-signals is discussed for performing operations over images such as the 2-D linear filtration in methods of transform-based image enhancement. Rather than process the image by traditional methods of the Fourier transform, we will process separately splitting-signals and then calculate and compose the 2-D DFT of the processed image, by new splitting-signals. Figure 1 shows a diagram of processing an image $\{f_{n,m}\}$ of size $N \times N$ by the Fourier transform. One of the selected splitting-signals $\{f_{p,s,t};\ t = 0 : (N-1)\}$ is calculated and processed by the method of 1-D α-rooting. The 1-D DFT

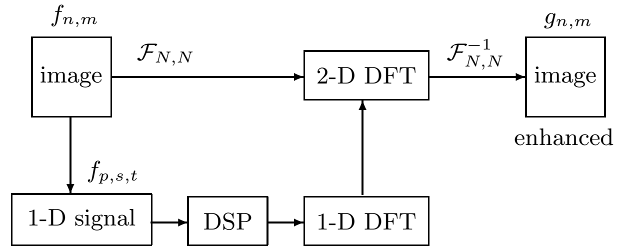


Fig. 1. Image processing by one splitting-signal.

of this signal is then calculated and recorded at the corresponding set of frequencies of the 2-D DFT of the image. The processed image $\{g_{n,m}\}$ is calculated by the 2-D inverse DFT. To estimate the quality of processed images, we consider the quantitative measure EME of image enhancement that relates to concepts of the Weber's and Fechner's laws of the human visual system. The detailed description of this measure and methods of finding optimal parameters of image enhancement can be found in [5]. Experimental results with different types of images, including aerial and medical images, show that a high quality enhancement can be achieved by processing only one or a few splitting-signals, and many arithmetic operations can be saved.

2. SPLITTING-SIGNALS

The $N \times N$-point DFT of an image f, accurate to the normalizing factor $1/N$, is defined by

$$F_{p,s} = (\mathcal{F}_{N,N} \circ f)_{p,s} = \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} f_{n,m} W^{np+ms} \qquad (1)$$

where $W = \exp(-2\pi j/N)$. Frequency-points (p, s) are from the set $X = \{(p, s);\ p, s = 0 : (N-1)\}$. The designation $p = 0 : (N-1)$ denotes p as an integer that runs from 0 to $(N-1)$. Set X can be covered by a family of subsets $\sigma = (T_k)_{k=1:l}$ to be defined, in a way that the 2-D Fourier transform of image f at subset T_k becomes an image of the 1-D N-point Fourier transform, $\mathcal{F}_N$, of an 1-D signal, $f^{(k)}$. In this case we say that the covering σ reveals the 2-D DFT, which means that:

(a) 1-D transforms $\mathcal{F}_N$ of $f^{(k)}$ compose a splitting of the 2-D DFT

$$\mathcal{F}[f] \leftrightarrow \left\{\mathcal{F}_N[f^{(1)}], \mathcal{F}_N[f^{(2)}], \ldots, \mathcal{F}_N[f^{(l)}]\right\}. \qquad (2)$$

(b) The set of splitting-signals $f^{(k)}$, $k = 1 : l$, define completely the image, $\{f^{(1)}, f^{(2)}, \ldots, f^{(l)}\} \leftrightarrow f$. Number l of splitting-signals equals $N + 1$, when N is a prime, and $3N/2$, when N is a power of two.

If σ is a covering revealing the 2-D DFT, then the algorithm of the transform $\mathcal{F}_{N,N} \circ f$ can be described by the following steps.

Step 1: Calculate splitting-signals f_T , $T \in \sigma$.

Step 2: Calculate 1-D transforms $\mathcal{F}_N \circ f_T$.

Step 3: Fill the 2-D DFT at frequency-points of subsets T, by using the calculated 1-D DFTs.

We are interested in a such set of splitting-signals that provides an effective performance of image enhancement through processing splitting-signals.

3. IMAGE TENSOR REPRESENTATION

The concept of tensor representation of the image relates to the covering $\sigma = (T)$ defined by the following cyclic groups with generators (p, s)

$$\begin{gathered} T_{p,s} = \big\{(0,0), (p,s), (\overline{2p}, \overline{2s}), \ldots, (\overline{kp}, \overline{ks})\big\} \\ k = card\, T - 1, \quad T_{0,0} = \{(0,0)\} \end{gathered} \tag{3}$$

where we denote by $\overline{p}$ the number $p \bmod N$. Points of $T_{p,s}$ lie on parallel lines at an angle of $\theta = \tan^{-1}(p/s)$ to the horizontal axis. The irreducible covering σ of set X, which is composed by groups $T_{p,s}$ is unique. For instance, the covering of the 3×3 set X is defined by $\sigma = (T_{1,1}, T_{0,1}, T_{2,1}, T_{1,0})$.

The following property holds for the 2-D DFT [3]

$$F_{\overline{kp},\overline{ks}} = \sum_{t=0}^{N-1} f_{p,s,t} W^{kt}, \quad k = 0\colon (N-1), \tag{4}$$

where

$$f_{p,s,t} = \sum_{V_{p,s,t}} f_{n,m}\,, \quad t = 0\colon (N-1). \tag{5}$$

Sets $V_{p,s,t}$ are defined by $\{(n,m);\ np + ms = t \bmod N\}$.

$V_{p,s,t}$, if it is not empty, is the set of points (n,m) along a maximum of $p + s$ parallel straight lines defined by the following equations

$$\left.\begin{array}{rcl} xp + ys & = & t \\ xp + ys & = & t + N \\ \cdots & . & \cdots \\ xp + ys & = & t + (p + s - 1)N. \end{array}\right\} \tag{6}$$

In the bounded domain $[0, N] \times [0, N]$, these parallel lines lie at angle $\psi = \tan^{-1}(s/p)$ to the horizontal axis.

Covering $\sigma = (T_{p,s})$ reveals the 2-D DFT, and the splitting-signals (or image-signals) are defined by

$$f_T = f_{T_{p,s}} = \{f_{p,s,0}, f_{p,s,1}, \ldots, f_{p,s,N-1}\}. \tag{7}$$

It means that for each set $T \in \sigma$

$$(\mathcal{F}_{N,N} \circ f)_{|T} = \mathcal{F}_N \circ f_T\,. \tag{8}$$

We use the notation $_{|T}$ for the restriction of data on T. The 2-D DFT is split into a set of the 1-D transformations $\{\mathcal{F}_N, \mathcal{F}_N, \ldots, \mathcal{F}_N\}$. The totality $\{f_T;\ T \in \sigma\}$ is called a tensor, or vectorial-representation of f with respect to the DFT, and the transformation $\chi : f \to \{f_T;\ T \in \sigma\}$ is called a tensor transformation.

Splitting-signal $f_{T_{p,s}}$ determines the 2-D DFT at frequency-points of the group $T_{p,s}$, and the following one-to-one correspondence holds

$$f_{T_{p,s}} \leftrightarrow \{F_{0,0}, F_{p,s}, F_{\overline{2p},\overline{2s}}, \ldots, F_{\overline{(N-1)p},\overline{(N-1)s}}\}.$$

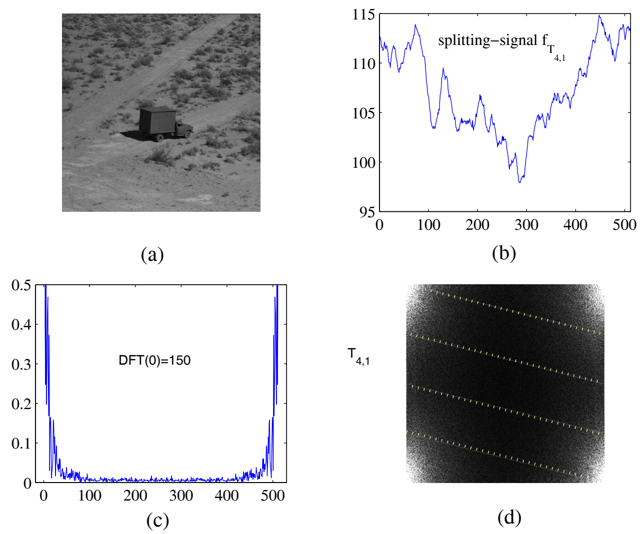


Fig. 2. (a) Truck image. (b) Splitting-signal $f_{T_{4,1}}$. (c) 1-D DFT (in absolute scale) of the splitting-signal. (d) Arrangement of values of the 1-D DFT in the 2-D DFT of the image at frequency-points of set $T_{4,1}$.

As an example, Figure 2 illustrates the truck image of size 512 × 512 in part a, along with splitting-signal $f_{T_{4,1}}$ of length 512 in (b), the 1-D DFT over this splitting-signal in (c), and frequency-points of group $T_{4,1}$ at which the 2-D DFT of the image is filled by the 1-D DFT in (d).

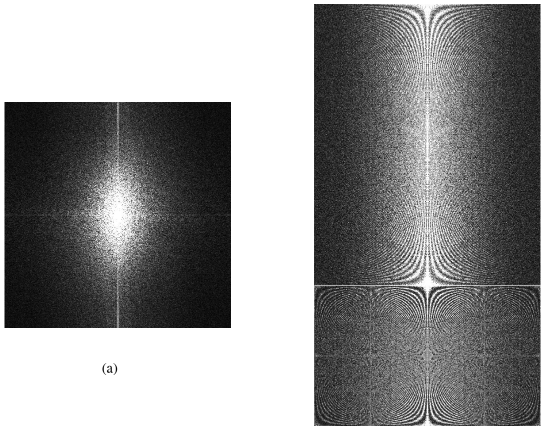
A processing of splitting-signal f_T yields a change in the Fourier spectrum at frequency-points of the corresponding group T. After performing the inverse 2-D discrete Fourier transform, such a change may be observed in the spatial domain at points along the parallel lines of sets $V_{p,s,t}$, $t = 0 : (N-1)$.

4. IMAGE ENHANCEMENT

In the tensor representation, an image is considered as the image of $3N/2$ splitting-signals and the 2-D DFT of the image as the set of $3N/2$ 1-D DFTs, when N is a power of two. Figure 3 shows the 2-DFT of the truck image (in absolute scale) in part a, along with the picture of 1-D DFTs performed over all splitting-signals of the image.

We can select splitting-signals by maximums of the energy they carry. By the Parseval's equality, the energy that splitting-signal $f_{T_{p,s}}$ carries is equal to

$$E_{p,s} = \frac{1}{N} \sum_{t=0}^{N-1} f_{p,s,t}^2 = \sum_{t=0}^{N-1} \mid F_{\overline{kp},\overline{ks}} \mid^2 . \tag{9}$$



(a) (b)

Fig. 3. (a) 2-D DFT of the truck image 512×512 and (b) 512-point 1-D DFTs of 768 splitting-signals.

Figure 4 shows the graph of function $E_{p,s}$ for all generators (p, s) of groups $T_{p,s}$ in the order given in the following construction of the covering $\sigma = \{\{T_{1,s}; s = 0 : N-1\}, \{T_{2p,1}; p = 0 : (N/2-1)\}\}$. The splitting-signal with the maximum energy 22.83 is $f_{T_{0,1}}$. The next five signals of high energy are $f_{T_{128,1}}$, $f_{T_{1,0}}$, $f_{T_{192,1}}$, $f_{T_{64,1}}$, and $f_{T_{1,256}}$. We recommend to use these splitting-signals for image enhancement.

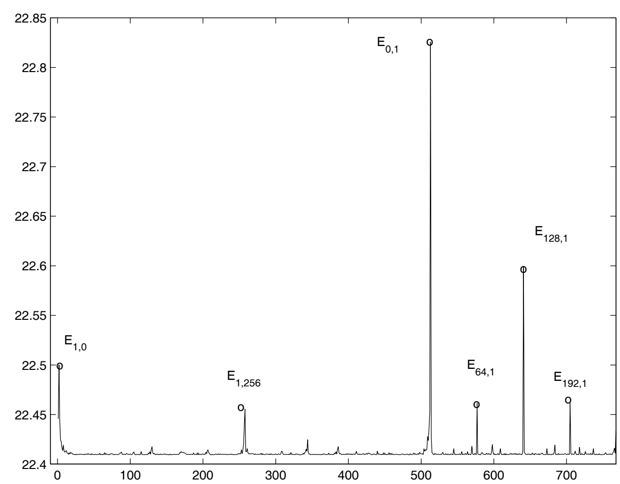


Fig. 4. The energy curve of 768 splitting-signals of the truck image.

The improvement in images after enhancement is very difficult to measure. The analysis of the existing transform-based image enhancement techniques developed in recent years show that to select optimal processing parameters and to measure the quality of images, the quantative measure EME of enhancement that relates to Weber's law of human visual system can serve as a building criterion for image enhancement [4, 5].

Such measure is defined as follows. Let g be the image obtained after processing image f by the Fourier transform-based enhancement algorithm with parameter α. In general, α may be a vector parameter. A discrete image g of size $N \times N$ is divided by k^2 blocks of size $L \times L$, where $L = N/k$. The quantative measure is calculated by

$$EME_\alpha(g) = \frac{1}{k^2} \sum_{m=1}^{k} \sum_{n=1}^{k} 20 \log \left[\frac{\max_{m,n} M_\alpha(g)}{\min_{m,n} M_\alpha(g)} \right] \qquad (10)$$

where $\max_{m,n} M_\alpha(g)$ and $\min_{m,n} M_\alpha(g)$ respectively are the maximum and minimum of image g inside the (m, n)th block. The value of $EME(f)$ is called the enhancement measure of the original image.

As an example, Figure 5(a) shows the measure of enhancement of the truck image in the curve described. The operation of a Fourier-transform-based image enhancement has been parameterized by a varying in the interval $[0, 1]$. The curve has a maximum at point $\alpha_0 = 0.92$. The experimental results show that the parameter α_0 corresponds to the best visual estimation of enhancement. Figures 5(b) and (c) illustrate the enhancement g of the original image f via the enhancement transform when $\alpha = 0.92$, which yields the enhancement $EME_{0.92}(g) - EME(f) = 17.43 - 9.81 = 7.62$. Blocks of size 7×7 are used in definition (10).

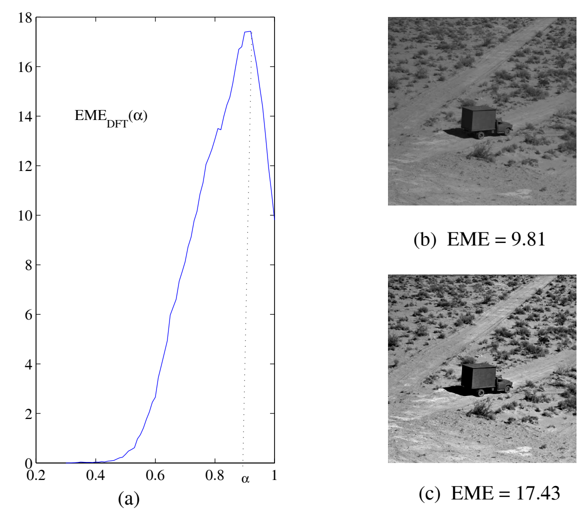


Fig. 5. Fourier transform image enhancement by α-rooting.

In the tensor representation, the Fourier transform method of image enhancement can be performed by processing one or a few splitting-signals $f_{T_{p,s}}$, $T_{p,s} \in \sigma$. As an application, we consider the α-rooting method of image enhancement [1, 2], when processing one selected splitting-signal.

Algorithm of Image Enhancement

Step 1: Perform the 1-D DFTs of the splitting-signal

$$f_{T_{p,s}} \to F_k = \sum_{t=0}^{N-1} f_{p,s,t} W^{kt}, \quad k = 0 : (N-1).$$

Step 2: Multiply the transform of the splitting-signal by coefficients $C_k = A|F_k|^{\alpha-1}$, $k = 0 : (N-1)$, where A is a constant.

Step 3: Fill (change) the 2-D DFT by the new 1-D DFT at frequency-points of subset $T_{p,s}$.

Step 4: Perform the inverse 2-D DFT.

In general, we can process all splitting-signals by the 1-D α-rooting method with a fixed parameter α, as well as process separately splitting-signals by different (optimal) values of parameter α, to achieve an optimal enhancement. The optimality is with respect to the enhancement measure EME. Thus, we may change Step 2 in the algorithm, by using different (or, optimal) $\alpha \in (0, 1]$ for splitting-signals.

As an example, Figure 6 shows the 513th splitting-signal $f_{T_{0,1}}$ in part a, along with the result g of image enhancement by this signal in b. The achieved enhancement equals $EME_\alpha(g) = 16.24$, when $\alpha = 0.96$. We here recall that the traditional α-rooting by the 2-D DFT yields the optimal value 0.92 with image enhancement 17.43. The 513th splitting-signal leads to the highest enhancement by EME, when considering parameter α to be equal 0.96.

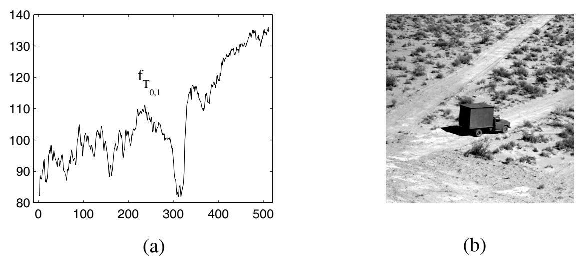


Fig. 6. (a) Splitting-signal $f_{T_{0,1}}$ and (b) image enhanced by this splitting-signal.

Other splitting-signals can also be used for effective enhancement of the truck image. Figure 7(a) shows the graph of the enhancement measure $EME(n; \alpha_o)$ calculated after processing only one, the nth splitting-signal for $\alpha_o = 0.95$, where $n = 0 : 767$. The splitting-signal $f_{T_{1,256}}$ is shown in b, along with coefficients C_k, $k = 0 : 511$ in c, and the enhanced image g in d. The enhancement equals $EME_{0.95}(g) = 13.45$, but it can be improved if we use the optimal value of α for this splitting-signal. Figure 8 shows the curve of function $EME(256, \alpha)$, when α varies in the interval $[0.6, 1]$. The value 0.97 is optimal for this splitting-signal $f_{T_{1,256}}$ which leads to image enhancement $EME_{0.97}(g) = 15.96$.

Thus, in the new view of the image processing, the image is represented as a set of 1-D splitting-signals and the problem of image enhancement in the frequency domain is reduced to processing one or a few splitting-signals. This approach allows us to achieve the enhancement and save many arithmetical operations when processing the 2-D DFT of the enhanced image.

5. REFERENCES

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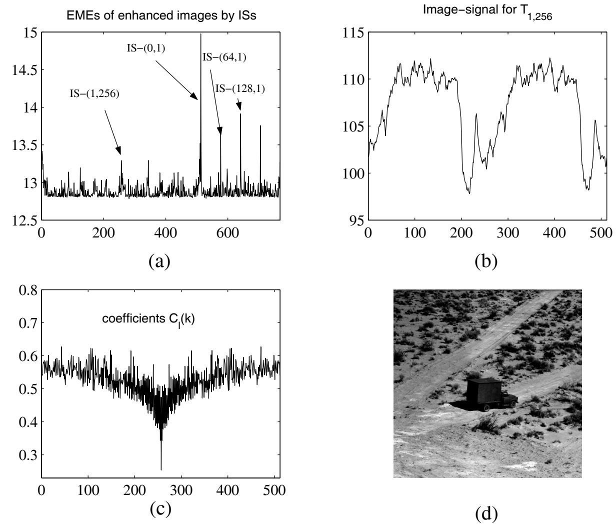

Fig. 7. (a) Enhancement measure function $EME(n, \alpha_o)$ for $\alpha_o = 0.975$, when $n = 0 : 767$. (b) splitting-signal $f_{T_{1,256}}$. (c) Coefficients $C_1(k)$, $k = 0 : 511$, of the one-dimensional α-rooting enhancement. (d) Truck image enhanced by the splitting-signal.

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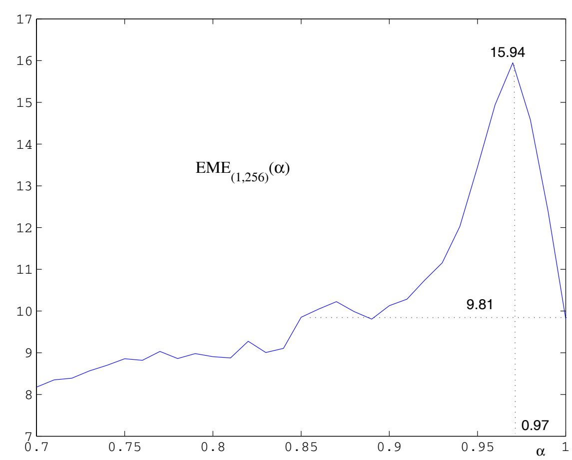

Fig. 8. Enhancement measure function $EME(256; \alpha)$.