VARYING WEIGHT TRIMMED MEAN FILTER FOR THE RESTORATION OF IMPULSE NOISE CORRUPTED IMAGES

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ABSTRACT

A weight function is introduced to the generalized version of the alpha-trimmed mean filter for the removal of impulse noise from corrupted images. Iterative implementation of the new filter-based switching scheme is proposed as an impulse detector to preserve the noisefree pixels. Numerical simulations show that the proposed filter is quite robust and efficient, and its performance compares favorably with many other wellknown nonlinear filtering algorithms.

1. INTRODUCTION

Impulse noise seriously affects the performance of image processing techniques, e.g., edge detection, pattern recognition, and image compression. One of the important tasks of image processing is to restore a high quality image from a corrupted one for use in subsequent processing. The goal of image filtering is to suppress the noise while preserving the integrity of significant visual information, such as textures, edges, details, etc. The most popular edge-preserving nonlinear filter is the median filter [1], which is known to have the required property of suppressing the impulse noise. However, it is also true that the median filter is not optimal. It suppresses the true signal as well as noise in many applications. For improving its filtering performance, many modifications and generalizations of the median filter have been proposed [2][3][4]. In [4], we have proposed a generalized version of the alpha-trimmed mean (\alpha-TM) filter [3]. We call it the general trimmed mean (GTM) filter, which has been shown to be extremely efficient in removing both additive noise and highly impulse noise from corrupted images. In the GTM filter, a "median basket" is employed to select a predetermined number of pixels on both sides of the median value of the sorted pixels in the moving window. The values of selected pixels and the center pixel in the window are then

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weighted and averaged to give the filtering output. As mentioned in [4], it is important to have the center pixel participate in the averaging operation when removing additive noise, but one usually sets the weight of the center pixel to zero when filtering impulse noise corrupted images.

In the GTM filter, the averaging weights are predetermined and are fixed throughout the filtering procedure. In the present work, a weight function is designed and applied to the GTM filter, which makes the averaging weights of the GTM filter vary with respect to the absolute difference between the pixel values in the median basket and the median value. We call it the varying weight trimmed mean (VWTM) filter.

2. VARYING WEIGHT TRIMMED MEAN FILTER

The pixels $\{I_1, I_2, \dots, I_{m-1}, I_m, I_{m+1}, \dots, I_N\}$ in the moving window associated with the center pixel I_c , have been sorted in an ascending (or descending) order, with I_m being the median value. The key generalization to the median filter which is introduced in the alpha-trimmed mean (α -TM) filter [3], is to employ a median basket in which one collects the same predetermined number of pixels above and below the median pixel. The values of these selected pixels are then averaged to give the filtering output, A_c , as an adjusted replacement to I_c , according to

$$A_{c} = \frac{1}{2L+1} \sum_{j=m-L}^{m+L} I_{j},$$
 (1)

where $L = \lfloor \alpha N \rfloor$, with $0 \le \alpha \le 0.5$. It is evident that a singleentry median basket (L=0) α -TM filter is equivalent to the median filter and a *N*-entry ($L = \lfloor 0.5N \rfloor$) median basket is equivalent to the simple moving average filter. The general trimmed mean (GTM) filter [4] uses a median basket in the same way as one does in the α -TM filter to collect the pixels. The pixel values in the median basket and the center pixel I_c in the moving window are then weighted and averaged to give the GTM filtering output:

$$G_{c} = \frac{w_{c}I_{c} + \sum_{j=m-L}^{m+L} w_{j}I_{j}}{w_{c} + \sum_{j=m-L}^{m+L} w_{j}},$$
(2)

where G_c is the GTM filtering output, and w_c and w_j 's are the averaging weights for the center pixel and the pixels in the median basket, respectively. When $w_c=0$ and all w_j 's are equal to each other (nonzero), the GTM filter becomes the α -TM filter. In the GTM filter, all the weights are predetermined and are fixed during filtering procedure. However, it is possible to further improve the GTM filter by varying the weights according to the absolute difference between the pixel values in the median basket and the median value. For the removal of impulse noise, we set $w_c=0$, and the varying weight trimmed mean (VWTM) filter is given by

$$V_{c} = \frac{\sum_{j=m-L}^{m+L} w(x_{jm}) I_{j}}{\sum_{j=m-L}^{m+L} w(x_{jm})},$$
(3)

where x_{jm} has a value in the range of [0,1], defined by

$$x_{jm} = \frac{|I_j - I_m|}{B},\tag{4}$$

with *B* being the maximum pixel value of a given type of image (e.g., B=255 for a 8-bit, gray-scale image). The weight w(x) in (3) is a decreasing function in the range [0,1] and is chosen to be

$$w(x) = \exp\left(-A\left(\frac{x}{x-1}\right)^2\right).$$
(5)

Notice that w(0)=1 and w(1)=0, so the median value always has the largest weight $(w(x_{mm})=1)$. The larger the absolute difference between the pixel values in the median basket and the median value, the smaller the weight will be. We anticipate that the VWTM filter will outperform both the median filter and the α -TM filter in suppressing impulse noise while preserving edges. As is well known, the median value has the least probability to be impulse noise corrupted because the impulses typically occur near the ends of the sorted pixels. However, although not corrupted, the median value may not be the optimal value to replace the center pixel value because it may differ significantly from the noise-free value. The α -TM filter itself will not perform better than the median filter when treating highly impulse noise corrupted images because corrupted pixels may also be included in the median basket for the averaging operation. In contrast, the VWTM filter can alleviate the shortcomings of both filters. The weight of the median value is the largest and the weights of other pixels in the median basket vary according to their difference from the median value. If an impulse noise corrupted pixel happens to be selected for

inclusion in the median basket, its contribution to the average will be small because x_{jm} is large. In general, the weight function can assist in eliminating impulse noise while providing a well-adjusted replacement value for the center pixel I_c .

3. THE VWTM FILTER-BASED IMPULSE DETECTOR AND THE ITERATIVE METHOD

Many algorithms have been proposed to detect and replace the impulse noise corrupted pixels [5][6][7]. In the present work, a simple but efficient switching scheme similar to that used in [5], but based on the VWTM filter, is employed to detect the impulse noise and recover the noise-free pixels. The filtering output I'_c is generated according to the following algorithm:

$$I_{c}^{'} = \begin{cases} I_{c}^{(i)}, \mid I_{c}^{(i)} - V_{c} \mid < T \\ V_{c}, \mid I_{c}^{(i)} - V_{c} \mid \geq T \end{cases},$$
(6)

where $I_c^{(i)}$ is the initial input value and V_c is the VWTM filtering result of the initial input value. The threshold *T* is chosen to characterize the absolute difference between $I_c^{(i)}$ and V_c . If the difference is larger the threshold, it implies that the pixel differs significantly from its neighbors. It is therefore identified as an impulse noise corrupted pixel, and is then replaced by V_c . If the difference is smaller than the threshold, it implies that the initial input value is similar to its statistical neighbors, and we identify it as noise-free pixel, and it therefore retains its original value.

Iteration of the above scheme will further improve the filtering performance, especially for images that are highly corrupted by impulse noise. The iteration procedure can be depicted as

$$I_{c}(t) = \begin{cases} I_{c}^{(i)}, \ |I_{c}^{(i)} - V_{c}(t)| < T \\ V_{c}(t), \ |I_{c}^{(i)} - V_{c}(t)| \ge T \end{cases},$$
(7)

where $I_c(t)$ is the system output at time *t* with $I_c(0) = I_c^{(i)}$, and $V_c(t)$ is the VWTM filtering output at time *t*, given by

$$V_{c}(t) = \frac{\sum_{j=m-L}^{m+L} w(x_{jm} \mid t-1) I_{j}(t-1)}{\sum_{j=m-L}^{m+L} w(x_{jm} \mid t-1)},$$
(8)

Note that it is important for the iterative procedure always to compare $V_c(t)$ with the initial input $I_c^{(i)}$ and to update output with $I_c^{(i)}$ when their absolute difference is less than the threshold *T*. In [8], Zhang and Wang proposed a median-based iterative scheme, which replaces $I_c^{(i)}$ and $V_c(t)$ with $I_c(t-1)$ and median filtering output at time *t*, respectively in equation (7). Test simulations show

that this iterative scheme is less efficient than the present algorithm.

4. NUMERICAL SIMULATIONS

The standard 8-bit, gray-scale "Lena" image (size 512×512) is used as an example to test the usefulness of our filtering algorithm. We degraded it with various percentages of fixed value (0 or 255) impulse noise. The proposed algorithm is compared with the median and α -TM filtering algorithms, and their peak signal-to-noise ratio (PSNR) performances are listed in Table 1. Both the direct and switch-based applications of these filters are presented in Table 1 for comparison, and all algorithms are implemented recursively in a 3×3 window for optimal PSNR performance. (The same iterative procedure is used for all the switch-based algorithms.) A 3-entry median basket (L=1) is used for both α -TM and VWTM filtering algorithms. The parameter A used for the weight function of VWTM filter is 2 for images with 15%, 20%, 25%, and 30% impulse noise, is 3 with 35% impulse noise, and is 4.5 with 40% impulse noise. (Test simulations show that the VWTM filtering performance is not very sensitive to the parameter A. For A=2, the PSNR of the VWTMswitch filtering results for 35% and 40% impulse noise corrupted images are 32.23 dB and 31.36 dB, respectively.) The threshold used for all the switch-based schemes is 28. From Table 1, it is easy to see that without the switching scheme, the α -TM filter performs even worse than the median filter. However, it performs better than the median filter when the switching scheme is used. This reflects the fact that although the α -TM filter may not perform well in impulse noise removal, it is a good impulse detector. The VWTM filter performs better than either the median filter or the α -TM filter, whether the switching scheme is used or not. It is simple, robust and efficient. The VWTM filter performs well in removing impulse noise, and is simultaneously a good impulse detector. It is especially efficient for using the VWTM filter to restore highly impulse noise corrupted images. Figure 1 shows the impulse noise corrupted Lena image (40% impulse noise), and the filtered results for several algorithms. One can observe from Figure 1 that even if there is no switching scheme employed, the VWTM filter performs better than the α -TM filter in terms of suppressing the noise and preserving the edges. The VWTM filter-based switching scheme provides a result that is almost the same as the original noise-free image.

5. CONCLUSIONS

This work presents a new nonlinear filtering algorithm for the removal of impulse noise from corrupted images. It is based on varying the weights of the generalized version of the trimmed mean filter continuously with respect to the absolute difference between the pixel values in the median basket and the median value. Simulations show that the VWTM filter is robust and efficient for both noise removal and impulse detection, and its performance compares favorably with many other nonlinear filtering techniques.

Table 1 Comparative filtering performances in PSNR (in dB) for Lena image corrupted with different percentage of fixed value impulse noise. All schemes are implemented recursively for best performance.

Scheme	15%	20%	25%	30%	35%	40%
Median	32.19	31.48	30.69	29.91	29.37	28.75
α-ΤΜ	32.09	31.30	30.39	29.28	28.38	27.49
VWTM	32.31	31.67	30.89	30.12	29.63	29.06
Median*	35.20	33.87	32.91	31.84	30.99	30.29
α-ΤΜ*	36.04	34.93	33.97	32.59	31.67	30.72
VWTM*	36.34	35.13	34.29	33.15	32.24	31.43

*Switching scheme







(b)



(c)



(d)

Figure 1 Image restoration from 40% impulse noise corrupted image. (a) Noisy Image, PSNR=9.29 dB; (b) α -TM filtering result (3×3), PSNR=27.49 dB; (c) VWTM filtering result, PSNR=29.06 dB; (d) VWTM-switch filtering result, PSNR=31.43 dB.

6. REFERENCES

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