JOINT DETECTION/ESTIMATION OF MULTIPATH EFFECTS FOR THE GLOBAL POSITIONING SYSTEM

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ABSTRACT

Multipaths cause major impairments to navigation with global positioning systems (GPS). Indeed, Non-Line-of-Sight (NLOS) propagation is well known to bias GPS position estimates. A recent methodology has been proposed to overcome this limitation by estimating simultaneously the kinematic states and the multipath biases all along the observation interval. However, multipaths clearly occur relatively infrequently during time intervals of fixed duration. This paper studies a particle filtering algorithm for joint detection and estimation of multipath biases. A Rao Blackwellized approach allows to estimate the kinematic states by extended Kalman filters whereas multipath detection is achieved by an appropriate fixed lag particle filter.

1. INTRODUCTION

The Global Positioning System (GPS) utilizes the concept of Time of Arrival (TAO) ranging to determine a vehicle position. Any user is equipped with a receiver that estimates the propagation delays of signals broadcast by GPS satellites. Four measurements (i.e. four distances between the vehicle and the GPS satellites) are necessary at each time instant to solve the navigation problem. Indeed, there are four unknowns for this problem: the position in 3 dimensions and the receiver clock offset with respect to GPS reference time. However, the performance of GPS is severely degraded in a multipath environment. Because of NLOS situations, the GPS signal is reflected several times before arriving to the receiver. A wrong timing information is measured and therefore the positioning solution is biased.

A new Kalman filter approach has been recently studied to track simultaneously the kinematics of the vehicle and the biases induced by multipaths [1]. The proposed algorithm consisted of augmenting the state vector with biases associated to the satellites subjected to multipath effects. An improved accuracy has been observed with this method for estimating the vehicle position. However, this approach does not allow to detect the presence of multipaths. Consequently, it is reliable in known multipath situations. This paper studies a new particle filtering algorithm that performs multipath detection and bias estimation. This algorithm allows to estimate the position of a vehicle in any multipath environment.

Section 2 formulates the GPS problem in the presence of multipaths. Section 3 studies the particle filter algorithm for joint detection and estimation of multipath effects. The proposed approach is based on an augmented state vector which contains multipath biases (as in [1]) but also indicators which reveal the presence or absence of multipaths. This augmented state vector is then estimated by a Rao-Blackwellized fixed lag particle filter. Section 4 is devoted to simulation results which illustrate the performance of the proposed algorithm. Conclusions and perspectives are reported in section 5.

2. MULTIPATH MODEL

2.1. Effects of Multipath on the GPS Measurements

GPS is based on Code Division Multiple Access (CDMA) techniques. The emitted signal is spread by pseudo-random sequences whose good autocorrelation properties allow to estimate the propagation delay. The maximum of the autocorrelation is obtained when the signal and its replica are in phase. As a consequence, the transmitted signal and the receiver local replica have to be synchronized to determine the propagation delay. Note that the interferences between the different satellites are weak since the transmission is based on nearly orthogonal spread spectrum sequences. In the presence of multipaths, the correlation peak is distorted yielding a biased estimation of the satellite to receiver range. The measurement errors are limited by the width of the correlation peak (typically 100m in civil applications). Several measurements can be affected at the same time depending on the relative geometry of the receiver and the satellites. The problem addressed in this paper is twofold: 1) detecting the presence of multipaths on all received GPS measurements, 2) estimating the biases induced by these detected multipaths. The state space model used to solve this problem is detailed in the next section.

2.2. State Space Model

We propose to formulate the multipath detection problem as an abrupt change detection problem. The state vector is classically composed of the unknown kinematic parameters (e.g. the position and its derivatives) in usual navigation applications. This paper shows that augmenting the dimension of the state vector allows joint detection/estimation of multipaths. We define for any vector $\boldsymbol{r}$ and p, q ($p < q$), $\boldsymbol{r}_p^q = (\boldsymbol{r}_p, \ldots, \boldsymbol{r}_q)$. The following GPS state space model is then considered:

$$\begin{aligned} \boldsymbol{X}_t &= F_t \boldsymbol{X}_{t-1} + B_t \boldsymbol{w}_t, && (1)\\ \boldsymbol{\lambda_t} &\sim p(\boldsymbol{\lambda_t}), && (2)\\ \boldsymbol{Y}_t &= h_t(\boldsymbol{X}_t, \boldsymbol{\lambda}_0^t) + D_t \boldsymbol{v}_t, && (3) \end{aligned}$$

where $(\boldsymbol{w}_t)_{t\geq 0}$ and $(\boldsymbol{v}_t)_{t\geq 0}$ are white noise sequences ($\boldsymbol{w}_t, \boldsymbol{v}_t \sim \mathcal{N}(\mathbf{0}, \mathrm{I})$, I being the identity matrix). Denote as n_s the number of visible satellites (or equivalently the number of available measurements at time t). $\boldsymbol{X}_t = (\boldsymbol{x}_t, \boldsymbol{b}_t, \boldsymbol{m}_t)^T$ ($\in \mathbb{R}^{n_s+8}$) is a *state vector* whose components are detailed below:

- $\boldsymbol{x}_t$ stands for the vehicle position (3 coordinates) and velocity (3 coordinates) in the Earth Centered Earth Fixed (ECEF) system of coordinates,
- $\boldsymbol{b}_t$ is composed of the GPS receiver clock offset and its derivative (2 components),
- $\boldsymbol{m}_t \in \mathbb{R}^{n_s}$ is the stacked vector of the multipath biases associated to the GPS measurements.

The dynamic behavior of the vehicle is conveniently represented by a position-velocity model where the acceleration is a white noise sequence of standard deviation σ_a. The multipath measurement biases are defined as random-walks monitored by a white noise of standard deviation σ_m. The drift characteristics of the GPS clock are described by the Allan variance parameters σ_b^2 and σ_d^2 [2]. Due to the independence between $\boldsymbol{x}_t$,$\boldsymbol{b}_t$ and $\boldsymbol{m}_t$, F_t and B_t are block-diagonal matrices such that:

$$F_t = \begin{pmatrix} A_t & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & C_t & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathrm{I} \end{pmatrix}, \quad B_t B_t^T = \begin{pmatrix} Q_t & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Sigma_t & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \sigma_m^2 \mathrm{I} \end{pmatrix}.$$

The diagonal blocks of F_t and $B_t B_t^T$ are recalled below:

$$C_t = \begin{pmatrix} 1 & \Delta t \\ 0 & 1 \end{pmatrix}, \quad A_t = \begin{pmatrix} C_t & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & C_t & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & C_t \end{pmatrix},$$

$$E_t = \begin{pmatrix} \sigma_a^2 \frac{\Delta t^3}{3} & \sigma_a^2 \frac{\Delta t^2}{2} \\ \sigma_a^2 \frac{\Delta t^2}{2} & \sigma_a^2 \Delta t \end{pmatrix}, \quad Q_t = \begin{pmatrix} E_t & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & E_t & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & E_t \end{pmatrix},$$

$$\Sigma_t = \begin{pmatrix} \sigma_b^2 \Delta t + \sigma_d^2 \frac{\Delta t^3}{3} & \sigma_d^2 \frac{\Delta t^2}{2} \\ \sigma_d^2 \frac{\Delta t^2}{2} & \sigma_d^2 \Delta t \end{pmatrix},$$

where Δt is the sampling period.

The *indicator vector* $\boldsymbol{\lambda}_t = (\boldsymbol{\lambda}_{t,1}, \ldots, \boldsymbol{\lambda}_{t,n_s})$ monitors the transitions induced by multipath on each measurement. More precisely, $\boldsymbol{\lambda}_{t,i} = 1$ indicates the presence of a change at time t for the ith measurement and $\boldsymbol{\lambda}_{t,i} = 0$ otherwise (there is a change when a multipath appears or disappears at time t). The indicator variable $\boldsymbol{\lambda}_{t,i}$ is classically assumed to be distributed according to a Bernoulli distribution:

$$P[\boldsymbol{\lambda}_{t,i} = 1] = \beta \text{ and } P[\boldsymbol{\lambda}_{t,i} = 0] = 1 - \beta. \tag{4}$$

The variables $\boldsymbol{\lambda}_{t,1}, \ldots, \boldsymbol{\lambda}_{t,n_s}$ are assumed to be independent.

The *observation vector* is formed of the GPS measurements which are the geometric ranges from the receiver to the satellites. These measures are corrupted by different sources of errors including the GPS receiver clock offset and the multipath biases:

$$\boldsymbol{Y}_{t,i} = \sqrt{\|\boldsymbol{x}_t - \boldsymbol{x}_{s,i}\|^2} + b_t + \boldsymbol{\varepsilon}_{t,i} \boldsymbol{m}_{t,i} + D_t \boldsymbol{v}_t,$$

where $\boldsymbol{x}_{s,i}$ denotes the position of the ith satellite in the ECEF frame, b_t is the receiver clock offset and $\boldsymbol{\varepsilon}_{t,i} = \oplus_{k=1}^{t} \boldsymbol{\lambda}_{k,i}$ is defined as follows ($\oplus$ is the exclusive or):

$$\begin{aligned} \boldsymbol{\varepsilon}_{t,i} &= 1 \text{ in the presence of a multipath bias,} \\ \boldsymbol{\varepsilon}_{t,i} &= 0 \text{ otherwise.} \end{aligned}$$

It is interesting to note that each GPS measurement is associated with a multipath bias, so that the dimension of the observation vector $\boldsymbol{Y}_t$ and the dimension of the bias vector $\boldsymbol{m}_t$ are both equal to the number of visible satellites n_s.

3. DETECTION/ESTIMATION METHOD

3.1. Sequential Monte Carlo Methods

Sequential Monte Carlo methods (SMC) provide a convenient framework to joint estimation and detection of multipath biases. These methods consist of simulating samples distributed according to *posterior* probability density functions (pdfs) such as $p(\boldsymbol{X}_0^t, \boldsymbol{\lambda}_0^t | \boldsymbol{Y}_1^t)$. The simulated samples can then be used to compute various quantities such as the minimum mean square error estimates of the states: $\mathrm{E}(\boldsymbol{X}_t | \boldsymbol{Y}_1^t)$ and $\mathrm{E}(\boldsymbol{\lambda}_t | \boldsymbol{Y}_1^t)$. A Rao-Blackwellized approach can be applied advantageously to make the most of the model structure. Indeed, given $\boldsymbol{\lambda}_0^t$, $p(\boldsymbol{X}_0^t | \boldsymbol{Y}_1^t, \boldsymbol{\lambda}_0^t)$ can be classically approximated by a Gaussian distribution whose parameters can be computed by an Extended Kalman Filter (EKF). The Rao-Blackwellized Particle Filter (RBPF) marginalizes $\boldsymbol{\lambda}_t$, so that the indicator states only are estimated by a particle filtering method. The variance of the estimates is then significantly reduced.

Particle filtering is based on a sequential importance sampling resampling combination. Importance sampling provides a solution to obtain a set of weighted samples, called particles, that represent the *posterior* pdf. The particles $(\boldsymbol{\lambda}_0^t)^i$, $i = 1, \ldots, N$, are generated from an arbitrary proposal distribution $q(\boldsymbol{\lambda}_0^t | \boldsymbol{Y}_1^t)$. They are then assigned importance weights to correct for the discrepancy between p and q. The weights are classically defined as:

$$w_t((\boldsymbol{\lambda}_0^t)^i) = \frac{p((\boldsymbol{\lambda}_0^t)^i | \boldsymbol{Y}_1^t)}{q((\boldsymbol{\lambda}_0^t)^i | \boldsymbol{Y}_1^t)}.$$

However, the variance of the importance weights is bound to increase until all but one particle have negligible weights. A resampling step is then introduced so that only the particles with high importance weights are propagated. The reader is invited to consult [3] for more details.

3.2. The Fixed Lag RBPF

GPS measurements are affected by multipaths during an unknown time interval. As a result, the detection performance should be significantly improved by considering near future observations to generate the particles $(\boldsymbol{\lambda}_0^t)^i$. In order to achieve this, we propose to use the standard proposal distribution $p(\boldsymbol{\lambda}_t | \boldsymbol{\lambda}_0^{t-1}, \boldsymbol{Y}_1^{t+\Delta})$. It is important to note that the indicator vector $\boldsymbol{\lambda}_t$ takes values in a finite set S of cardinal $|S| = 2^{n_s}$. Consequently, the future state space can be fully explored to compute analytically the proposal distribution:

$$p(\boldsymbol{\lambda}_t | \boldsymbol{\lambda}_0^{t-1}, \boldsymbol{Y}_1^{t+\Delta}) = \sum_{\boldsymbol{\lambda}_{t+1}^{t+\Delta}} p(\boldsymbol{\lambda}_t, \boldsymbol{\lambda}_{t+1}^{t+\Delta} | \boldsymbol{\lambda}_0^{t-1}, \boldsymbol{Y}_1^{t+\Delta}).$$

However, such a procedure turns out to be highly computationally demanding since the sum covers $|S|^{\Delta} = 2^{n_s \times \Delta}$ terms. Different solutions can be used to reduce this computational cost. Wang *et als.* [4] proposed to explore only the most likely future paths by means of pilots in the delayed pilot sampling algorithm. This paper proposes a simpler procedure which takes advantage of the sparseness of multipath events. This procedure assumes that at every time instant t an absence of transition between time $t+1$ and time $t+\Delta$ is far more likely than all other possibilities. The proposal distribution can then be approximated as follows:

$$P[\boldsymbol{\lambda}_t = s_j | \boldsymbol{\lambda}_0^{t-1}, \boldsymbol{Y}_1^{t+\Delta}] \simeq P[\boldsymbol{\lambda}_t = s_j, \boldsymbol{\lambda}_{t+1}^{t+\Delta} = \mathbf{0} | \boldsymbol{\lambda}_0^{t-1}, \boldsymbol{Y}_1^{t+\Delta}],$$

where $(s_j)_{j=1,\ldots,|S|} \in S$ is a possible value of the indicator vector $\boldsymbol{\lambda}_t$. The proposed algorithm consists of generating particles according to the approximated discrete distribution $P[\boldsymbol{\lambda}_t = s_j | \boldsymbol{\lambda}_0^{t-1}, \boldsymbol{Y}_1^{t+\Delta}]$ and computing the corresponding importance weights.

3.3. Algorithm description

This section details the different steps of the fixed lag RBPF at a given time instant t. The importance sampling is performed as follows:
For each particle $(\boldsymbol{\lambda}_t)^i$, $i = 1, \ldots, N$:

- For each element $(s_j) \in S$,
  set $(\boldsymbol{\lambda}_0^t)_j^i = \left(\left(\boldsymbol{\lambda}_0^{t-1}\right)^i, (\boldsymbol{\lambda}_t)^i = s_j, \left(\boldsymbol{\lambda}_{t+1}^{t+\Delta}\right)^i = \mathbf{0} \right)$.
- Run $|S| \times (\Delta + 1)$ steps of the extended Kalman filter ($k = 0, \ldots, \Delta$ and $j = 1, \ldots, |S|$), which yield the *a posteriori* estimates $\widehat{\boldsymbol{X}}_{t+k,j}^i$ and the associated error covariance matrices $P_{t+k,j}^i$ such that:

$$p(\boldsymbol{x}_{t+k} | (\boldsymbol{\lambda}_0^{t+k})_j^i, \boldsymbol{Y}_1^{t+k}) \sim \mathcal{N}(\widehat{\boldsymbol{X}}_{t+k,j}^i, P_{t+k,j}^i),$$

  The *innovation* pdfs $p(\boldsymbol{Y}_{t+k} | (\boldsymbol{\lambda}_0^{t+k})_j^i, \boldsymbol{Y}_1^{t+k-1})$ are also computed.
- Generate the new particle according to the proposal distribution: $q((\boldsymbol{\lambda}_t)^i | (\boldsymbol{\lambda}_0^{t-1})^i, \boldsymbol{Y}_1^{t+\Delta}) = \sum_{j=1}^{|S|} \gamma_j^i \delta((\boldsymbol{\lambda}_t)^i - s_j)$, where δ denotes the Dirac distribution,

$$\begin{aligned} \gamma_j^i &= \tilde{\gamma}_j^i \left(\sum_{k=1}^{|S|} \tilde{\gamma}_k^i \right)^{-1} \simeq P[(\boldsymbol{\lambda})_t^i = s_j | \boldsymbol{\lambda}_0^{t-1}, \boldsymbol{Y}_1^{t+\Delta}], \\ \tilde{\gamma}_j^i &= \prod_{k=0}^{\Delta} p(\boldsymbol{Y}_{t+k} | (\boldsymbol{\lambda}_0^{t+k})_j^i, \boldsymbol{Y}_1^{t+k-1}) p((\boldsymbol{\lambda}_{t+k})_j^i). \end{aligned}$$

  and $p((\boldsymbol{\lambda}_{t+k})_j^i)$ is obtained from the Bernouilli priors defined in 4. The innovation pdfs $p(\boldsymbol{Y}_{t+k} | (\boldsymbol{\lambda}_0^{t+k})_j^i, \boldsymbol{Y}_1^{t+k-1})$ are evaluated from the Kalman runs.
- Compute the importance weights. For $(\boldsymbol{\lambda}_t)^i = s_j$, the weights can be updated as follows:

$$w_t((\boldsymbol{\lambda}_0^t)^i) \propto w_{t-1}((\boldsymbol{\lambda}_0^{t-1})^i) \frac{p(\boldsymbol{Y}_t | (\boldsymbol{\lambda}_0^t)_j^i, \boldsymbol{Y}_1^{t-1}) p((\boldsymbol{\lambda}_t)_j^i)}{\gamma_j^i}.$$

A classical approximation of the filtering distribution $p((\boldsymbol{\lambda}_0^t)^i | \boldsymbol{Y}_1^t)$ can be obtained by the random measure $(w_t((\boldsymbol{\lambda}_0^t)^i), (\boldsymbol{\lambda}_0^t)^i))$. However, the algorithm can be improved by introducing a discount factor α [5]. This factor allows newer measurements to affect the state estimates more than older measurements. The discounted weights are then computed as follows:

$$w_t((\boldsymbol{\lambda}_0^t)^i) \propto [w_{t-1}((\boldsymbol{\lambda}_0^{t-1})^i)]^\alpha \frac{p(\boldsymbol{Y}_t | (\boldsymbol{\lambda}_0^t)_j^i, \boldsymbol{Y}_1^{t-1}) p((\boldsymbol{\lambda}_t)_j^i)}{\gamma_j^i},$$

with $0 < \alpha < 1$. The lower α, the less effect the past has on the future. This parameter has been adjusted from several experimental results.

3.4. Indicator and State estimates

Indicators and states are estimated using the *maximum a posteriori* and the *minimum mean square error* rules, respectively. These estimates could be computed from the random measure $(w_t((\boldsymbol{\lambda}_0^t)^i), (\boldsymbol{\lambda}_0^t)^i))$. However the estimation performance is improved by computing smoothed estimates:

$$\begin{aligned} \widehat{\boldsymbol{\lambda}_t} &= \underset{(\boldsymbol{\lambda}_t)^i}{\text{argmax}} \left\{ \tilde{w}_t((\boldsymbol{\lambda}_0^t)^i) \right\}_{i=1:N}, \\ \widehat{\boldsymbol{X}_t} &= \sum_{i=1}^{N} \tilde{w}_t((\boldsymbol{\lambda}_0^t)^i) \widehat{\boldsymbol{X}}_t^i, \end{aligned}$$

where $\widehat{\boldsymbol{X}}_t^i = \widehat{\boldsymbol{X}}_{t,j}^i$ for $(\boldsymbol{\lambda}_t)^i = s_j$. The smoothing importance weights $\tilde{w}_t((\boldsymbol{\lambda}_0^t)^i)$ are obtained as:

$$\tilde{w}_t((\boldsymbol{\lambda}_0^t)^i) \propto w_{t-1}((\boldsymbol{\lambda}_0^{t-1})^i) \sum_{k=1}^{|S|} \tilde{\gamma}_k^i.$$

4. SIMULATION RESULTS

Many simulations have been conducted to validate the previous theoretical results. This paper considers a classical navigation scenario. The vehicle trajectory is simulated according to the position-velocity model with an approximate speed of 10km/h. Satellite orbits are generated from real data to compute the vehicle to satellite rangings. These distances are then corrupted by the simulated GPS receiver clock offset and the GPS noise (the standard deviation is approximately 10 meters) to yield GPS measurements. Finally, multipath biases are added randomly on the different measurements all along the trajectory. It is important to note that, at each time instant, the number of measures affected by multipath ensures the overall system observability: 7 GPS measurements are still in sight during the simulation and only 3 of them can be affected simultaneously by multipath. This hypothesis makes sense since the satellites affected by NLOS propagation have low elevation angles. The change probability (appearance or disappearance of a multipath) is set to $\beta = 0,03$. The different simulation parameters are summarized below:

Parameters	N	α	σ_a	σ_m
Values	500	0.5	0.1 m/s^2	0.1 m

The multipath biases are reinitialized to zero each time a change has been detected. Equivalently, the kth component of $\widehat{\boldsymbol{m}}_t^i((\boldsymbol{\lambda}_0^t)^i)$ is set to 0 when the kth component of the ith particle $\boldsymbol{\lambda}_t^i$ equals 1 (we recall here that $\widehat{\boldsymbol{X}}_t^i((\boldsymbol{\lambda}_0^t)^i) = (\widehat{\boldsymbol{x}}_t^i((\boldsymbol{\lambda}_0^t)^i), \widehat{\boldsymbol{b}}_t^i((\boldsymbol{\lambda}_0^t)^i), \widehat{\boldsymbol{m}}_t^i((\boldsymbol{\lambda}_0^t)^i)^T)$. The covariance matrix $P_t^i((\boldsymbol{\lambda}_0^t)^i)$ has also to be reinitialized as follows:

$$\mathrm{E}\left((\boldsymbol{m}_t - \widehat{\boldsymbol{m}}_t)(\boldsymbol{m}_t - \widehat{\boldsymbol{m}}_t)^T \right) = \begin{cases} \sigma_j^2 & \text{if multipath,} \\ 0 & \text{otherwise.} \end{cases}$$

The parameter σ_j allows to adjust the change amplitudes ($\sigma_j = 10$ meters in this implementation).

The following figures illustrate the performance of the algorithm. Figure 1 shows typical detection probabilities $P[\boldsymbol{\lambda}_t = 1 | \boldsymbol{Y}_1^{t+\Delta}]$ obtained in absence of smoothing (i.e. for $\Delta = 0$). The same probabilities are plotted in figure 2 for a fixed lag $\Delta = 5$.

The change instants are represented by vertical bars on these figures. Some changes cannot be detected in the absence of smoothing ($\Delta = 0$). The results are clearly better when the algorithm uses a fixed lag ($\Delta = 5$). Note, however, that a false detection still occurs around the iteration 65. These results emphasize the interest of using the fixed lag approach.

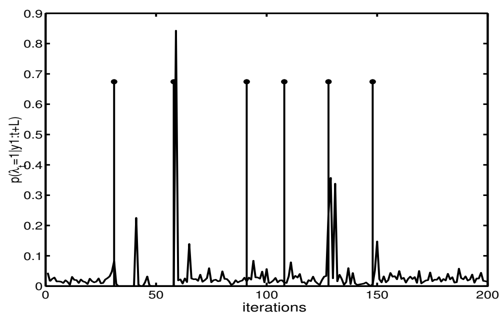

Fig. 1. Detection Probability for $\Delta = 0$

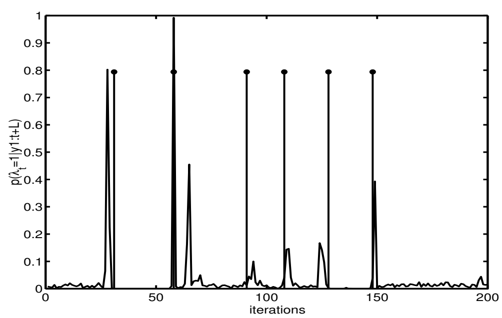

Fig. 2. Detection Probability for $\Delta = 5$

The figures 4 and 4 show the estimated multipath biases. These biases are well estimated when the fixed lag smoothing procedure is used, except at time 65, where a false detection has been previously noticed.

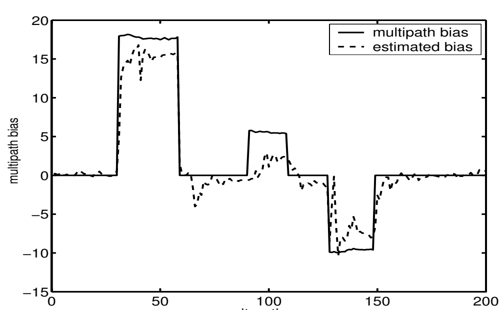

Fig. 3. Bias Estimation in Absence of Smoothing ($\Delta = 0$).

The last simulation compares the results obtained with a standard extended Kalman filter that takes into account multipath biases all along the simulation. The average positioning mean square errors obtained with the extended Kalman filter and the smoothed RBPF are depicted on figure 5. This figure shows that the particle filter allows to improve the estimation performance.

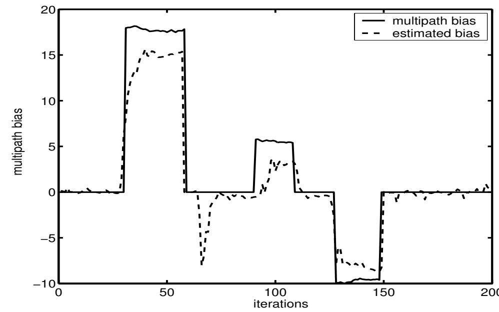

Fig. 4. Bias Estimation for a Fixed Lag ($\Delta = 5$).

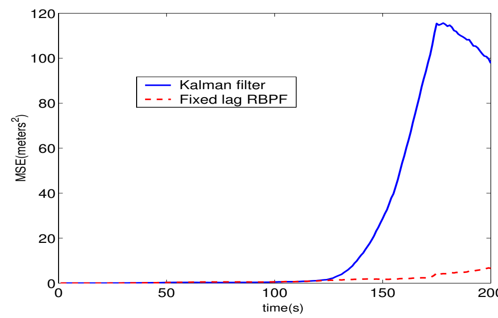

Fig. 5. Positioning Estimation Error.

5. CONCLUSION

This paper studied a Rao Blackwellized Particle Filter for joint detection and estimation of multipath biases. The proposed algorithm allowed to detect the presence or absence of multipaths contrary to existing algorithms. The impact of the algorithm on the position estimation accuracy should be further investigated. The challenging problem of detecting interferences affecting GPS measurements should also be addressed by using a similar approach.

6. ACKNOWLEDGMENT

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