ITERATIVE EQUALIZATION FOR CHAOTIC COMMUNICATIONS SYSTEMS

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ABSTRACT

Many previously proposed communications systems based on chaos disregard common channel distortions and fail to work under realistic channel conditions. In recent years, a sequential equalization algorithm based on the Viterbi algorithm showed good performance to combat the channel distortions. However,the amount of computation needed for the sequential equalization algorithm increases sequentially with the number of channel parameters and precision of the estimated chaotic sequence. In this paper, a lower complexity, iterative equalization algorithm for chaotic communications system is developed. In this scheme, the trellis size is greatly reduced by implementing the channel equalization and chaotic demodulation separately but connecting them iteratively. The proposed algorithms are simulated under both time-invariant and time-varying channel distortions. Bit-error rate graphs are illustrated for each case.

1. INTRODUCTION

A number of properties of chaotic systems, including low-power implementation, noise-like appearance, broadband spectra, and nonlinearity, are especially appealing for communications applications and can be exploited for secure communications [1, 2, 3]. A variety of approaches to chaotic communications have recently been proposed, including chaotic modulation, masking, and spread spectrum. Such communications systems offer the promise of inherent security, resulting from the broadband and 'noise-like' appearance of chaotic signals, and efficiency since systems could be allowed to operate in their natural nonlinear states. Even simple one-dimensional maps can produce random-like yet deterministic signals.

A generic chaotic communications system based on chaotic modulation is shown in Figure 1. In such a system, the information bits to be transmitted must first be encoded in the chaotic waveform using the chaotic system's *symbolic dynamics*. Rather than using structured signals such as rectangular pulses or sinusoids, to denote "0"s and "1"s (or other information symbols), these communications systems embed the information in the time evolution of the transmitted signal. Regions of the state space formed by the chaotic system's time evolution are designated to represent different symbols. The process of mapping information bits to the state of a chaotic system through its symbolic dynamics is termed chaotic modulation. This assignment of information bits to state is not arbitrary, and the greatest efficiency is achieved when the information transmission rate matches the topological entropy of the chaotic system [2]. Here, the symbolic dynamics representation of a chaotic system is exploited to implement the chaotic modulation described later in Section 2.

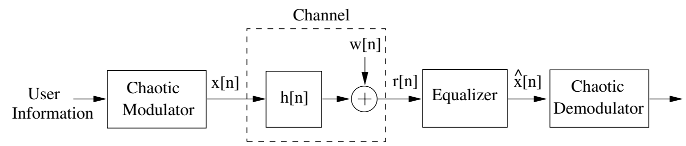

Fig. 1. A generic chaotic communication system.

Once the information sequence is encoded by the chaotic system, the modulated signal is then transmitted through the communication channel. In a real communications scenario, the communications channel distorts the transmitted signal. The channel distortion can be in the form of noise or intersymbol interference plus noise. To recover the transmitted information at the receiver, one must first undo the distortions before chaotic demodulation, which is the process of obtaining information symbols from the chaotic signal, is done.

For channel distortions where intersymbol interference as well as noise is imposed on the transmitted sequence, channel equalizers, typically implemented as linear adaptive filters, are usually employed to remove the distortion. There has been some research into equalization algorithms for chaotic communications systems [4, 5]. This scheme utilizes synchronizing discrete-time chaotic systems. In [4], a self-synchronization-based channel equalizer was proposed. However, this approach has limited utility because most chaotic systems do not self-synchronize. In [5], a technique for the blind identification of autoregressive (AR) systems driven by a chaotic signal was proposed. The algorithm is based on a minimum phase-space technique and can be applied for channel equalization of an AR system with a chaotic input. However, channel distortion is generally modeled as a FIR filter and can be time-varying.

In [6], a sequential equalization algorithm based on the Viterbi algorithm was developed. First, the symbolic dynamic representation of chaotic systems was first exploited to represent chaotic dynamics with an equivalent trellis diagram. Then, the trellis diagram was expanded to accommodate the finite impulse response the channel model. Finally, the viterbi algorithm was used to estimate the transmitted chaotic sequence and track changes in the channel. This algorithm was shown to perform well under realistic channel distortions. However, the amount of computation needed for the sequential equalization algorithm was mainly determined by the number of states in the trellis diagram. It was pointed out that the trellis diagram for the equalization algorithm had 2^{N_p+L} states where N_p represents the number of precision bits and $L+1$ is the length of the FIR channel model. For long channel models and/or for a large number bits per chaotic output (i.e., high precision), the computation load may become a problem. In this paper, a lower complexity, iterative equalization algorithm for chaotic communications system is developed.

Iterative (turbo) equalization algorithms were first proposed for conventional communications systems [**?**, **?**]. In a conventional iterative equalization scheme, the channel equalizer and channel decoder are separate algorithms but are jointly implemented in an iterative fashion. After each iteration, the extrinsic information obtained from the channel decoder about the transmitted signal is fed into the channel equalizer for the next iteration to further remove the ISI and, therefore, to better estimate the user information. This process is repeated iteratively. In the literature, iterative equalization is commonly referred to as *turbo equalization* because of its similarity to the turbo decoding process.

Here, the same principle is applied to chaotic communications systems where the trellis size is greatly reduced by implementing the channel equalization and chaotic demodulation separately but connecting them iteratively. Improvement is achieved by exploiting the information obtained from the chaotic demodulator. This information is used in channel equalization during the next iteration to better estimate the channel parameters and, therefore, to further remove the ISI at the output of the equalizer. This will in return improve the estimate of the transmitted user information. This process is repeated iteratively.

The remaining of the paper is organized as follows. Background information about symbolic dynamics and chaotic modulation is given in Section 2. The iterative equalization algorithm is explained in Section 3. Simulations results for the proposed algorithm are illustrated in Section 4. Concluding remarks are given in Section 5.

2. SYMBOLIC DYNAMICS AND CHAOTIC MODULATION

Discrete-time signals generated by first order chaotic systems satisfy the dynamical equation

$$x[n] = f(x[n-1]),\ x[n] \in I, \tag{1}$$

where $f(\cdot)$ is a nonlinear function mapping points from an interval I onto the same interval and possessing certain properties, such as sensitivity to initial conditions. Once the nonlinear dynamics, $f(\cdot)$, and initial condition, $x[0]$, are known, then it is straightforward to generate a chaotic sequence. Higher order chaotic systems obey a similar equation where $x[n]$ is now a vector representing the state of the system.

Rather than generating the chaotic sequence directly by iterating (1), one can exploit symbolic dynamics to obtain an equivalent chaotic sequence through an alternate process[6]. Symbolic dynamics also provide a means of examining the real dynamics with finite precision. This finite precision symbolic dynamics representation is the key to efficient chaotic modulation and demodulation. Chaotic modulation and demodulation through mapping functions are illustrated in Figure 2. The mapping function may not be known for certain chaotic systems and may not exist in some cases. However, it may still be possible to encode the information through symbolic dynamics.

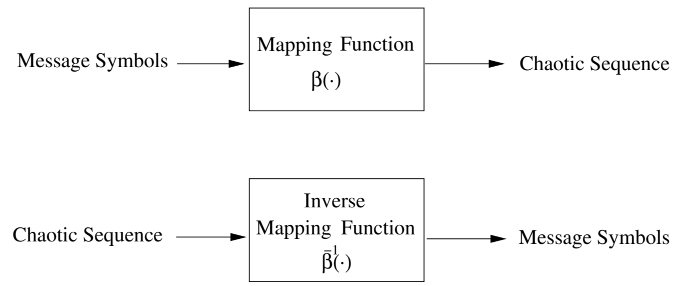

Fig. 2. Chaotic modulation and demodulation through symbolic dynamics.

The ability to use a finite number of symbols to generate chaotic sequences allows us to represent chaotic dynamics with an equivalent trellis diagram, since the symbol set is also finite. This trellis diagram representation of chaotic systems was the key for the development of the optimal estimation and channel equalization algorithms developed in [6].

3. ITERATIVE EQUALIZATION ALGORITHM

The chaotic communication system depicted in Figure 1 is considered here. Once the information sequence is encoded by the chaotic system, the modulated signal is then transmitted through the communication channel. The distortion introduced by the channel is modelled by a linear filter with impulse response $h[n]$ and additive white Gaussian noise $w[n]$ with variance of σ_w^2. The received signal, $r[n]$, can then be written as

$$r[n] = h[n] * x[n] + w[n], \tag{2}$$

where $*$ represents the convolution operation. To recover the transmitted information, one must undo these distortions before chaotic demodulation, which is the process of obtaining an estimate of the transmitted symbol sequence from the received signal. The goal of the equalizer is to obtain an accurate estimate of transmitted signal $x[n]$ from the observed signal $r[n]$.

In iterative equalization, channel equalization and chaotic demodulation are jointly performed in a repetitive manner. The iterative equalizer scheme is depicted in Figure 3 where delays are equal to the latency of each module. Each iteration $p; p = 1, \ldots, P$ is carried out by a module fed in by both samples of the received signal, represented as $r^{p-1}[n]$ and the estimated chaotic signal, represented as $\hat{x}^{p-1}[n]$, originating from the module $p-1$.

Each module consists of an equalizer, a chaotic demodulator, and depending on whether the transmitted signal has been interleaved, a deinterleaver and an interleaver. Figure 4 depicts module $p-1$ of the iterative scheme, where each module consists of an adaptive equalizer and a chaotic demodulator. Each module provides an estimate of the transmitted chaotic sequence, $\hat{x}^p[n]$. This information is then used by the adaptive equalizer of the next module.

The adaptive equalizer is implemented as an adaptive linear FIR filter. The equalizer structure is shown in Figure 5. The linear filter parameters represented as $q[n]$ are updated using the least mean square (LMS) algorithm. The parameter update equation at time k for each module (superscript p has been dropped for conve-

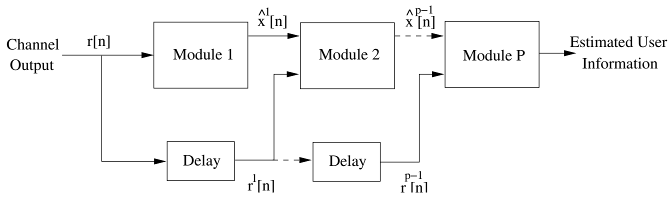

Fig. 3. Iterative equalization principle.

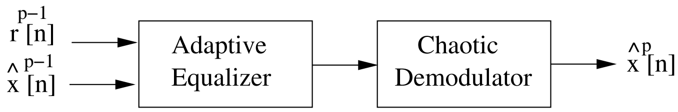

Fig. 4. Iterative equalization of module $p-1$.

nience) is as follows

$$q_k[j] = q_{k-1}[j] + \mu \cdot e[n]r[n-k], \tag{3}$$

where μ is the adaptation step size and $r[n]$ is the received signal. The the error sequence $e[n]$ is calculated as

$$e[n] = \hat{x}[n] - \tilde{x}[n], \tag{4}$$

where $\hat{x}[n]$ is the estimate of the transmitted sequence obtained from the output of the chaotic modulator at the previous module, and $\tilde{x}[n]$ is the output of the equalizer. The initial estimate of channel parameters are obtained using a training sequence.

Assuming that the adaptive equalizer removes ISI reasonably, the signal at the output of the equalizer, $\tilde{x}[n]$, could be thought of as a noisy observation of the transmitted chaotic signal, $x[n]$. Therefore, the optimal estimation algorithm developed in [6] can be used as a chaotic demodulator to estimate the transmitted chaotic sequence and to obtain the user information. Since the estimation algorithm exploits the knowledge of the system dynamics, the estimated chaotic sequence will satisfy the system dynamics. This estimated signal is used in the next module to better update the equalizer parameters and, therefore, to further reduce ISI. When this process is repeated iteratively, the performance is expected to improve after each iteration.

Since the initial estimate of the transmitted signal, $\hat{x}[n]$ is not available at the first iteration, the channel parameters are updated using estimates from the chaotic demodulator corresponding to a finite trellis depth as was done for the parameter update procedure in the sequential equalization algorithm in [6].

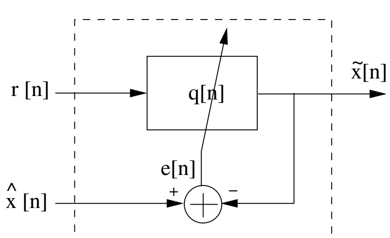

Fig. 5. Adaptive equalizer structure.

4. SIMULATIONS

Performance of the iterative equalization algorithm was simulated by encoding user information with the sawtooth map for a 10-bit precision. The adaptive equalizer was implemented with a 10-tap FIR filter. The trellis depth was set to 10 for the first iteration. It was also assumed that the information symbols were transmitted in a block of 1000. The channel distortion was first modeled with a 3-tap multipath fading channel where $f_m \cdot T$ was 1e-4. The bit error rate performance of the iterative equalization is illustrated in Figure 6 for the first 5 iterations, i.e., modules 1 through 5, and compared with the sequential equalization algorithm.

The iterative equalization algorithm was also simulated for the time-invariant raised cosine channel model given by equation

$$h[n] = \begin{cases} 1/2 + 1/2\cos[2\pi(n-2)/3] & \text{for n=1,2,3} \\ 0 & \text{otherwise} \end{cases} . \tag{5}$$

(It was also assumed that the channel adds white Gaussian noise to the signal. The bit-error rate performance of the iteration equalizer is plotted in Figure 7 and compared with the sequential equalization algorithm. It can easily be concluded that the performance of iterative equalization doesn't improve significantly after each iteration for the time-invariant channel for the sawtooth chaotic map.

In a conventional iterative equalization algorithm, an interleaver and deinterleaver are commonly used to whiten the noise at the output of the linear equalizer and to split the error series. The effect of the interleaver for the iterative equalization algorithm developed here was also simulated. In this case, it was assumed that the signal at the output of the chaotic modulator was interleaved before being transmitted. At the receiver, a deinterleaver was used after the linear equalizer. The estimated signal at the output of the chaotic demodulator was interleaved before being fed into the equalizer of the next iteration. Here, a random block interleaver with a block size of 1000 was used. The effect of the interleaver for the time-varying channel model, simulated by Jake's fading method for an $fm \cdot T$ value of 1e-4, is shown in Figure 9. In this figure, the iterative equalizer with an interleaver is again compared to the no interleaver scheme. It can be observed that surprisingly using an interleaver slightly degrades the performance of the equalizer in both cases; thus justifying not using an interleaver in the earlier simulations of this chapter

5. CONCLUSIONS

This paper has described an iterative equalization algorithm for chaotic communications systems. In this scheme, channel equalization and chaotic demodulation have been jointly implemented to improve the global performance of the algorithm. The iterative equalization algorithm has lower complexity than the sequential equalization algorithm. Simulation results showed that the performance of the iterative equalization algorithm improved significantly after the first iteration for time-varying channel models but didn't improve much for the time-invariant channel model. It was also shown that interleaving did not improve the performance.

6. REFERENCES

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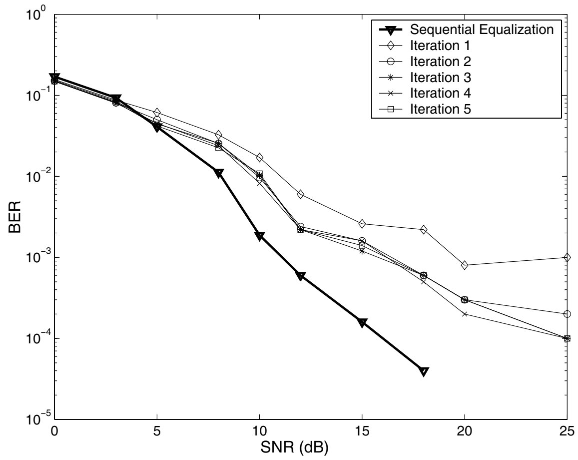

Fig. 6. Bit-error rate performance of the iterative equalization algorithm for the first 5 iterations as compared to the sequential equalization algorithm for a multipath fading channel with $f_m \cdot T$ equal to 1e-4.

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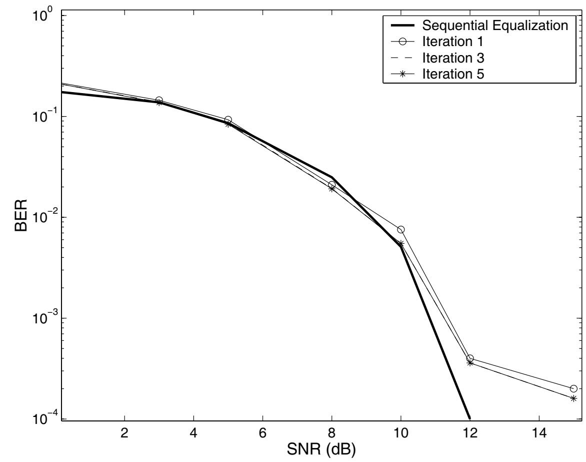

Fig. 7. Bit-error rate performance of the iterative equalization algorithm for the first 5 iterations as compared to the sequential equalization algorithm for the raised cosine channel model.

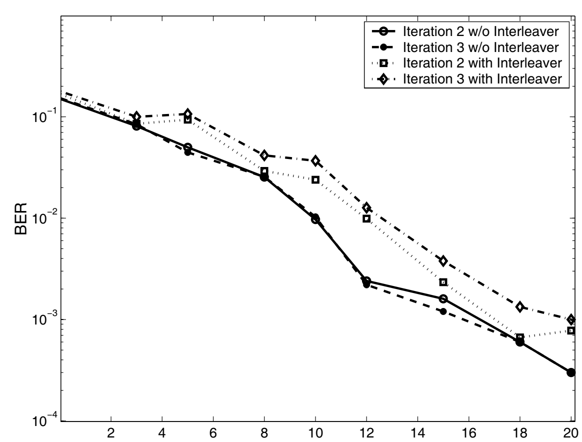

Fig. 8. Bit-error rate performance of the second and third iterations of the iterative equalization algorithm with an interleaver as compared to second and third iterations of no interleaver case for multipath fading channel with a $f_m \cdot T$ value of 1e-4.