SIGNAL RECOVERY DUE TO ROTATIONAL PIXEL MISALIGNMENT

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ABSTRACT

Pixel misalignments such as two-dimensional lateral shifts, rotation and magnification are one of the common sources of distortion in optical data storage systems like volume holographic memories. In this paper we formulate the channel model for rotational misalignment of the detector grid array with respect to the transmitter, about the center of the optical axis and derive an expression for the number of transmitted bits that can be recovered losslessly. Finally we outline an algorithm for recovering the transmitted bits from the detector array in the presence of additive white Gaussian noise and rotational misalignment. The method is applicable to holographic memories and other imaging systems.

1. INTRODUCTION

The advent of optical data storage systems like holographic memories [4] for high-density volumetric storage and fast page-oriented data access has evoked interest in constrained coding and signal processing for recording and reading two-dimensional data. Two-dimensional coded modulation schemes and improved signal processing techniques for handling noise and inter pixel interference can greatly enhance the data storage density in such optical memories.

The detection and imaging process in most systems is not perfect. The inherent effects of band-limiting aperture, diffraction, misfocus and optical aberrations [4] lead to inter-pixel interference. Further, effects such as mechanical motion of optical components and defects in optical imaging have a serious impact leading to shifts in the detector pixel with respect to the intended transmitted pixel. Thus we need signal processing algorithms to remove the residual energy from unintended pixels and recover the transmitted data.

There are signal-processing algorithms [3], [5] for reducing inter-pixel interference, for correcting pixel blurs and for recovering the signal from a combination of known pixel patterns. However there are very few algorithms [1], [2], [5] which correct pixel shifts. Burr developed signal reconstruction algorithms [1], [2] for compensating fractional lateral shifts that are constant in two dimensions. Rotational misalignments lead to fractional shifts that are not constant over pixels. In other words, pixels that are farther away from the center suffer more severe distortion than those at the center. Our work in this paper is motivated to extend the idea of handling fractional shifts for rotational distortion. Our algorithm is in general applicable for rotationally misaligned systems with square apertures and for holographic systems with low fill factors in the transmitter and detector arrays.

The paper is organized as follows. In Section 2, we formulate the channel model with rotational misalignment and derive an expression for the number of transmitted bits that can be recovered losslessly. In Section 3 we outline an algorithm for recovering bits due to rotational misalignment in the presence of detector noise and present simulation results to evaluate the performance of the algorithm. Finally we conclude in Section 4.

2. CHANNEL MODEL WITH ROTATIONAL MISALIGNMENT

We will briefly review the 4-F optical system [1] that forms the basis of the channel model. The system consists of two identical lenses separated by the sum of their focal lengths. A square aperture of dimensions D^2 is placed in the common focal plane of the two lenses. The input transmitter array is a spatial light modulator (SLM) and is assumed identical in dimensions to the output detector array, which is typically a charge-coupled device (CCD).

The spatial sampling rate is determined by the spacing of the pixels in the SLM and we assume this to be identical to the aperture width D. The pixel-spread function is the convolution of the space-invariant impulse response (due to the aperture) with the original pixel shape. The space-invariant impulse response is determined by the continuous space Fourier transform of the aperture shape. With a square aperture, the impulse response is a 2-D separable sinc function in the x-y plane given by,

$$h(x,y)=c^2\int_{-0.5g_{SLM}}^{0.5g_{SLM}}\frac{\sin(\pi(x-x'))}{\pi(x-x')}dx'\int_{-0.5g_{SLM}}^{0.5g_{SLM}}\frac{\sin(\pi(y-y'))}{\pi(y-y')}dy' \quad (1)$$

where, g_{SLM} is the SLM fill factor where most of the intensity is captured, the variables x, x', y and y' are in the units of the pixel dimensions and the normalizing constant c is chosen so that $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h^2(x,y)dxdy=1$.

We note that $h(x,y)=1$ when evaluated at the center of the SLM pixel and is oscillatory decaying along both the axes.

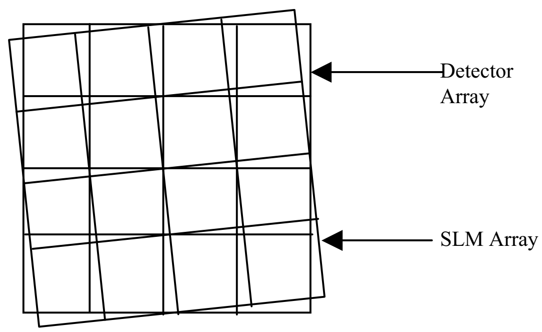

Figure 1: Rotational Misalignment of the SLM array with respect to the detector

Figure 1 shows the schematic of a rotationally misaligned SLM array with respect to the detector about the optical axis. The angle of rotation α is positive in the clockwise direction. The coordinates of the SLM with respect to the detector can be obtained by the rotational transform R given by,

$$R=\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix} \quad (2)$$

The received signal at the detector pixel $d(m,n)$ is given by,

$$d(m,n)=\int_{-0.5g_{CCD}}^{0.5g_{CCD}}\int_{-0.5g_{CCD}}^{0.5g_{CCD}}\left(\sum_{m_i,n_i}g_{m_i,n_i}(x,y)\sqrt{t(m_i,n_i)}\right)^2 dxdy \quad (3)$$

where, g_{CCD} is the CCD fill factor where the most of the intensity at the detector pixel is captured. The term $t(m_i,n_i)$ denotes the binary signal from the SLM pixel with discrete index (m_i,n_i) that overlaps with the detector pixel with discrete index (m,n). We note that for small angles α, the indices (m_i,n_i) of the SLM pixels contributing to the cross talk terms in the detected signal $d(m,n)$ are due to $(\lceil m\cos\alpha+n\sin\alpha\rceil,\lceil -m\sin\alpha+n\cos\alpha\rceil)$ and its three neighbors on the left, bottom and left-diagonal. Referring to Figure 1, let us fix the origin as the center of the grid array. The signal at the detector pixel with right top corner coordinates $(1,2)$ is indexed as $d(1,2)$ and has energy mainly contributed by the SLM pixels $t(2,2), t(2,1), t(1,2)$ and $t(1,1)$.

The kernel $g_{m_i,n_i}(x,y)$ in (3) is the rotated version of the function $h(x,y)$ and is given by,

$$g_{m_i,n_i}(x,y)=\frac{1}{|R|}h(x-a)h(y-b) \quad (4)$$

where, $(a,b)=(m_i-0.5,n_i-0.5)R^t$. Also $|R|=1$ since R is orthonormal.

Figure 2 shows the sketch of the kernel $g(x,y)$ centered at the point $(0.5,0.5)$. We can imagine a tiling of such kernels at the center of each SLM pixel. These kernels low pass filter the transmitted signal causing pixel blur. The effect of rotation causes non-uniform inter-pixel interference at the intended detector pixel.

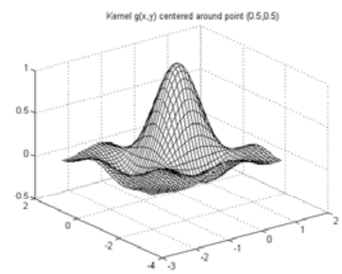

Figure 2: Schematic of the rotated two-dimensional kernel $g(x,y)$

The goal of the problem is to recover the transmitted bits from the detected signal $d(m,n)$.

Without any loss of generality, assume that the detector grid array and the SLM array are of size $2m\times 2m$ and each square is of unit area. Without any coding, every element of this uniformly spaced grid array is an equally likely binary symbol. Without any misalignment, $4m^2$ information bits can be stored. Let A_t and A_d denote the areas of the transmitted and the detected arrays respectively. As a result of rotational misalignment, the number of bits lost is given the fraction of the area that does not overlap between the transmitted and detected arrays. This fact can be interpreted that the portion of the channel not containing $A_t\cap A_d$ is lost due to misalignment. Thus the number of transmitted bits lost is given by,

$$T_{bits\_lost}=4m^2\left(\frac{A_t-A_t\cap A_d}{A_t}\right) \quad (5)$$

From simple coordinate geometry, we can compute the overlapping areas of the detected and SLM arrays and obtain an upper bound on the number of transmitted bits that can be recovered losslessly. The result is stated in Fact 1 and we avoid the derivations for the sake of brevity.

Fact 1: For $2m \times 2m$ equally likely binary bits of transmitted signal, at most $4m^2(1-T_{loss})$ bits can be recovered from rotational misalignment. The parameter T_{loss} is given by,

$$
\begin{aligned}
T_{loss} &= \frac{1}{2}pq \\
p &= 1+\cot\alpha - \sin\alpha - \cot\alpha\cos\alpha \qquad (6)\\
q &= 1-\cos\alpha + \tan\alpha - \sin\alpha\tan\alpha
\end{aligned}
$$

We note that the loss function T_{loss} is periodic with $\pi/2$ and the maximum loss is around 0.18 bits per pixel and occurs at angle $\alpha = \pm n\pi/4$. Figure 3 shows a plot of the loss function for $0 \le \alpha \le \pi/2$.

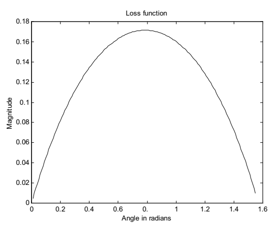

Figure 3: Loss as a function of the angle of rotation

However in the presence of noise, the transmitted pixels can suffer errors. In such cases we can model the channel as a binary symmetric channel with capacity C_{BSC}. The number of bits that can be recovered losslessly is now at most $4m^2(1-T_{loss})C_{BSC}$. It must be noted that the number of transmitted bits lost due to misalignment is not significant for small angles. However the inter-pixel interference with additive noise makes the problem difficult for signal recovery.

3. ALGORITHM FOR SIGNAL RECOVERY

We will now consider the problem of recovering the input pixels from the detected pixels. We consider small angles of rotational misalignment i.e., less than around 3 degrees from a practical perspective. Large angles of rotation can always be compensated by carefully aligning the parts till the point where it is difficult to fine-tune the angle alignment. Without loss of generality, we assume the detector array to be a square grid of size $2m \times 2m$. We define a coordinate system of the detector array as follows. The coordinate of the center of the grid array is designated as the origin $(0,0)$. The index for a detector pixel is identified by the coordinate of its right top corner.

From the structure of equation (3) we observe that the system is fundamentally non linear and anti causal. The non-linearity is due to the cross terms in the squaring process. The non-causality arises because we cannot initiate a recursion without the knowledge of a few transmitted bits.

In order to facilitate a recursion, we need to initialize the SLM pixels in the boundary layers to zero. To determine the number of such layers that are initialized to zero, we compute the coordinates of the SLM array that just exceeds the range of the detector array. Consider the column of pixels at the right most ends. The coordinate of their right top corner $(m, y)^T$ after rotational transform is obtained as $(m\cos\alpha - y\sin\alpha, -m\sin\alpha + y\cos\alpha)^T$. The point where is the ordinate y exceeds m is given by,

$$-m\sin\alpha + y\cos\alpha > m \qquad (7)$$

From (7) we infer that we need to set the top $\lfloor m(1-\sin\alpha)/\cos\alpha \rfloor$ layer of SLM pixels to zero. Similarly by symmetry we set the bottom, left and right most $\lfloor m(1-\sin\alpha)/\cos\alpha \rfloor$ layer of SLM pixels to zero.

Before beginning with the algorithm, we need to first estimate the angle of rotation. By sending a known preamble of pixel pattern and measuring the detected signal at a predesignated location we can estimate the angle of rotation. This measurement is used subsequently in the reconstruction algorithm. As an illustration, consider Figure 1. By presetting the transmitted SLM array as an all zero pattern except at location $t(1,1)$ and by measuring the detected signals $d(1,2), d(1,1)$, we can estimate the angle of rotation.

We assume that the SLM and CCD fill factors are low [3] so that we can get rid of the integrals in equations (1) and (3). The integrands will be evaluated at the center of the detector pixel. We can take the square root of the detected signal and do linear processing [3] for signal recovery. With the above framework, we will now outline an algorithm for signal recovery.

The recovery process is done in two blocks. The first block comprises of all the detector pixels towards the right half plane of the detector array. The second block consists of all pixels in the left half plane.

For the first block, detector pixels are sequentially scanned starting from the top most row until all the transmitted SLM bits are sequentially decoded from right to left along this row. The scanner moves to the next row and repeats the process of decoding all the row bits before starting the next row. This process iterates until all the bits in the first block are decoded. The idea is illustrated in Figure 4. For the second block the scanner starts from the bottom most row in the left half plane, decodes all the bits from left to right along that row, moves to next top row and iterates the process till all the bits are recovered.

We can also do the decoding by processing the array of detected signals in four blocks corresponding to each of the four quadrants and then average the results obtained from the two block scanning and decoding process illustrated in Figure 4. This averaging technique will be helpful especially in the presence of severe detector noise when the decoding errors tend to propagate.

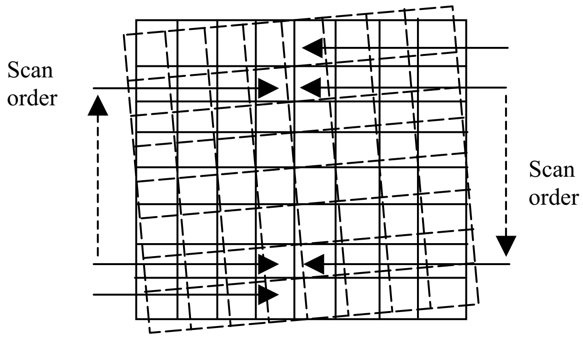

Figure 4: Scanning and decoding process

We now outline the steps for the decoding of the first block in the form of an algorithm described below. The decoding algorithm for the second block follows anti symmetrically in exactly the same way as the first block.

Outline of the Algorithm

Introduce the following definitions:
s : Array representing the decoded bits.
d : Array holding the detected signal values.
Initialize:

- Set all the pixels of the array s along the rows from $\lfloor m(1-\sin\alpha)/\cos\alpha \rfloor$ to m as zero. Also set all the pixels of the array s from columns $\lfloor m(1-\sin\alpha)/\cos\alpha \rfloor$ to m as zero.

1. Set the detector index to the top right corner $(r,c) = (m,m)$
2. Obtain the SLM pixel indices that overlap with (r,c) as $(a,b),(a-1,b),(a,b-1)$ and $(a-1,b-1)$ where $(a,b) = (\lceil r\cos\alpha + c\sin\alpha \rceil, \lceil -r\sin\alpha + c\cos\alpha \rceil)$.
3. Evaluate the components of the kernel $g_{a-1,b}(x,y)$, $g_{a,b}(x,y)$ and $g_{a,b-1}(x,y)$ at the center of the detector pixel $x = r-0.5, y = c-0.5$.
4. Obtain the component of the signal energy for SLM pixel $(a-1,b-1)$ as $\sqrt{d(r,c)} - q$ where, $$q = g_{a,b}\sqrt{s(a,b)} - g_{a-1,b}\sqrt{s(a-1,b)} - g_{a,b-1}\sqrt{s(a,b-1)}$$
5. Compute $\left(\dfrac{\sqrt{d(r,c)} - q}{g_{a-1,b-1}(r-0.5,c-0.5)} \right)^2$.
6. If $\left(\dfrac{\sqrt{d(r,c)} - q}{g_{a-1,b-1}(r-0.5,c-0.5)} \right)^2 > 0.5$, decode $s(a-1,b-1) = 1$. Else decode $s(a-1,b-1) = 0$.
7. $c \leftarrow c-1$. Loop back to Step 2 till $c = 1$.
8. $r \leftarrow r-1$. Loop back to Step 2 till $r = -m+2$

It is interesting to note that the decoding process is simple and the algorithm has no extra storage overheads. The time complexity of the algorithm is linear in the number of pixels decoded. We use threshold detection at Step 6 to circumvent round off errors and for handling detector noise.

To test the performance of the algorithm, several pages of 100x100 pixel arrays were modeled to mimic the detector output with 3 degrees rotational misalignment. White Gaussian noise was added to the detector output. The noise variance was varied to obtain the different SNRs. Table I shows the average bit error rate (BER) versus signal to noise ratio (SNR). It is interesting to note that the decoding algorithm performs well with high SNR and is fairly robust with SNRs around 30-40dB intended for practical scenarios.

Table I: Bit Error rate versus Signal to Noise ratio

SNR (dB)	BER
∞	0
60	0.001
20	0.25
10	0.4034

4. CONCLUSIONS

We formulated the problem of rotational pixel misalignment leading to non-uniform inter pixel interference in optical imaging systems. The misalignment induced inter-pixel interference can be interpreted as reduction of SNR in the optical domain proportional to the amount of overlap between the SLM and detector. We proposed an algorithm for signal recovery in the presence of noise. The algorithm is robust for high SNRs and is applicable for practical scenarios.

5. REFERENCES

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