AUXILIARY PARTICLE FILTERING FOR DETECTION IN ARCH NOISE

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ABSTRACT

In recent years there has been great interest in particle filtering as a sequential Bayesian estimation method for dynamic systems. In this paper, the problem of detection in flat fading channels in the presence of Autoregressive Conditional Heteroscedasticity (ARCH) noise is addressed. The Auxiliary particle filtering (APF) algorithm is presented for the estimation of transmitted sequence and ARCH noise variance. We introduce the ARCH process for the purpose of noise modelling. The applicability of the model is based on a variance change over time compatible to a large extent with non-stationarity and non-Gaussianity of wireless fading channels. We show through simulation the superiority of the proposed algorithm over the bootstrap particle filter (PF) even in the presence of the new noise model; i.e., the ARCH process.

1. INTRODUCTION

The ever-increasing movement towards the world of wireless communication encourages us to find more efficient and robust detection algorithms at the receiver. The next generation wireless technology is going to operate at higher bit rates, and in such circumstances the multipath fading is a serious problem specially for indoor communication. This fact and the need for high bit rates makes us utilize more robust and efficient algorithms which also give us the real time processing capability. Optimal or Bayesian filtering [1] has found a variety of applications for nonlinear and non Gaussian problems. Bayesian filtering addresses the problem of estimating recursively in time the posteriori probability of a hidden state space variable from an a priori probability distribution function. To achieve this goal a set of simulation-based methods known as Sequential Monte Carlo SMC methods are used [1, 3]. Particle filtering as a SMC method approximates the posteriori distribution of a desired state variable by a set of particles with associated probability weights allowing estimation of the posteriori distribution recursively in time. The particle filtering algorithm is an attractive tool for estimating the posteriori distribution given increased computational powers in today's technology. Auxiliary particle filtering [2] attempts to reduce the variability of the importance weights at time $t-1$ by performing particle filtering in a higher dimension. In other words by changing the sampling mechanism of PF a more efficient algorithm is obtained [2]. Non-Gaussian impulsive noise has attracted considerable attention from researchers in various fields of applied signal processing due to their close fit to a variety of underlying physical processes. Such is the case in communication channels for additive ambient noise and noises being generated from different natural and man made sources and their propagation effects to the receiver. Besides, in wireless channels factors such as the birth and death of users, electromagnetic interferences, co-channel and co-site interference may vary dynamically in time and space; i.e., they are non-stationary and non-Gaussian characteristics. Therefore it is a more realistic assumption to consider the $volatility$ nature of the received signal; i.e., we assume time-varying variance for additive noise. In the last decade, after the seminal work by Engle [7], there has been a growing interest in time series models with time varying variance or volatility. These models have found a great application in non-stationary and non-Gaussian financial time series analysis. Autoregressive Conditional Heteroscedasticity (ARCH) [7] is a time series modelling technique which introduces heteroscedasticity. A series is said to be heteroscedastic if its variance changes over time. ARCH models account for two main characteristics, excess kurtosis (heavy tail probability distribution) and volatility clustering (large changes tend to follow large changes and small changes tend to follow small ones). Therefore, ARCH process can successfully model non-stationarity and non Gaussianity of wireless channel noise [4]. In addition, it is clear that the probability distribution function of ARCH noise exhibits heavier tails than those of the standard normal distribution compatible to a large extent with impulsive noise which is a commonly used model for additive noise in wireless channels. In this paper as an application to improve, and make more suitable APF algorithm in non-Gaussian, and non stationary environment; i.e., a realistic scenario for a wireless communication channel, we consider the problem of detection in ARCH noise whose non constant variance is unknown . This paper is organized as follows: Section 2 is devoted to model presentation and mathematical formulation of the fading channel along with non-Gaussian ARCH noise. In section 3 an overview of particle filtering is presented first and then auxiliary particle filtering algorithm is proposed. The new detection in flat fading channel in the presence of ARCH(1) noise is discussed in section 4. Section 5 contains simulation results in presence of ARCH(1) noise. In section 6 concluding remarks are provided.

2. SYSTEM MODEL AND PROBLEM FORMULATION

In this section we present the model for the wireless fading channel and ARCH noise. Given the information from the channel denoted by $\mathcal{F}_{t-1}$ we model the additive noise of the channel by ε_t, where the dependence of this noise and the information from the channel, i.e., $\mathcal{F}_{t-1}$, is modelled by a Gaussian random variable. Let ε_t be a real valued discrete time stochastic process and let $\mathcal{F}_{t-1}$ be the set of all information available through the observed data up to time $t-1$. Then ARCH(p) is given by [7]:

$$\varepsilon_t|\mathcal{F}_{t-1} \sim \mathcal{N}(0, \sigma_t^2) \tag{1}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \tag{2}$$

where,

$$\alpha_0 > 0\ ,\ \alpha_i \geq 0\ , \qquad i = 1, ..., p, \quad p > 0 \tag{3}$$

For $p = 0$, ε_t reduces to white noise. In order to show the impulsive nature of the ARCH process, one can obtain the second and fourth moments of the ARCH(1) process as

$$\begin{aligned} \sigma_{\varepsilon_t}^2 &= E(\varepsilon_t^2) = \frac{\alpha_0}{1-\alpha_1} \\ E(\varepsilon_t^4) &= 3\left(\frac{\alpha_0^2}{1-\alpha_1}\right)\left(\frac{1+\alpha_1}{1-3\alpha_1^2}\right) \end{aligned}$$

The unconditional kurtosis of ε_t becomes:

$$\frac{E(\varepsilon_t^4)}{\sigma_{\varepsilon_t}^2} = 3\frac{1-\alpha_1^2}{1-3\alpha_1^2} > 3$$

It can be seen that the tail distribution of ε_t is heavier than that of a normal distribution which shows suitable modelling for the impulsive nature of wireless noise. We now consider a wireless communication channel where d_t denotes multiplicative fading disturbance resulting from the multipath phenomenon, and ε_t as ARCH(1) additive noise. Therefore the observed signal can be shown as

$$y_t = d_t s_t + \varepsilon_t, \tag{4}$$

where s_t and y_t are modulated and observation signals, respectively. Suppose d_t to be Rayleigh distributed; i.e., flat fading channel and modelled by an ARMA(r,r) [6].

$$d_t = \boldsymbol{a}^T d_{t-1:t-p} + \boldsymbol{b}^T \upsilon_{t-1:t-p}$$

where $\upsilon_t \sim \mathcal{N}_c(0,1)$ is a complex white Gaussian noise with identically independently distributed zero-mean real and imaginary parts. Filter parameters $\boldsymbol{a}$ and $\boldsymbol{b}$ both are $(r+1)\times 1$ vectors, and are known for a determined Doppler frequency and symbol rate. In order to represent the problem in state space framework we write the ARCH variance in the form shown below

$$\begin{aligned} \sigma_t^2 &= \alpha_0 + \alpha_1 \sigma_{t-1}^2 u_t \\ \varepsilon_t &= \sigma_t z_t \end{aligned}$$

where $u_t = z_{t-1}^2$, $z_t \sim \mathcal{N}(0,1)$. Now, we can develop the whole problem in a State Space (SS) framework of the following form

$$\begin{aligned} \mathbf{x}_t &= H\mathbf{x}_{t-1} + G\upsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \sigma_{t-1}^2 u_t \\ y_t &= F_t \mathbf{x}_t + \sigma_t z_t \end{aligned}$$

where u_t is independent from z_t and $\mathbf{x}$ is a $(r+1)\times 1$ state vector. The matrix H has dimensions $(r+1)\times(r+1)$ and is constructed from the coefficients of the ARMA process as follows

$$H = \begin{bmatrix} a_1 & a_2 & \cdots & a_r & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

and $F_t = s_t \boldsymbol{b}^T$, $G = [1, 0 \cdots, 0]^T$ with dimensions $(r+1)\times 1$. The model parameters of ARCH(1) process; i.e., α_0 and α_1 are assumed known in our analysis. However, in general cases such as blind detection, they can be estimated by maximum likelihood method. More accurate results can be obtained if they are estimated in particle filtering framework.

3. PARTICLE FILTERING

Consider a dynamic state space model of the following form

$$\begin{aligned} \mathbf{x}_t &= H_t(\mathbf{x}_{t-1}, \upsilon_t) \\ \mathbf{y}_t &= F_t(\mathbf{x}_t, \varepsilon_t) \end{aligned}$$

where $\mathbf{x}_t$ is the latent state variable and unknown to us, υ_t and ε_t are state noise and observation noise respectively. In general $H_t(\mathbf{x}, \upsilon)$ and $F_t(\mathbf{x}, \varepsilon)$ are nonlinear functions of the state and noise inputs.Given observation y_t, particle filters recursively in time approximate the random variable $\mathbf{x}_t|\mathcal{F}_t = (y_1, ..., y_t)'$ by a set of particles $\mathbf{x}_t^{(i)}$, $i = 1, ..., N$, with probability masses $w_t^{(i)}$, $i = 1, ..., N$. In particular, if the particles $\{\mathbf{x}_{0:t-1}^{(i)}, w_{0:t-1}^{(i)}\}$ are distributed approximately according to $p\ (\mathbf{x}_{1:t-1}|y_{1:t-1})$, then one can update the importance weights as [5]

$$w_t^{(i)} = w_{t-1}^{(i)} \frac{p\ (y_t|{x_t}^{(i)}) p\ ({x_t}^{(i)}|x_{t-1}^{(i)})}{q({x_t}^{(i)}|x_{0:t-1}^{(i)}, y_{1:t})} \tag{5}$$

where $q({x_t}^{(i)}|x_{0:t-1}^{(i)}, y_{1:t})$ is known as proposal distribution. If posteriori distribution is used as a proposal distribution $q({x_t}^{(i)}|x_{0:t-1}^{(i)}, y_{1:t}) = p({x_t}^{(i)}|x_{0:t-1}^{(i)}, y_{1:t})$, then importance weights can be updated as follows [5]

$$w_t^{(i)} = w_{t-1}^{(i)} p\ (y_t|x_{0:t-1}^{(i)}, y_{0:t-1}). \tag{6}$$

On the other hand if the prior distribution $q({x_t}^{(i)}|x_{0:t-1}^{(i)}, y_{1:t}) = p({x_t}^{(i)}|x_{t-1}^{(i)})$ is chosen as the proposal distribution, then the weight updating equation becomes

$$w_t^{(i)} = w_{t-1}^{(i)} p\ (y_t|x_{0:t}^{(i)}, y_{0:t-1}) \tag{7}$$

It is clear that the likelihood function $p\ (y_t|x_{0:t}^{(i)}, y_{0:t-1})$ in case of linear state space and Gaussian noise can be computed using Kalman filtering. Once the weights are updated, in order to prevent from degeneracy, a resampling step must be performed. In resampling step particles with low associated weights are discarded and particles with high weights are multiplied. In order to improve the efficiency of particle filtering some new algorithms have been suggested [2]. As mentioned, APF tries to reduce the variability of the importance weights by resampling the particles at time $t-1$ with probability close to $p(x_{t-1}|y_{1:t})$. In other words, in auxiliary particle filtering a mechanism of adaptation is added to standard particle filtering. In practice, running time for the APF algorithm is a little bit more than for the standard particle filtering. That's because we are modifying the weights of the past to reduce the weights' variations. Assume that θ_t is the desired state vector at time t to be estimated by APF algorithm. First we make a prediction $\hat{\theta}_t^{(i)}$ of the past trajectories $\theta_t^{(i)}$ by means of the mode, then resampling according to their associated weights are performed. Finally, the algorithm proceeds as in basic particle filter. Thus, for $t > 1$ the APF algorithm can be written as follows

- Predict the state variable by the mode of $\theta_{t-1}^{(i)}$ to obtain $\hat{\theta}_t^{(i)}$, $i = 1, \cdots, N$
- Compute $p(y_t|y_{1:t-1}, \hat{\theta}_{1:t}^{(i)})$ using Kalman filtering.
- $w_t^{(i)} \propto w_{t-1}^{(i)} p(y_t|y_{1:t-1}, \hat{\theta}_{1:t}^{(i)})$, $\sum_{i=1}^N w_t^{(i)} = 1$, $i = 1, \cdots, N$
- Resample $\theta_{t-1}^{(i)}$ with respect to the probabilities $w_t^{(i)}$
- $\theta_t^{(i)} \propto q(\theta_t^{(i)}|\theta_{0:t-1}^{(i)}, y_{0:t})$
- $w_t^{(i)} \propto \frac{p(y_t|y_{1:t-1}, \theta_{1:t}^{(i)})}{p(y_t|y_{1:t-1}, \hat{\theta}_{1:t}^{(i)})}$
- Normalize the new weights,$\sum_{i=1}^N w_t^{(i)} = 1, i = 1, \cdots, N$

Next, we introduce the new auxiliary particle filtering strategy in presence of a possible non-Gaussian and/or non-stationary interference and noise.

4. AUXILIARY PARTICLE FILTERING IN THE COMMUNICATION CHANNEL

Assume that symbols s_t are transmitted from a Markovian source with transition probability function $p\ (s_t|s_{t-1})$. In the state space model we presented for the problem of detection of which the transmitted symbols s_t, noise variance σ_t^2 and the latent variable $\mathbf{x}_t$ are unknown to us. In order to achieve better estimation of desired parameters in particle filtering it is important to marginalize out undesired parameters. Therefore, marginalizing out the latent variable $\mathbf{x}_t$ to reduce importance weights variance, we can define the state variable to be estimated as $\theta_t = (s_t, \sigma_t^2)$. The importance function from which one has to generate samples of the above mentioned state variable is

$$\begin{aligned} q\ (\theta_t|\theta_{0:t-1}^{(i)}, y_{0:t}) \quad &\propto \quad p(y_t|\theta_t, \theta_{0:t-1}^{(i)}, y_{0:t-1}) \\ &\times \quad p\ (\sigma_t^2|\sigma_{t-1}^2) p\ (s_t|s_{t-1}) \end{aligned} \tag{8}$$

Assuming ARCH(1)and by referring to (2), one can see that the distribution of noise variance $p\ (\sigma_t^2|\sigma_{t-1}^2)$ is related to $\chi^2(1)$ and can be calculated analytically as

$$p\ (\sigma_t^2|\sigma_{t-1}^2) = [2\pi\alpha_1 h_{t-1}(h_t - \alpha_0)]^{-\frac{1}{2}} \exp\left(-\frac{h_t - \alpha_0}{2\alpha_1 h_{t-1}}\right) \tag{9}$$

In addition the transition distribution $p\ (s_t|s_{t-1})$ is known to us and the likelihood function $p(y_t|\theta_t, \theta_{0:t-1}^{(i)}, y_{0:t-1})$ is normal with mean and covariance can be calculated by Kalman filtering. During the algorithm implementation particles are sampled from (8) and their associated weights are calculated from the likelihood function. Once the particles are obtained MAP estimator is used to estimate s_t i.e.,

$$\hat{s_t} = \arg\max p\ (s_t|y_{0:t}) \tag{10}$$

and,

$$p\ (s_t|y_{0:t}) \quad = \quad \sum_{i=1}^{N} w_t^{(i)} \delta(s_t^{(i)}) \tag{11}$$

where $\delta(.)$ is dirac delta function.

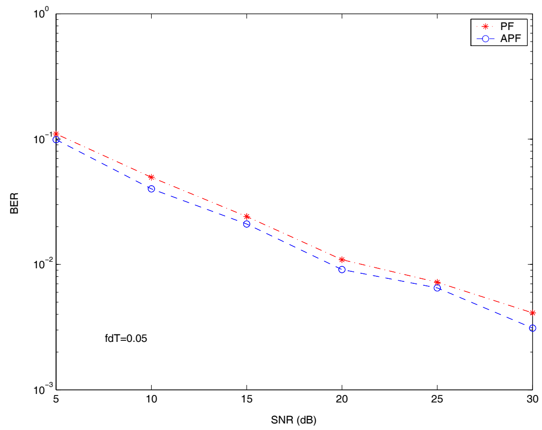

Fig. 1. APF and PF algorithms in additive ARCH(1) noise and $f_d T = .05$ fading characteristic

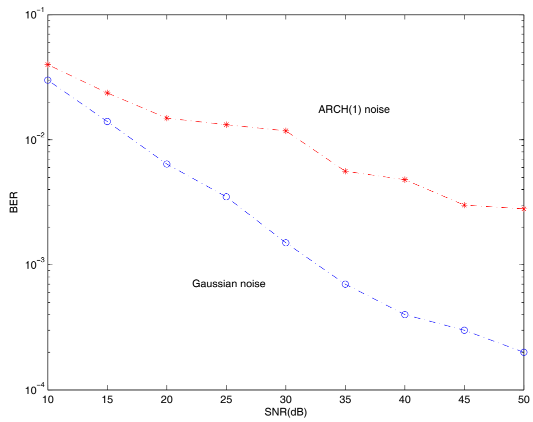

Fig. 2. Performance of APF algorithm in ARCH(1) and Gaussian noise

5. SIMULATION

Bit Error Rate (BER) simulation results are presented in this section. An ARMA(2) model with complex Gaussian noise input is used for simulating fading disturbance. For additive noise ARCH(1) is chosen. The number of particles is $N = 500$. Both PF and APF algorithms are applied to a BPSK modulated signal. Figure (1) illustrates the result for $f_d T = 0.05$ fading characteristic and additive ARCH(1) noise. One can see the effectiveness and the superiority of APF over particle filtering, this is expected because of the fact that in APF the effect of observations is included. In other words by adding a forecast step to PF algorithm we have the possibility of updating weights according to the probabilities obtained from this forecast. Figure (2) shows the performance evaluation of APF in the presence of both additive ARCH(1) noise and Gaussian noise with $f_d T = 0.02$ fading characteristic. We see that the APF algorithm is doing well and is still robust in case of ARCH noise, keeping the BER in a reasonable range. Hence, the introduced auxiliary particle filtering method can be utilized in a more realistic wireless channel environment.

6. CONCLUSION

In this paper, we presented an auxiliary particle filtering scheme for detection in wireless fading channels. ARCH noise with the characteristic of having changing variance over time is introduced as a more realistic assumption for non-Gaussian/non stationary wireless channels in conjunction with the introduced auxiliary particle filtering. The APF algorithm is presented for the introduced model in case of unknown time-varying variance. The proposed algorithm was applied to ARCH(1) noise and its efficiency and superiority over standard particle filter is verified through simulation results.

7. REFERENCES

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